

Curvature Radiation from a quantum-electrodynamics point of view

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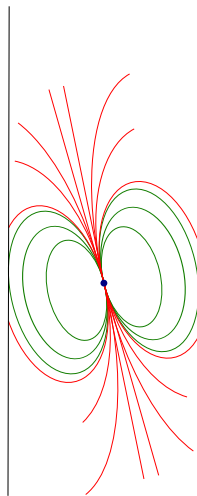
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Collaboration with Silvano Bonazzola and Fabrice Mottez

Part I

High energy radiation mechanisms in pulsar magnetospheres

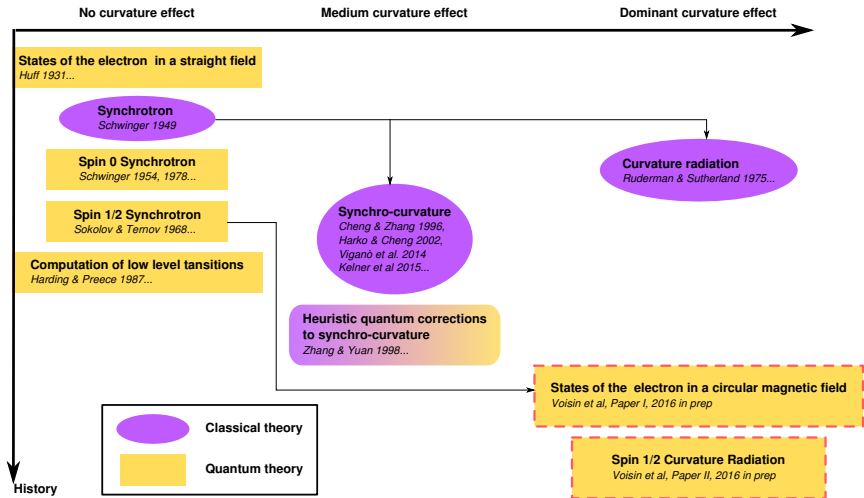
A pair production factory



Radiation processes , a QED zoo

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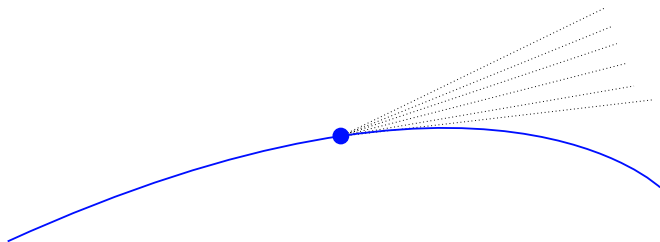
- ▶ Synchrotron
- ▶ Curvature radiation
- ▶ Quantum transitions (LMXBs, Maser?...)
- ▶ Compton and Inverse Compton scatterings
- ▶ $\gamma + \gamma \rightarrow e_+ + e_-$
- ▶ $\gamma + \vec{B} \rightarrow e_+ + e_-$
- ▶ ...



Curvature radiation: classical picture

Proposed by Ruderman and Sutherland (1975).

- ▶ Classical curvature peak : $E_{\max} = \gamma^3 \hbar c / \rho \sim 20 \gamma_7^3 \text{ GeV}$
- ▶ Total power radiated : $W = \frac{e^2 c}{6 \pi \epsilon_0 \rho^2} \gamma^4 \sim 2 \cdot 10^8 \gamma_7^4 \text{ GeV/s}$
- ▶ Emission time scale $\tau_e \sim E_{\max} / W = 10^{-7} \text{ s}$
- ▶ **But unphysical trajectory** : $\vec{v} \parallel \vec{B} \Rightarrow \vec{v} \wedge \vec{B} = \mathbf{0}$!



Parameters : $\gamma \sim 10^7$, $B \sim 10^8 \text{ T}$, $\rho \sim 10^4 \text{ m}$

Synchro-curvature radiation

This regime has been studied by several authors (Cheng and Zhang (1996), Harko and Cheng (2002), Kelner et al. (2015), Viganò et al. (2014)). The last gives :
The maximum Lorentz factor :

$$\gamma_{\max} = \left(\frac{3}{2} \frac{E_{\parallel} \rho}{e} \right)^{1/4} \quad (1)$$

The pitch angle :

$$\sin \alpha = \sin \alpha_0 \exp \left(-\frac{t}{\tau_{\alpha}} \right) \quad (2)$$

With :

$$\tau_{\alpha} = \frac{\gamma_{\max} m_e c}{e E_{\parallel}} \quad (3)$$

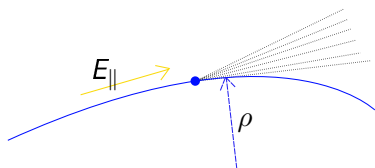


Figure : A particle pushed by an electric field E_{\parallel} , along a magnetic field line of curvature radius ρ .

Synchro-curvature radiation

This regime has been studied by several authors (Cheng and Zhang (1996), Harko and Cheng (2002), Kelner et al. (2015), Viganò et al. (2014)). The last gives :
The maximum Lorentz factor :

$$\gamma_{\max} \simeq 2 \cdot 10^7 E_{\parallel 12}^{1/4} \rho_4^{1/4} \quad (4)$$

The pitch angle :

$$\sin \alpha = \sin \alpha_0 \exp \left(-\frac{t}{\tau_\alpha} \right) \quad (5)$$

With :

$$\tau_\alpha = 2 \cdot 10^{-8} \gamma_{\max 7} E_{\parallel 12}^{-1} \text{ s} \quad (6)$$

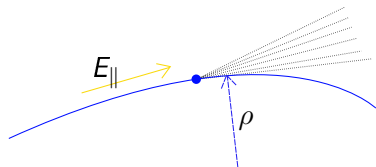


Figure : A particle pushed by an electric field E_{\parallel} , along a magnetic field line of curvature radius ρ .

But... Energy levels become quantized

Energy states of an electron in a straight magnetic field :

$$E = \sqrt{m^2 c^4 + \underbrace{\hbar \omega_c m c^2 n}_{\text{Perpendicular momentum}} + \underbrace{(c p_{\parallel})^2}_{\text{Parallel momentum}}} \quad (7)$$

"Pitch angle" corresponding to the first Landau level :

$$\sin \alpha_1 \sim 2 \cdot 10^{-9} B_8 \gamma_7^{-1} \quad (8)$$

The first Landau state is reached in 10 meters !

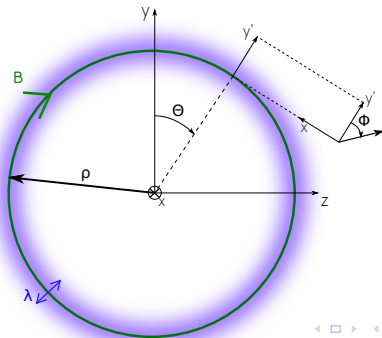
Part II

State of an electron in a circular
magnetic field (Voisin et al., Paper I,
2016, in prep.)

Symmetries of an electron in a circular magnetic field

- ▶ Circular static magnetic field.
- ▶ Locally homogeneous on a length scale :

$$\lambda = \left(\frac{2\hbar}{eB}\right)^{1/2} \simeq 10^{-12} B_8^{-1/2} \text{ m}$$
- ▶ Invariance by rotation around the axis of the circle and the magnetic field.



Equation and Approximation

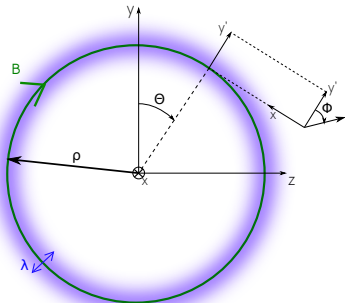
Find proper states of the following set of operators :

$$\hat{J}_x \text{ and } \hat{J}_\theta$$

$$\hat{H}/c = \alpha^r(-i\hbar\partial_r) + \alpha^\theta(-i\hbar\partial_\theta) + \alpha^\phi(-i\hbar\partial_\phi + eA_\phi(r)) + \beta mc \quad (9)$$

We seek solutions at zeroth order
in ϵ :

$$\epsilon = \frac{\lambda}{\rho} \simeq 10^{-16} B_8^{-1/2} \rho_4^{-1} \quad (10)$$



States of an electron in a circular magnetic field :

$$\Psi_{n,l_{\perp},l_{\parallel},a}(r,\theta,\phi) \propto e^{-\frac{r^2}{2\lambda^2}} e^{i\theta l_{\parallel}} e^{i\phi l_{\perp}} \left[L_{n-l_{\perp}}^{l_{\perp}} \left(\frac{r^2}{\lambda^2} \right) \chi_{\uparrow}^a(\theta) + e^{i\phi} L_{n-l_{\perp}}^{l_{\perp}+1} \left(\frac{r^2}{\lambda^2} \right) \chi_{\downarrow}^a(\theta) \right] \quad (11)$$

$$E = \sqrt{m^2 c^4 + \hbar \omega_c m c^2 n + (\hbar \Omega)^2 l_{\parallel}^2} \quad (12)$$

With the pulsation around the circle : $\Omega = c/\rho$

- ▶ n : main perpendicular quantum number.
- ▶ $l_{\perp} + 1/2$: angular momentum around the magnetic field (\hat{J}_{θ}).
- ▶ l_{\parallel} : angular momentum around the axis of the circle (\hat{J}_x).
- ▶ a : spin orientation

Part III

Radiation of an ultra-relativistic electron
in a circular magnetic field (Voisin et al.,
Paper II, 2016, in prep.)

Defining curvature radiation from a quantum point of view

Proposition of definition

Curvature radiation results of transitions between the most localized quantum states that allow all spin orientations.

States concerned

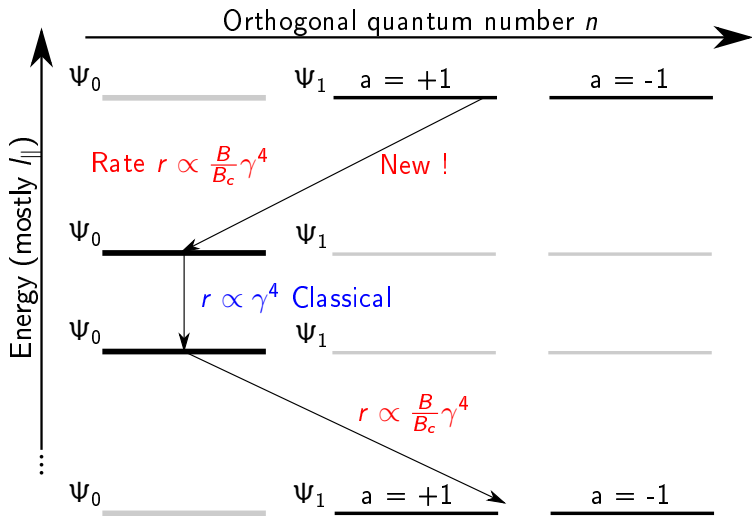
- ▶ Fundamental state : $l_{\parallel}, n = 0, l_{\perp} = 0, a = 1$, spin anti-aligned.

$$\Psi_0 = \Psi_0^{\downarrow} \quad (13)$$

- ▶ First excited state : $l_{\parallel}, n = 1, l_{\perp} = 0, a = \pm 1$, mixed spins.

$$\cos(\xi)\Psi_{1,a=1} + \sin(\xi)\Psi_{1,a=-1} = \underbrace{\Psi_1^{\uparrow}}_{l_{\perp}+1}(\xi) + \underbrace{\Psi_1^{\downarrow}}_{l_{\perp}}(\xi) \quad (14)$$

Allowed transitions and main rates

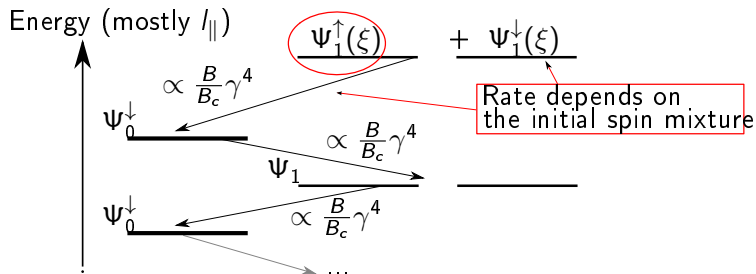


Spin loves flip-flops if $B \geq B_c = 4.4 \cdot 10^9 \text{T}$

The transition rates read :

$$dw_{fi} = \|M_{fi}\|^2 2\pi \hbar \delta(E_f + \hbar\omega - E_i) \frac{\omega^2 d\omega d\Omega}{c^3 (2\pi)^3 / V} \quad (15)$$

$$\|M_{fi}\|^2 \propto \left\| \int \bar{\Psi}_f \gamma^\mu e_\mu \Psi_i e^{-i\vec{k}\cdot\vec{x}} d^3x \right\|^2 \quad (16)$$



Spectrum expressions

Classical curvature radiation is identical to its classical version up to high energy corrections. **Spin-flip expressions are very similar.**

$$\frac{d^2 I_{\downarrow\downarrow}^{\parallel}}{d\omega d\Omega} = \frac{1}{2\pi\Omega} \frac{e^2 \omega^2}{12\pi^3 \epsilon_0 c} \left(\kappa^2 + \frac{1}{\tilde{\gamma}^2} \right)^2 K_{2/3}^2(\xi)$$

$$\frac{d^2 I_{\downarrow\downarrow}^{\perp}}{d\omega d\Omega} = \frac{1}{2\pi\Omega} \frac{e^2 \omega^2}{12\pi^3 \epsilon_0 c} \kappa^2 \left(\kappa^2 + \frac{1}{\tilde{\gamma}^2} \right) K_{1/3}^2(\xi)$$

$$\frac{d^2 I_{\uparrow\downarrow}^{\parallel}}{d\omega d\Omega} = \frac{1}{2\pi\Omega} \frac{e^2 \omega^2}{12\pi^3 \epsilon_0 c} \left(\kappa^2 + \frac{1}{\tilde{\gamma}^2} \right) \frac{1}{2\tilde{\gamma}} \frac{B}{B_c} K_{1/3}^2(\xi)$$

$$\frac{d^2 I_{\uparrow\downarrow}^{\perp}}{d\omega d\Omega} = \frac{1}{2\pi\Omega} \frac{e^2 \omega^2}{12\pi^3 \epsilon_0 c} \left(\kappa^2 + \frac{1}{\tilde{\gamma}^2} \right) \frac{1}{2\tilde{\gamma}} \frac{B}{B_c} \left(K_{1/3}^2(\xi) + \kappa^2 \left(\kappa^2 + \frac{1}{\tilde{\gamma}^2} \right) \right)$$

Polarizations and symbols above are within the conventions of Jackson (1998)

Total radiated power

Spin-flip curvature radiation is mostly unpolarized :

$$I_{\downarrow\downarrow}^{\parallel} = \frac{1}{\epsilon_0 c} \left(\frac{\Omega e}{16\pi} \right)^2 \frac{28}{3} \tilde{\gamma}^4 \quad (17)$$

$$I_{\downarrow\downarrow}^{\perp} = \frac{1}{\epsilon_0 c} \left(\frac{\Omega e}{16\pi} \right)^2 \frac{4}{3} \tilde{\gamma}^4 \quad (18)$$

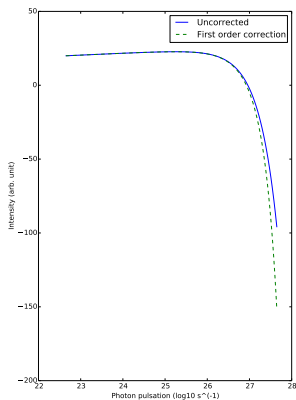
$$I_{\uparrow\downarrow}^{\parallel} = \frac{1}{\epsilon_0 c} \left(\frac{\Omega e}{16\pi} \right)^2 \frac{16}{3} \frac{B}{B_c} \tilde{\gamma}^4 \quad (19)$$

$$I_{\uparrow\downarrow}^{\perp} = \frac{1}{\epsilon_0 c} \left(\frac{\Omega e}{16\pi} \right)^2 \frac{B}{B_c} \left(\frac{16}{3} \tilde{\gamma}^4 + \frac{14}{3} \tilde{\gamma}^2 \right) \quad (20)$$

Polarizations and symbols above are within the conventions of Jackson (1998)

Corrections at high energies

$$\hbar\omega \rightarrow \hbar\omega \left(1 + \frac{\hbar\omega}{E} \right) \quad (21)$$



Conclusion

- ▶ We prove that **curvature radiation can be derived in a fully consistent way** using quantum electrodynamics in a high-magnetic-field, ultra-relativistic regime.
- ▶ We show that **spin-flip transitions amount to 10% in "normal" 10^8 Teslas pulsars and dominate for magnetic fields $\gtrsim B_c = 4.4 \cdot 10^9$ Teslas. : spin-flip curvature radiation should not be neglected in young pulsars (Crab-like), and becomes dominant in magnetars.**
- ▶ For both we worked out **high energy quantum corrections**.
- ▶ **The spectra are very similar in shape** to the usual curvature/ synchrotron but differ in intensity (for spin-flip).
- ▶ Contrary to constant-spin (classical) radiation, **spin-flip curvature radiation is mostly unpolarized**.
- ▶ In the longer term, **"quantum synchro-curvature radiation"** should also be investigated.

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