Equation of State for Astrophysical Applications

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Meudon, France

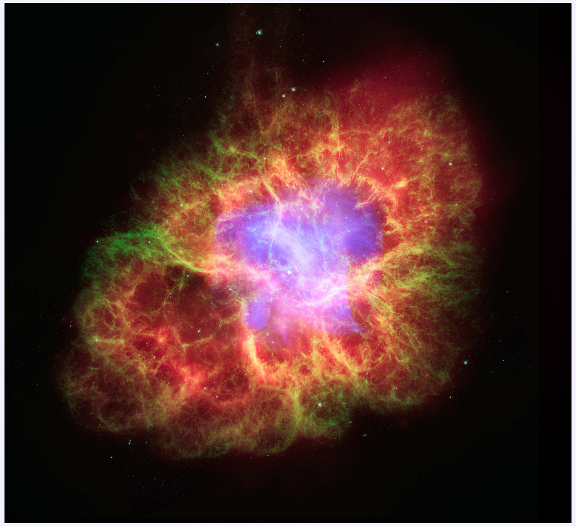
Outline

- **Introduction**
  Astrophysics and EoS, Theoretical Approaches, EoS for Astrophysical Applications, Correlations, Constraints
- **Generalized Relativistic Density Functional**
  Details of gRDF Model, Properties of Nuclei, Nuclear Matter Parameters, Symmetry Energy, Stellar Matter, Low-Density Limit, Mass Shifts, Constraint from Heavy-Ion Collisions, EoS Table, Neutron Star Matter, Low-Temperature Limit, Hyperon Puzzle, Optical Potential Constraint
- **Conclusions**

Introduction

Astrophysics and Equation of State

- essential ingredient in astrophysical model calculations:
  Equation(s) of State of dense matter
  ⇒ dynamical evolution of core-collapse supernovae, neutron star mergers
  ⇒ static properties of neutron stars
  ⇒ conditions for nucleosynthesis
  ⇒ energetics, chemical composition, transport properties, . . .



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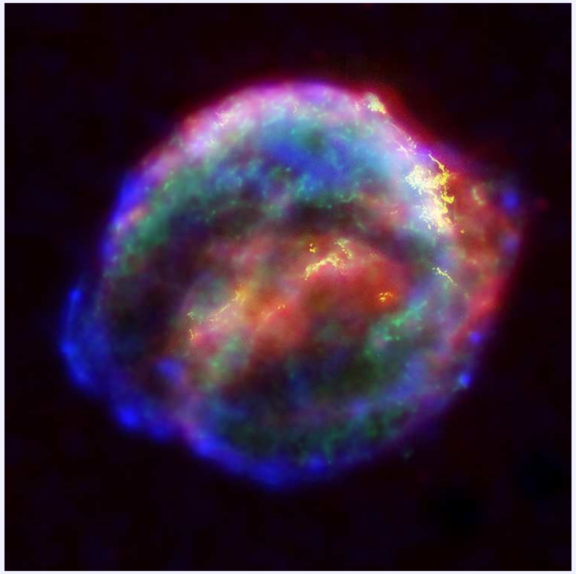

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  ⇒ equilibrium (thermal, chemical, . . .)
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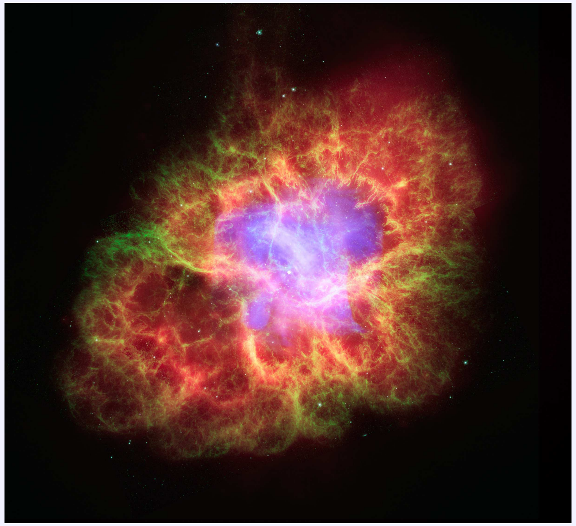

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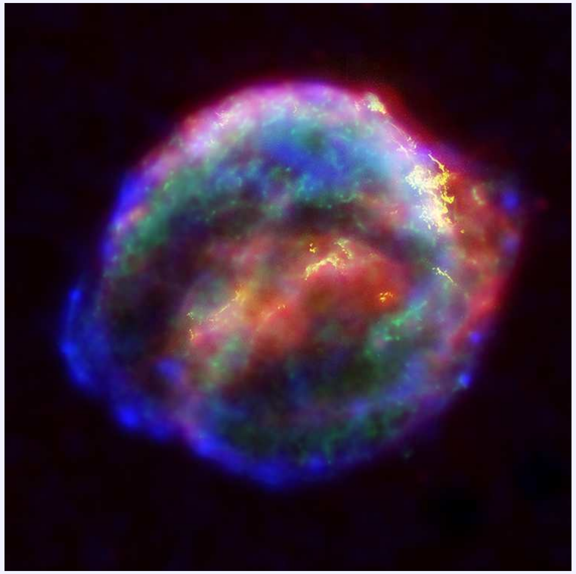

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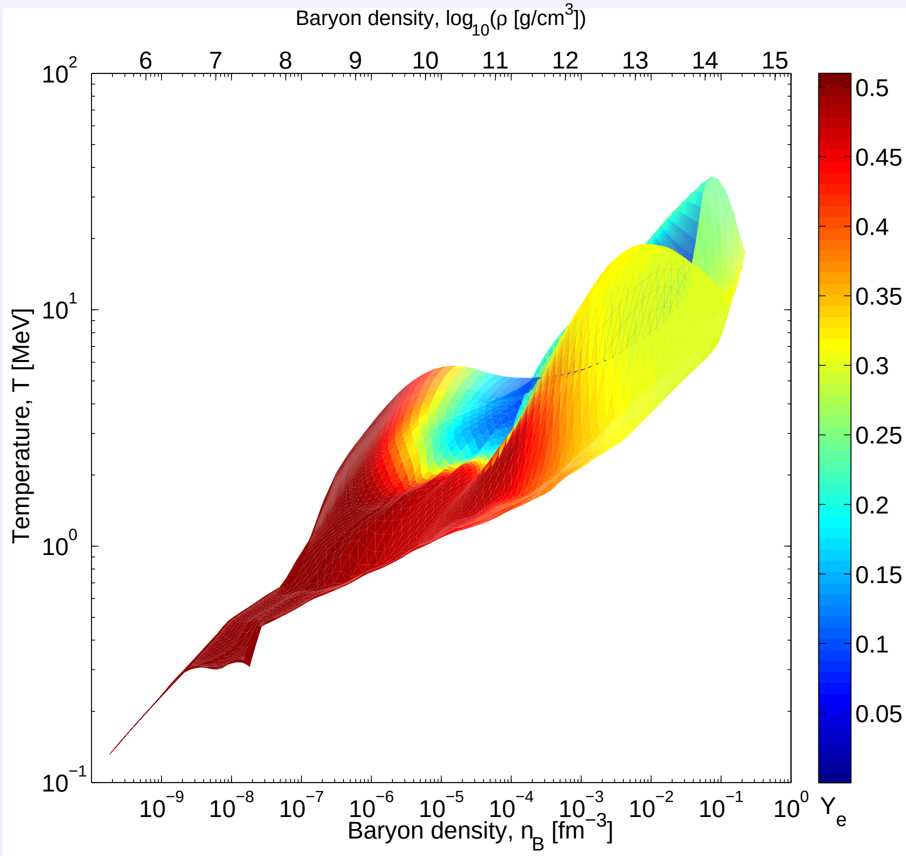
- **wide range of thermodynamic variables**
  (temperature, density, isospin asymmetry)
  ⇒ **global, multi-purpose EoS** required

simulation of core-collapse supernova



T. Fischer, Uniwersytet Wrocławski

Theoretical Approaches

- **hadronic 'ab-initio' methods with realistic interactions**
  - interactions: potential models, meson-exchange, chiral forces, RG evolved (Argonne, Urbana, Tucson-Melbourne, Nijmegen, Paris, Bonn, . . .)
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only **phenomenological models** for global EoS at present

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  - HS (TM1,TMA,FSUgold,NL3,DD2,IUFSU) M. Hempel, J. Schaffner-Bielich,
    NPA 837 (2010) 210
  - SHT (NL3) G. Shen, C.J. Horowitz, S. Teige,
    PRC 82 (2010) 015806, 045802, PRC 83 (2011) 035802
  - SHO (FSU1.7, FSU2.1) G. Shen, C.J. Horowitz, E. O'Connor, PRC 83 (2011) 065808
  - SFHo/SFHx A.W. Steiner, M. Hempel, T. Fischer, ApJ 774 (2013) 17
  - recently many more, also with additional degrees of freedom (hyperons, quarks)

Correlations

- **low densities** (much below nuclear saturation density)
  two-, three-, . . . many-body correlations due to short-range NN interaction
  ◦ modification of thermodynamic properties
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- **at low temperatures**
  inhomogeneous matter
  - **nuclear matter:** "liquid-gas" phase transition
    (no Coulomb interaction, no electrons, no charge neutrality)
  - **stellar matter**: formation of lattice structures, clustering, "pasta phases"
    (charge neutrality, interplay of surface effects, long-range Coulomb interaction, entropy)

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- from **laboratory experiments**
  - properties of nuclei:
    masses, charge/diffraction radii, surface properties, giant resonances, . . .
  - characteristic nuclear matter parameters (mostly indirect):
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- from **astronomical observations**
  - properties of neutron stars:
    masses, radii, rotation, cooling, . . .
  - core-collapse supernovae:
    explosion dynamics, neutrino signal, nucleosynthesis, . . .

Generalized Relativistic Density Functional

Generalized Relativistic Density Functional I

- extension of relativistic mean-field (RMF) models
  - basic constituents: nucleons (n,p), mesons (ω, σ, ρ), photons (γ), hyperons (optional)
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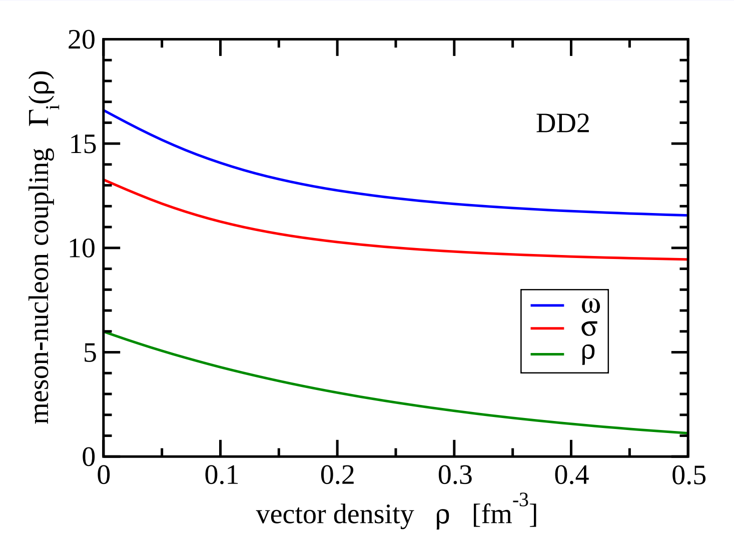

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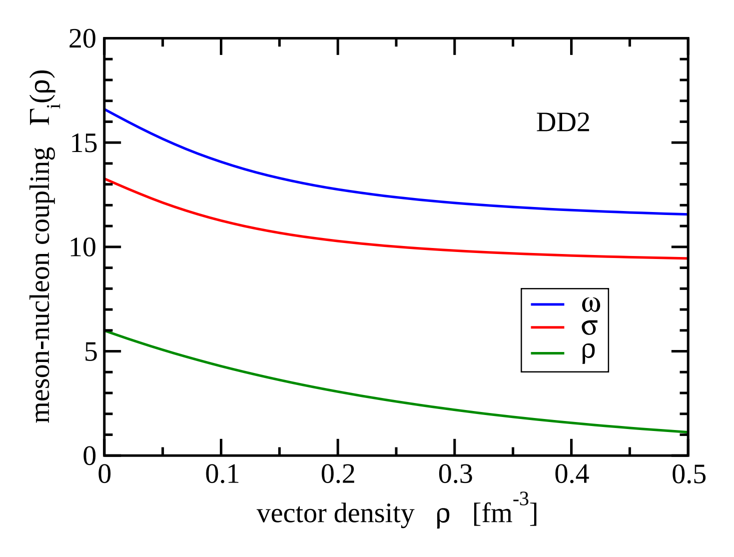

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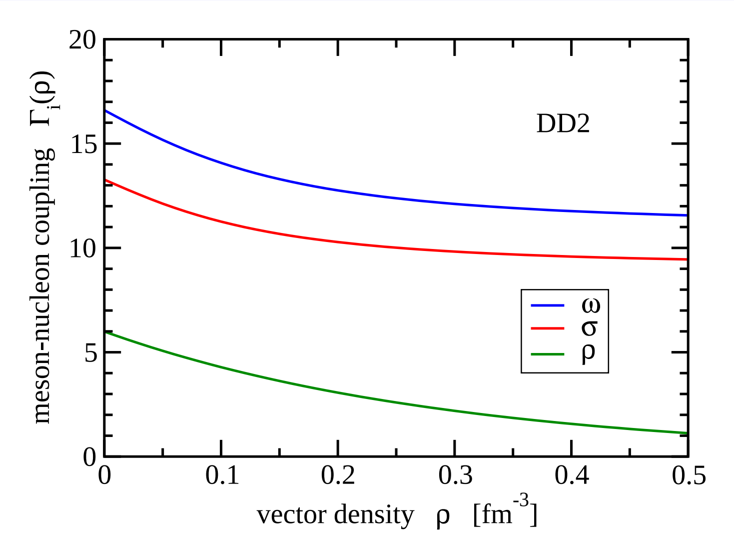

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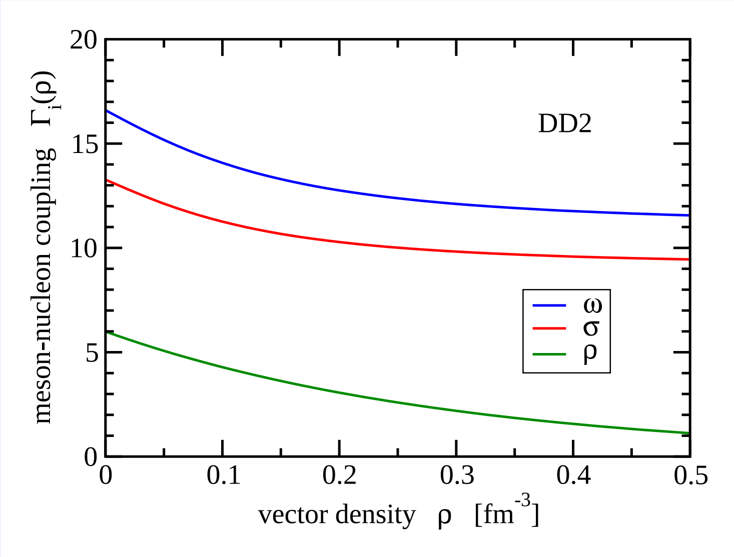

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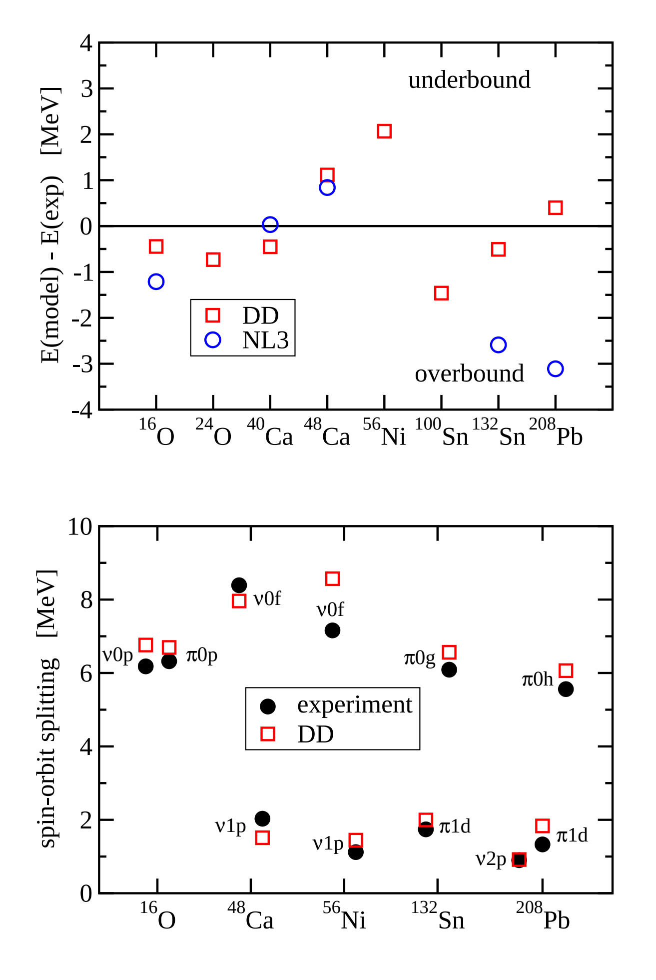
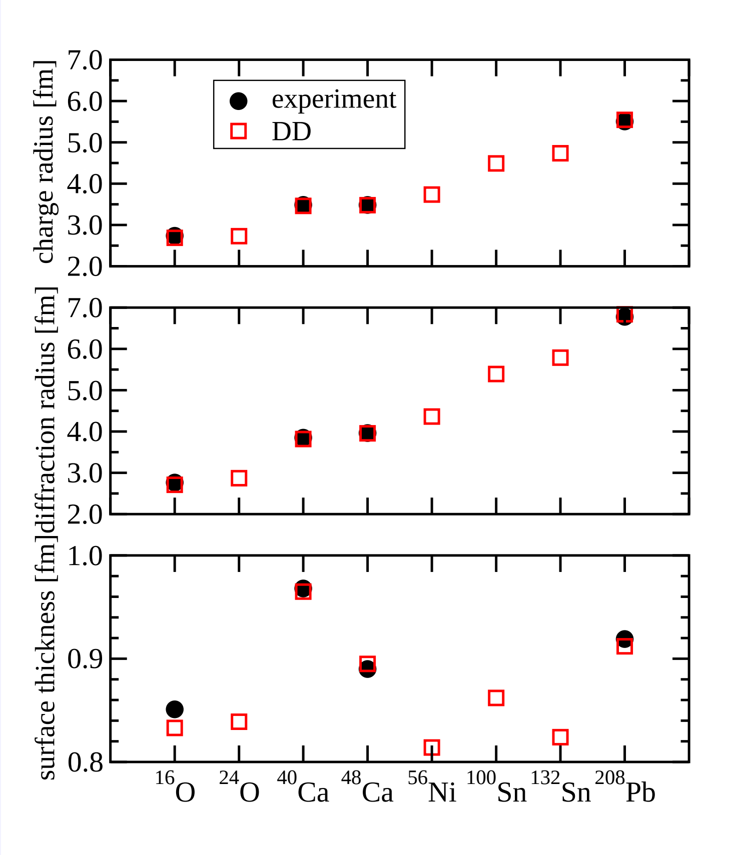
- phenomenological approach with 9 parameters
  - determined from fit to properties of finite nuclei

Properties of Nuclei

- used in the fit of the RMF parameters:
  - binding energies, spin-orbit splittings
  - properties of charge form factor (charge radius, diffraction radius, surface thickness)





DD: S. Typel, Phys. Rev. C 71 (2005) 064301

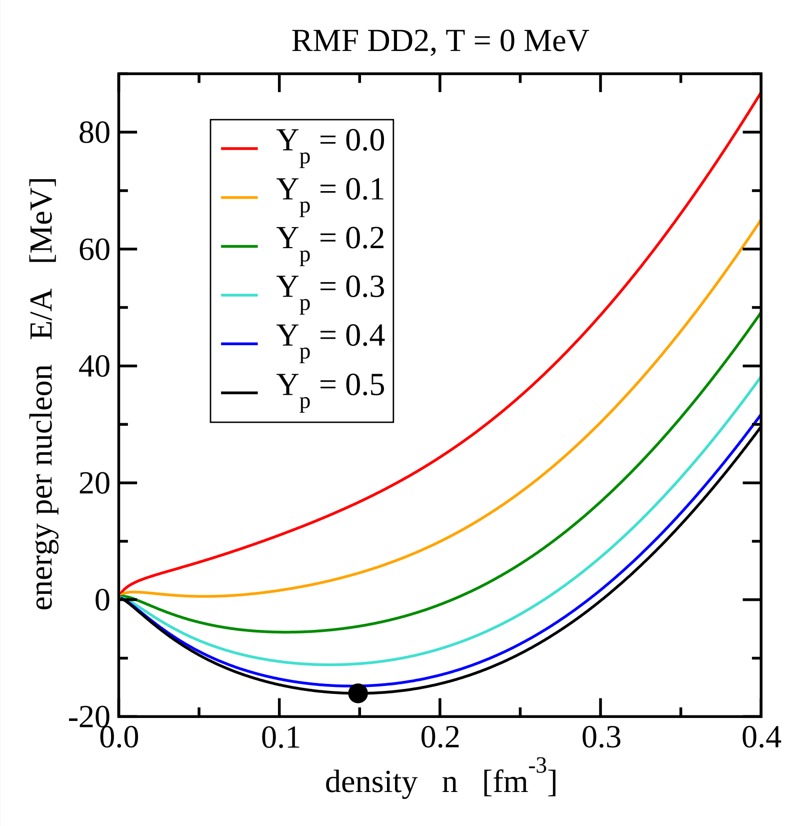
NL3: G. A. Lalazissis, J. König, P. Ring, Phys. Rev. C 55 (1997) 540

Nuclear Matter Parameters

- **energy per nucleon** near saturation:

$$\frac{E}{A}(n,\beta) = \frac{\varepsilon}{n} - m = -B_{\text{sat}} + \frac{K}{18}x^2 - \frac{K'}{162}x^3 + \beta^2\left(J + \frac{L}{3}x + \ldots\right) + \ldots$$

with $x = (n - n_{\text{sat}})/n_{\text{sat}}$, asymmetry $\beta = 1 - 2Y_p$ and



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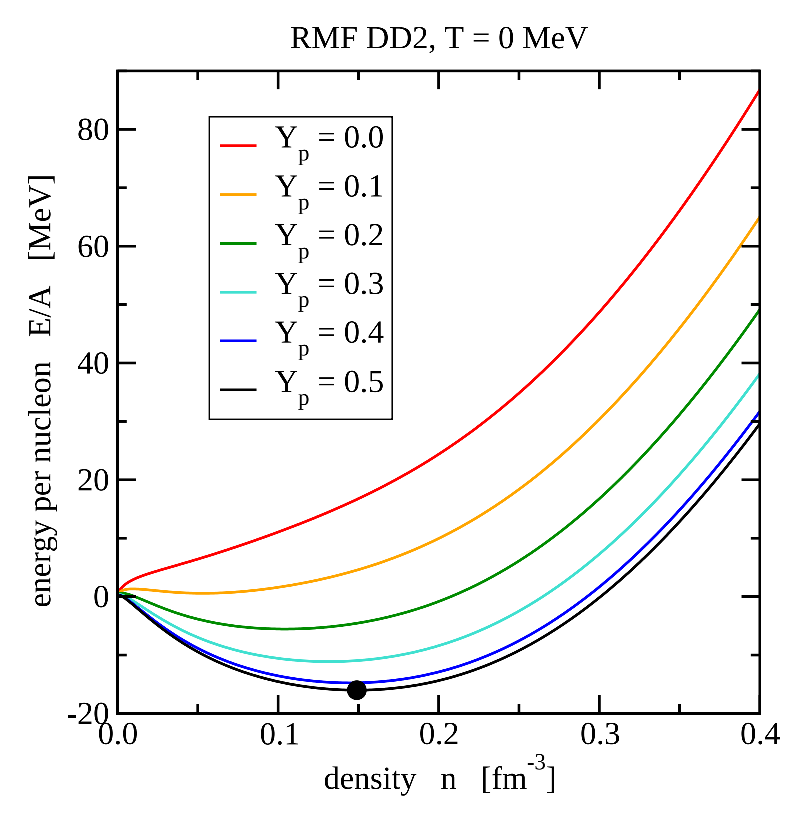
- **nuclear matter parameters**
  - n_{sat} saturation density
  - B_{sat} bulk binding energy
  - K incompressibility
  - K' skewness
  - J bulk symmetry energy
  - L slope of symmetry energy



DD2: Typel et al., Phys. Rev. C 81 (2010) 015803,

refit of DD with experimental nucleon masses

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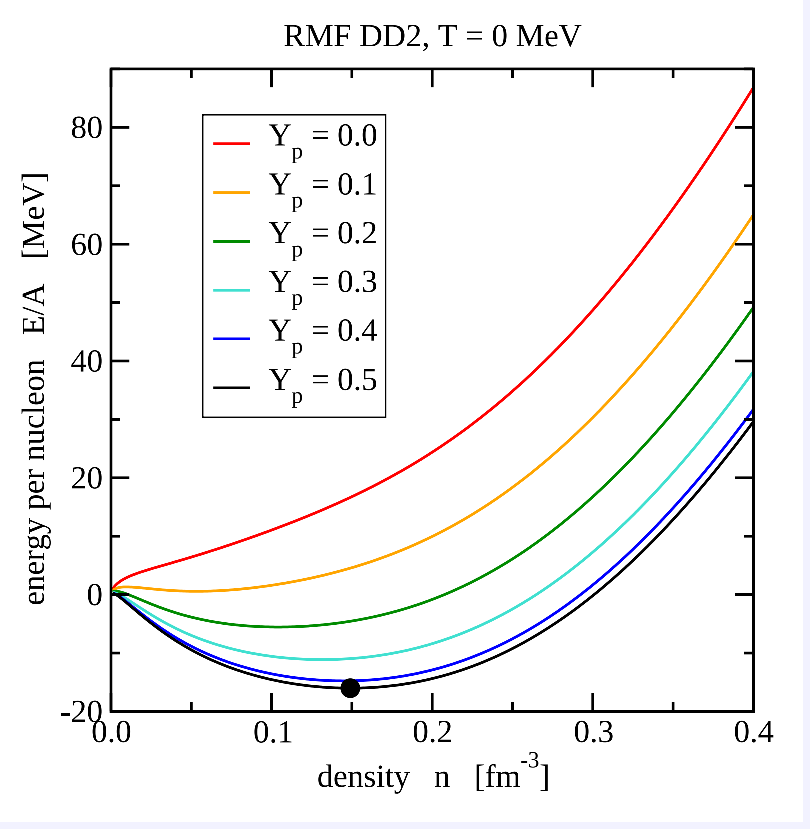
- **nuclear matter parameters**
  - $n_{\rm sat} \Rightarrow$ size of nuclei
  - $B_{\rm sat}$, $J = a_S \Rightarrow$
    general trend of binding energies
    cf. Bethe-Weizsäcker mass formula
  - K, $K' \Rightarrow$ giant resonances,
    ratio surface tension/surface thickness
  - $L \Rightarrow$ neutron skin thickness



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Constraints on Symmetry Energy

density dependence of symmetry energy $E_s(n)$ in nuclear matter

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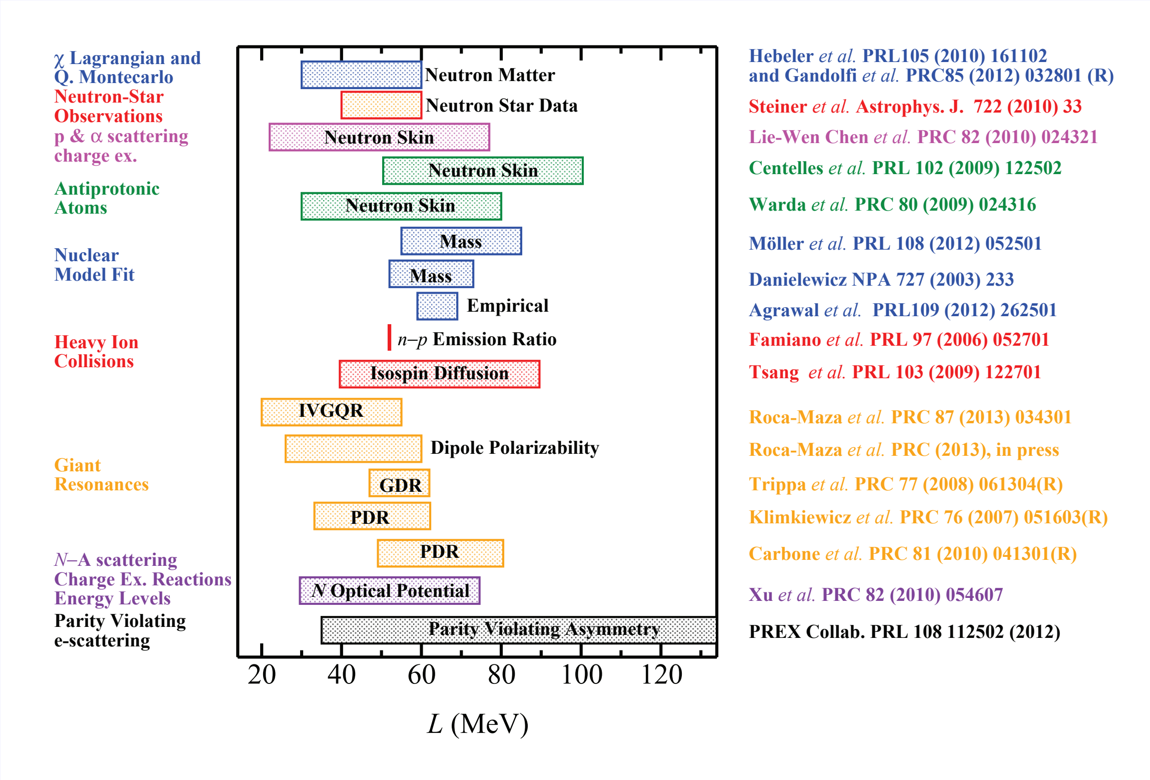
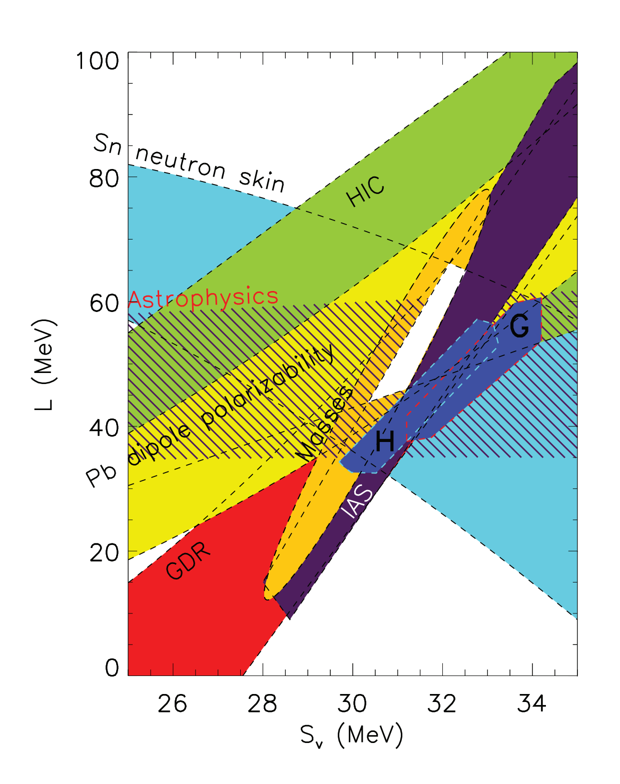
- many efforts to determine $J = S_v$ and L experimentally



(X. Viñas et al., EPJA 50 (2014) 27)



(J.M. Lattimer, A.W. Steiner, EPJA 50 (2014) 40)

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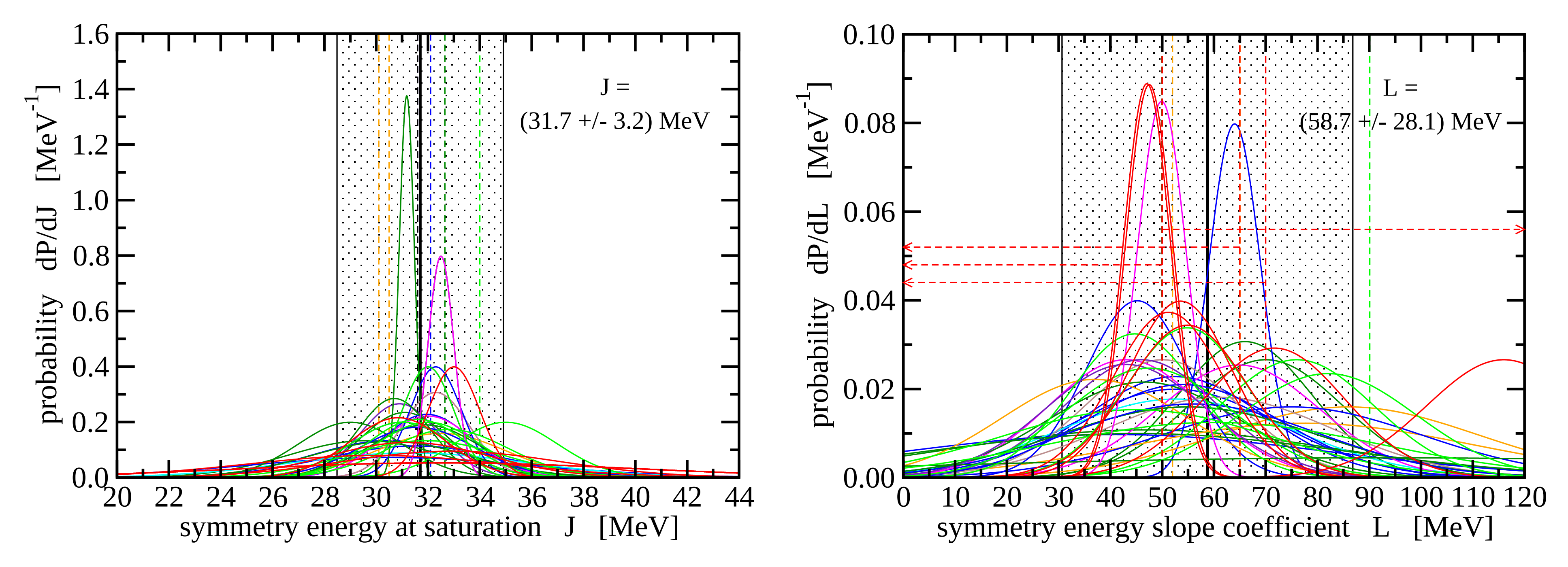


(M. Oertel, M. Hempel, T. Klähn, S. Typel, in preparation)

Symmetry Energy and Neutron Skins of Nuclei

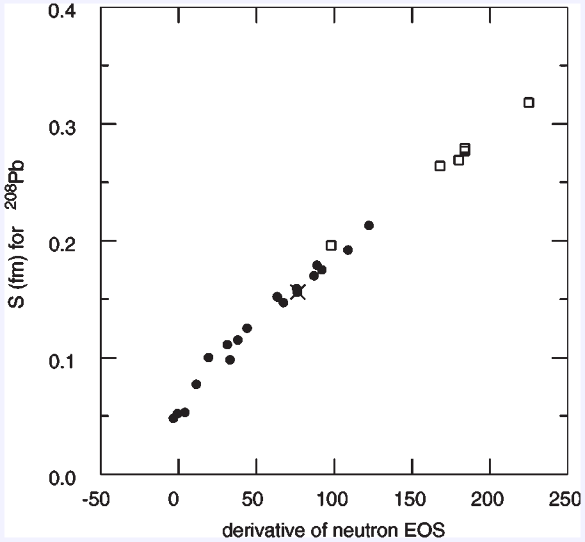
- **correlation**: neutron skin thickness

$$\Delta r_{np} = S = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

$\Leftrightarrow$ derivative of neutron matter EoS

B.A. Brown, PRL 85 (2000) 5296

S. Typel and B. A. Brown, PRC 64 (2001) 027302



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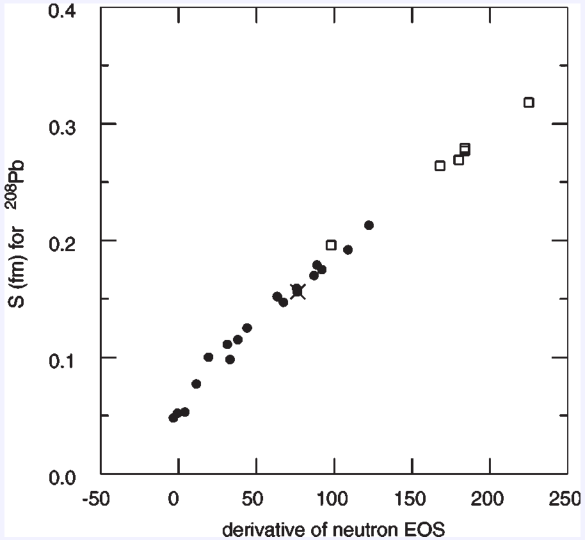

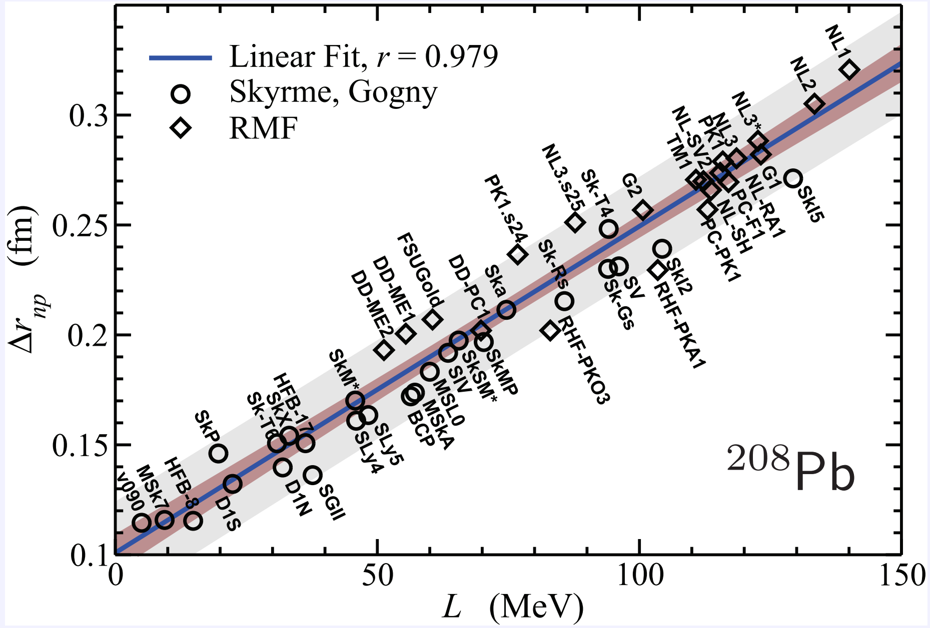

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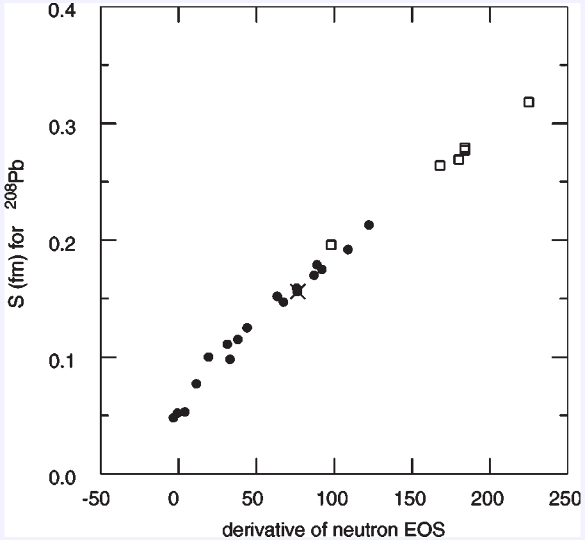
  $\Leftrightarrow$ symmetry energy slope parameter L

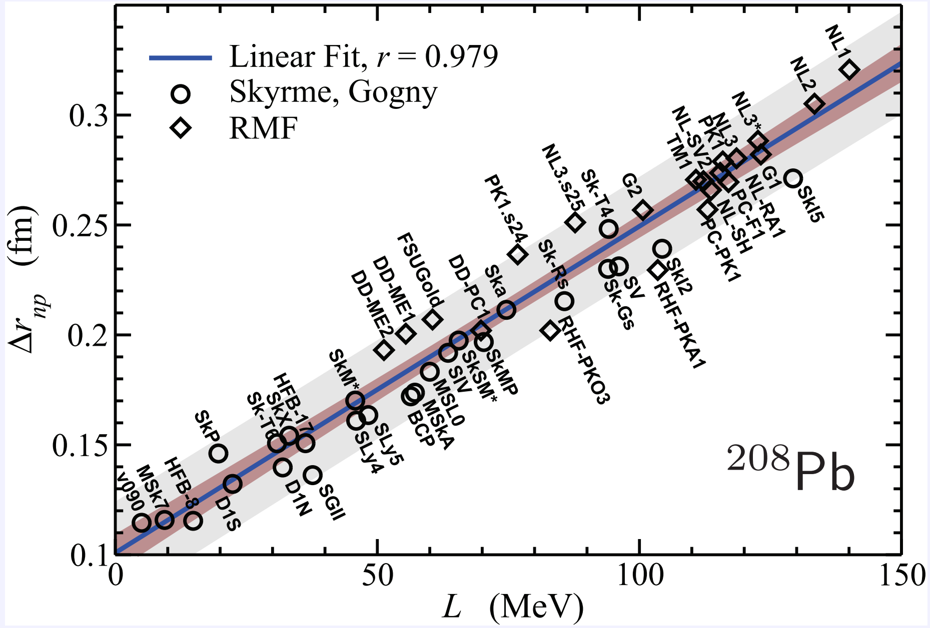
- determine L from experimental measurement of Δr_{np}
  - ◦ parity violation in electron scattering

    PREX@Jefferson Lab, C.J. Horowitz et al.,

    PRC 63 (2001) 025501, PRC 85 (2012) 032501
  - ◦ coherent pion photoproduction

    MAMI@Mainz, C. Tarbert et al., PRL 112 (2014) 242502





(X. Viñas et al., Eur. Phys. J. A50 (2014) 27)

Symmetry Energy and Neutron Skins of Nuclei

- **correlation**: neutron skin thickness

$$\Delta r_{np} = S = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

$\Leftrightarrow$ derivative of neutron matter EoS

B.A. Brown, PRL 85 (2000) 5296

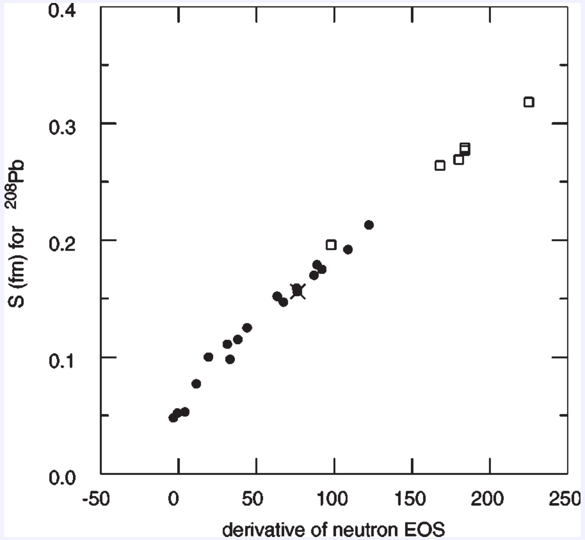
S. Typel and B. A. Brown, PRC 64 (2001) 027302

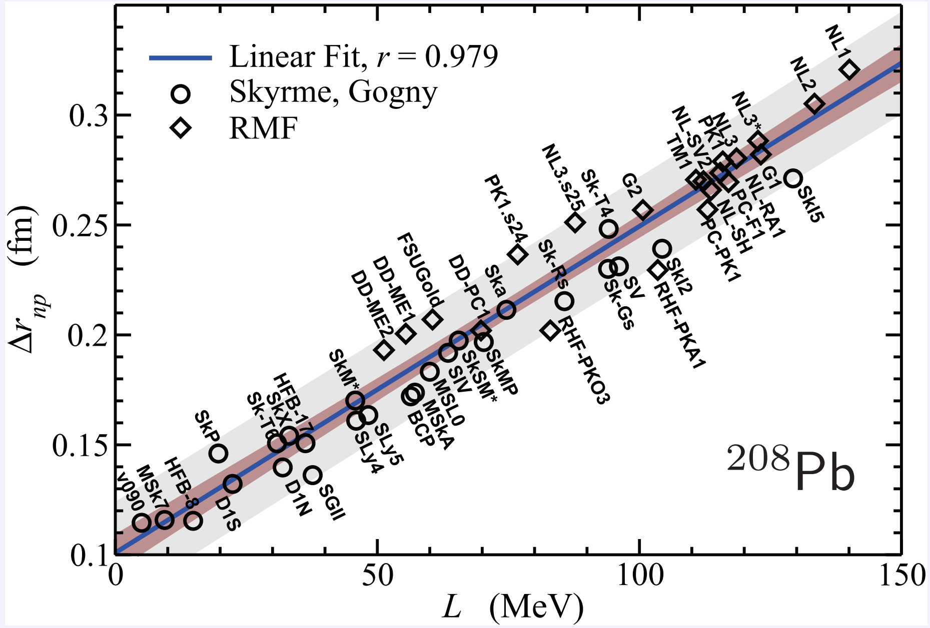
$\Leftrightarrow$ symmetry energy slope parameter L

- determine L from experimental measurement of Δr_{np}
  - ○ parity violation in electron scattering
    PREX@Jefferson Lab, C.J. Horowitz et al.,
    PRC 63 (2001) 025501, PRC 85 (2012) 032501
  - ○ coherent pion photoproduction
    MAMI@Mainz, C. Tarbert et al., PRL 112 (2014) 242502

- correlation based on mean-field models, low densities at nuclear surface
  $\Rightarrow$ effects of correlations?

(see S. Typel, PRC 89 (2014) 064321)





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Nuclear Matter Parameters II

- density-dependent meson-nucleon couplings with parametrization DD2 (refit of parametrization DD with experimental nucleon masses)

  S. Typel et al., Phys. Rev. C 81 (2010) 015803

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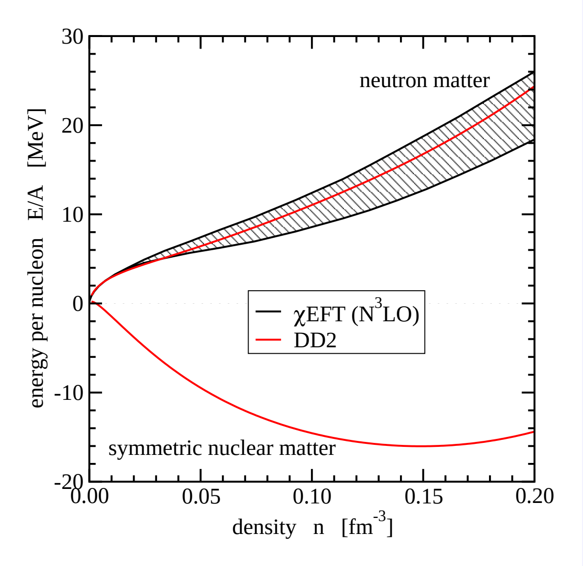

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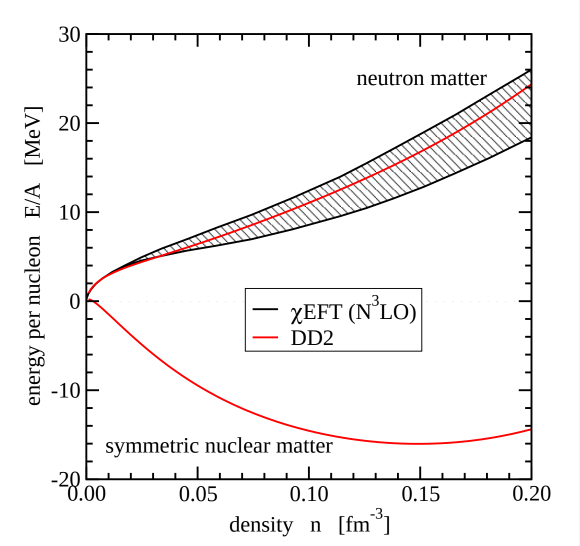

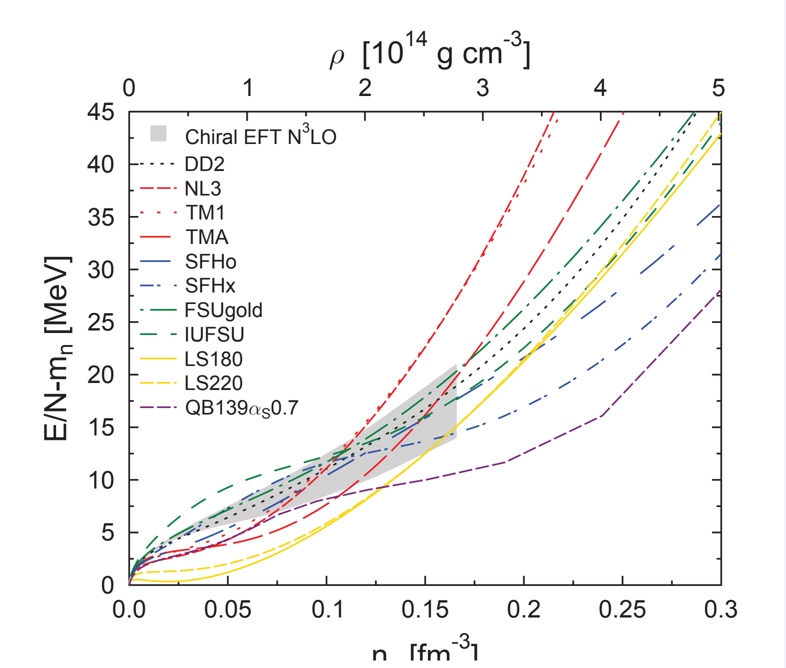

T. Fischer et al., EPJA 50 (2014) 46

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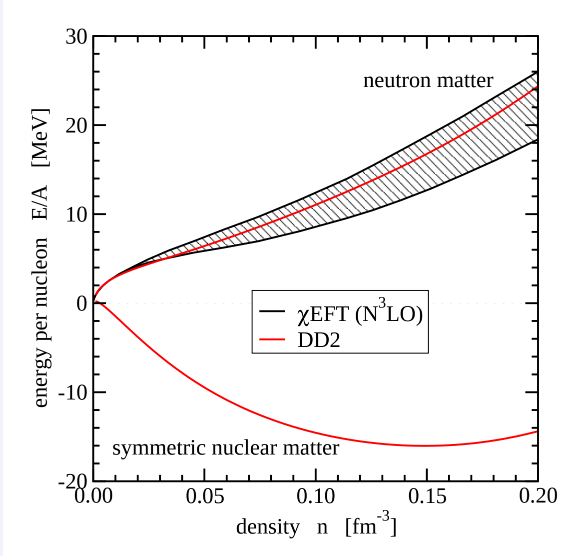

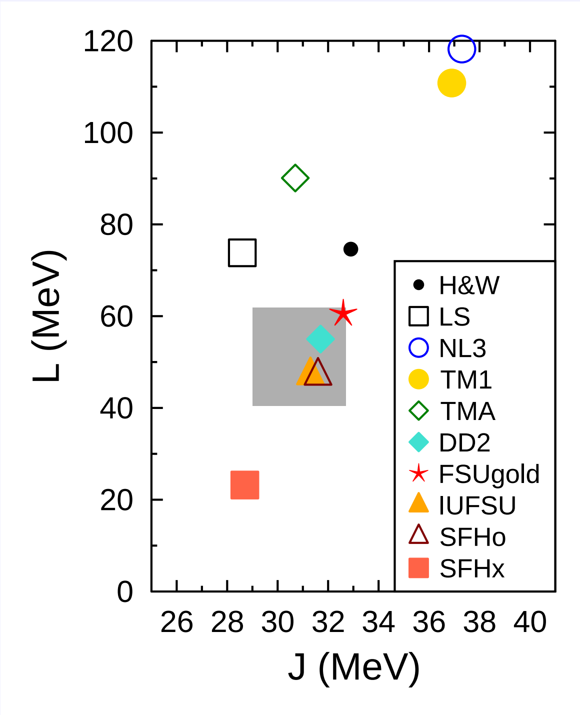

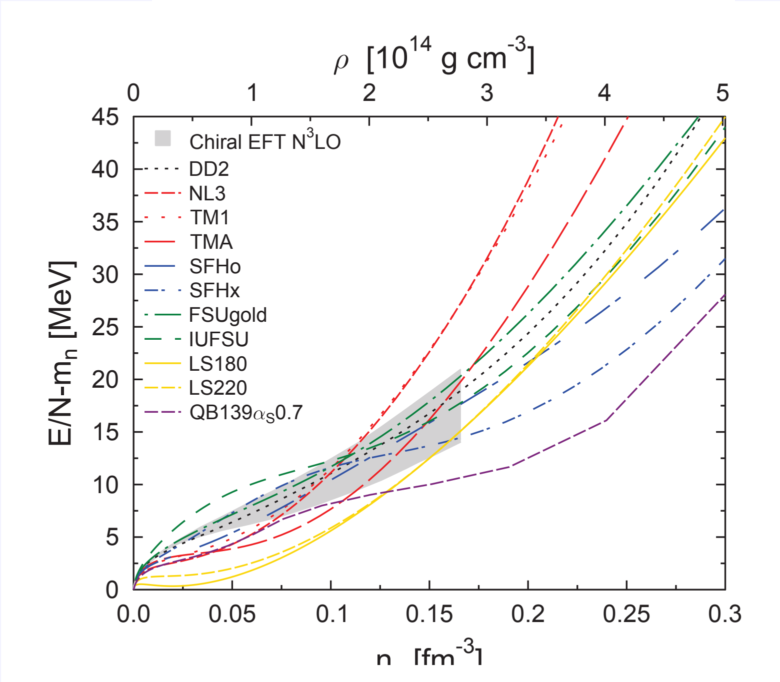

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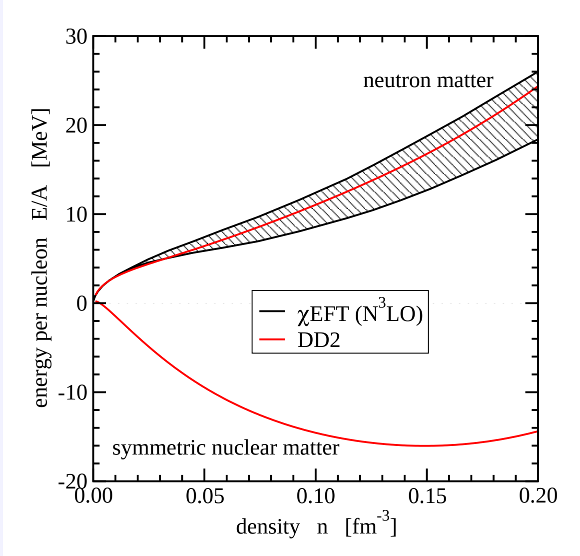
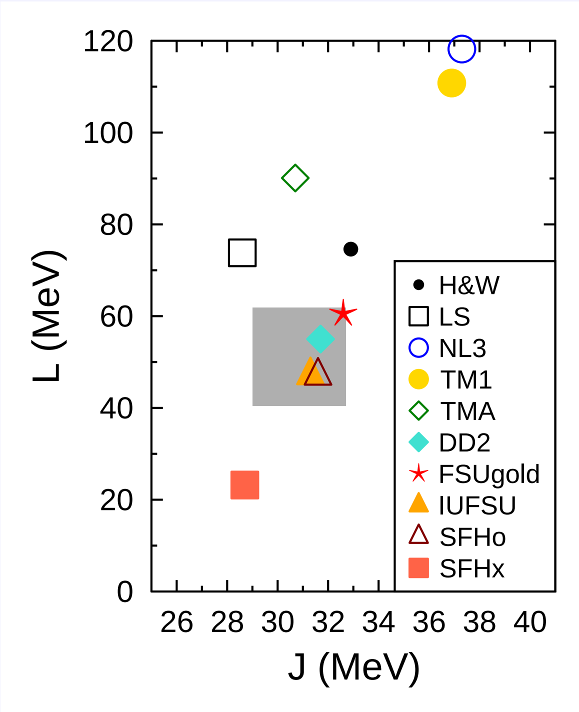
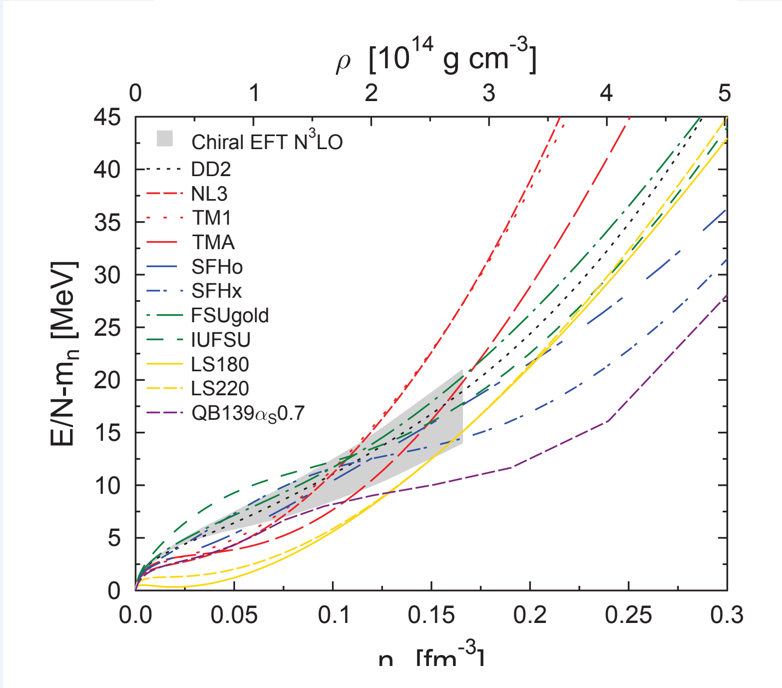


- DD2:
  very reasonable nuclear matter parameters

$$n_{\text{sat}} = 0.149\ \text{fm}^{-3}$$
$$a_V = 16.02\ \text{MeV}$$
$$K = 242.7\ \text{MeV}$$
$$J = 31.67\ \text{MeV}$$
$$L = 55.04\ \text{MeV}$$





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Generalized Relativistic Density Functional II

stellar matter

- strong and electromagnetic interaction
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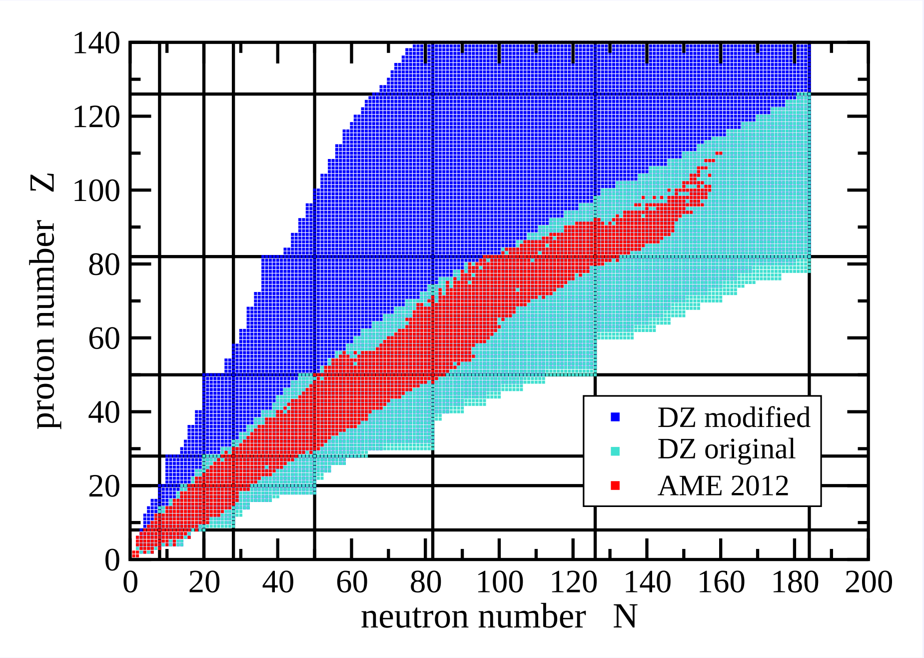

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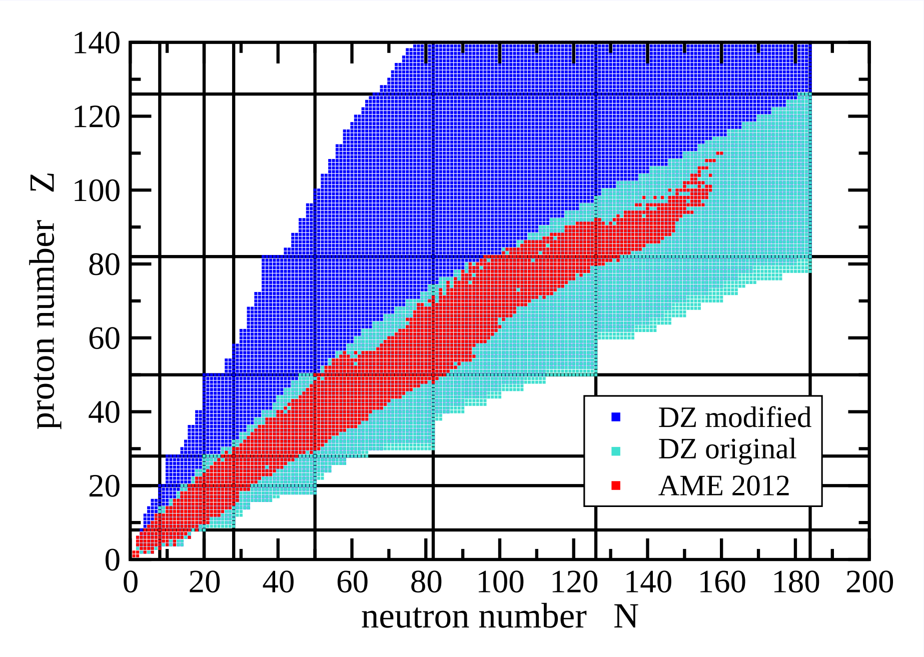

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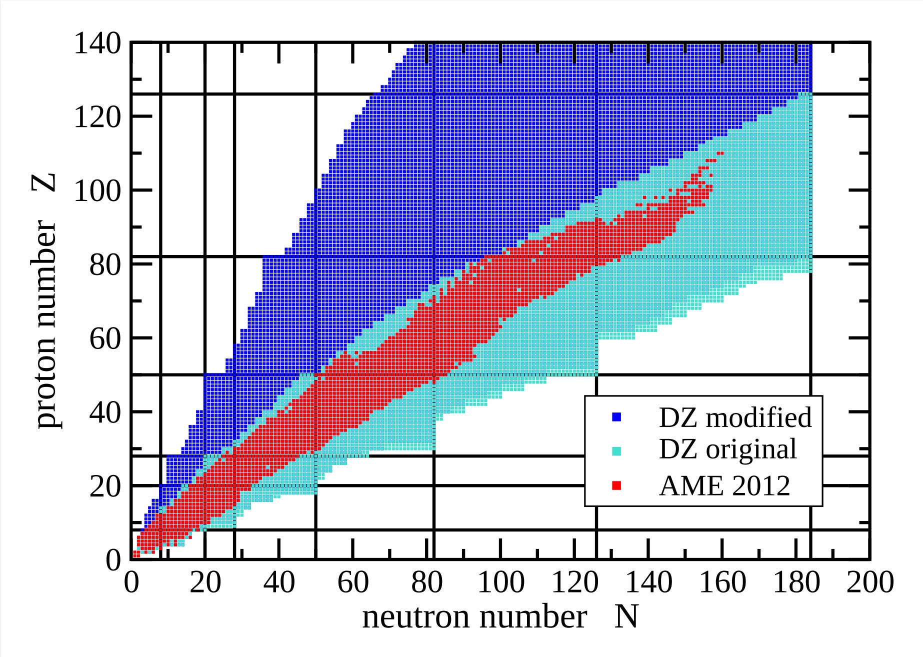

  $\Rightarrow$ shell effects included, not only average heavy nucleus
  - ◦ considered as quasi-particles with scalar and vector potentials
  - ◦ additional medium modifications of composite particles
    (mass shifts, internal excitations) $\Rightarrow$ dissolution of nuclei, Mott effect
  - ◦ NN scattering correlations included $\Rightarrow$ correct low-density limit, virial EoS

Details: S. Typel et al., Phys. Rev. C 81 (2010) 015803, Eur. Phys. J. A 50 (2014) 17,
M.D. Voskresenskaya and S. Typel, Nucl. Phys. A 887 (2012) 42, M. Hempel et al., Phys. Rev. C 91 (2015) 045805

Low-Density Limit I

- **comparison** of generalized relativistic density functional
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  $\Rightarrow$ conventional mean-field models don't reproduce effect of correlations
  at very-low densities

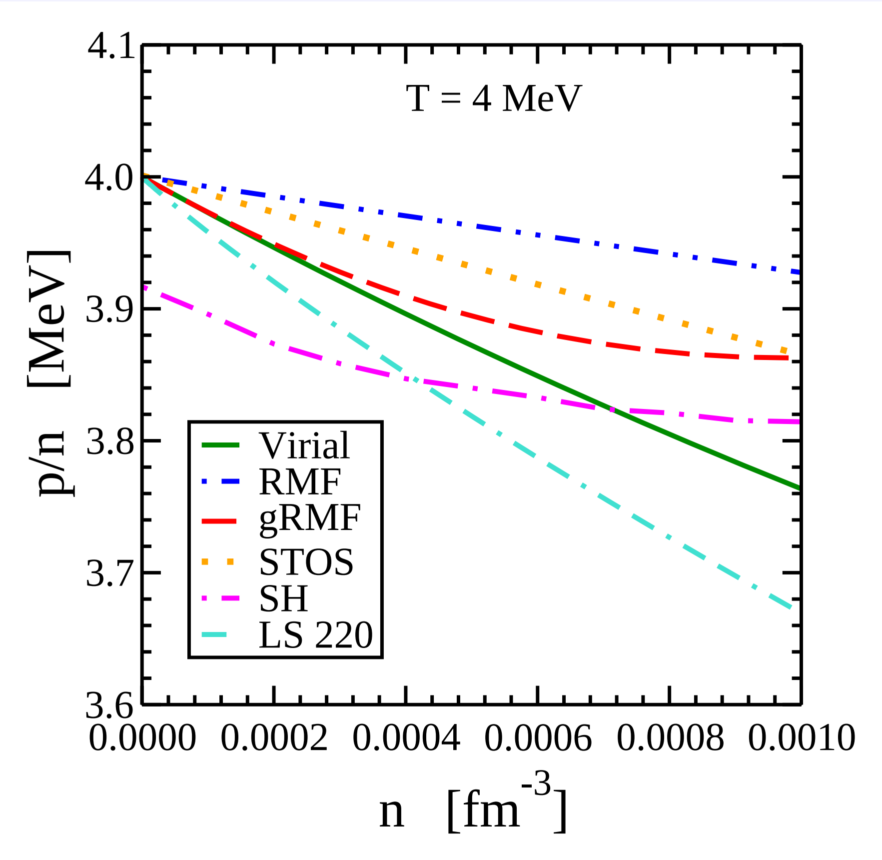
  $\Rightarrow$ introduce continuum correlations,
  represented by effective resonance energies $E_{ij}(T)$ $(i, j = n, p)$
  with effective degeneracy factors $g_{ij}^{(\mathrm{eff})}(T)$

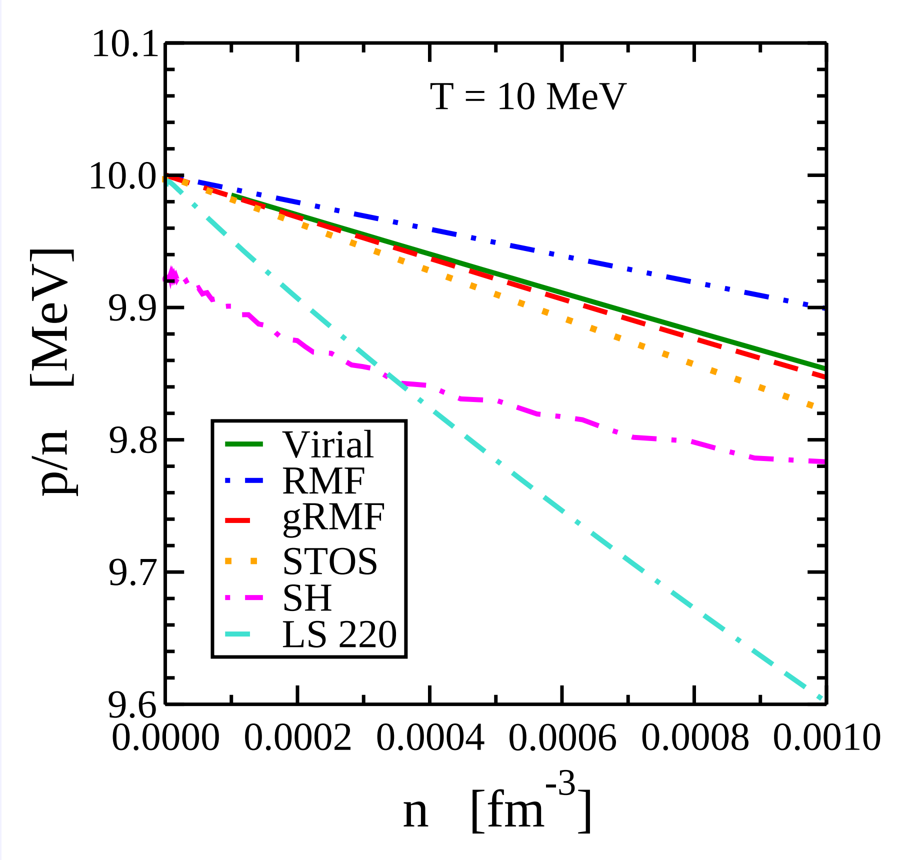
  $\Rightarrow$ relativistic corrections

(M.D. Voskresenskaya and S. Typel, Nucl. Phys. A 887 (2012) 42)

Low-Density Limit II

comparison: p/n in different models for neutron matter (ideal gas: $p/n = T$)





STOS: H. Shen et al., Nucl. Phys. A 637 (1998) 435 (TM1)

SH: G. Shen et al., Phys. Rev. C 83 (2011) 065808 (FSUGold)

LS 220: J.M. Lattimer et al., Nucl. Phys. A 535 (1991) 331 ($K = 220$ MeV)

Mass Shifts I

- concept applies to composite particles: **clusters**
  - light and heavy nuclei
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    - $\Rightarrow$ reduction of binding energies
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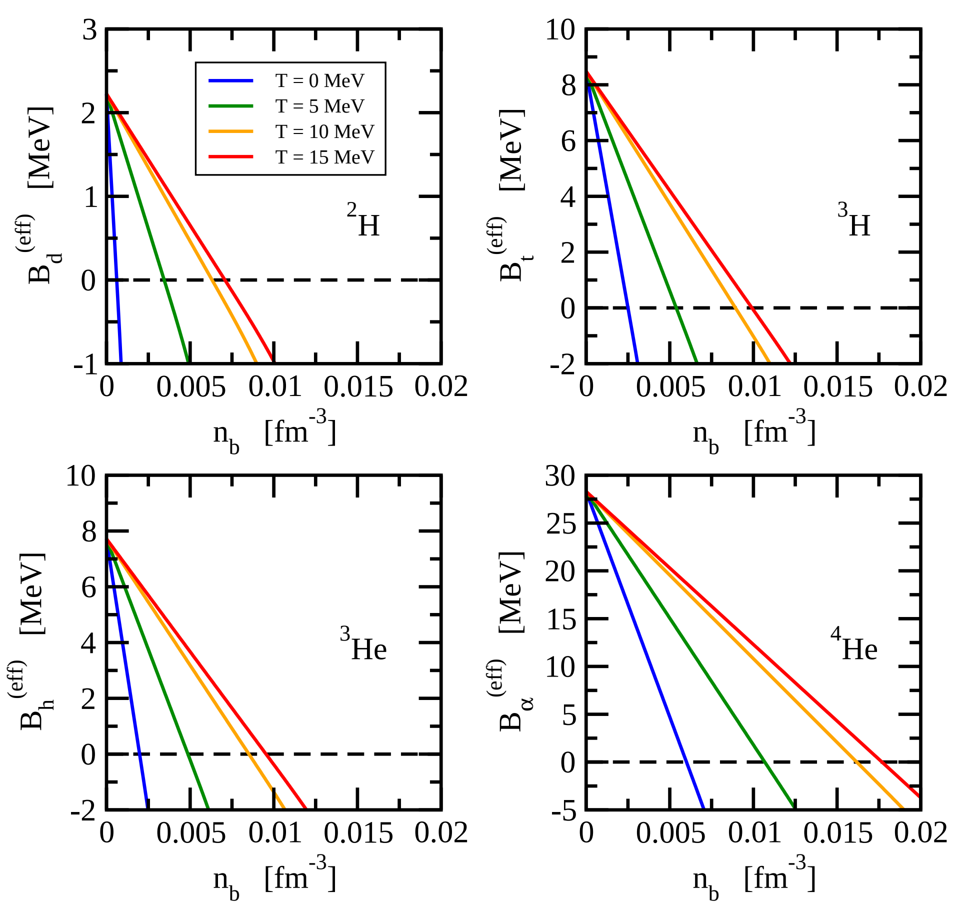
- **electromagnetic shift** $\Delta E_i^{(\mathrm{Coul})}$ (in stellar matter)
  - electron screening of Coulomb field
    - $\Rightarrow$ increase of binding energies

Mass Shifts II

- light nuclei: parametrization from G. Röpke, simplified and modified for high densities and temperatures
- NN scattering states: as for deuteron

effective binding energies $B_i^{(\mathrm{eff})} = B_i^{(0)} - \Delta m_i^{(\mathrm{strong})}$



Mass Shifts II

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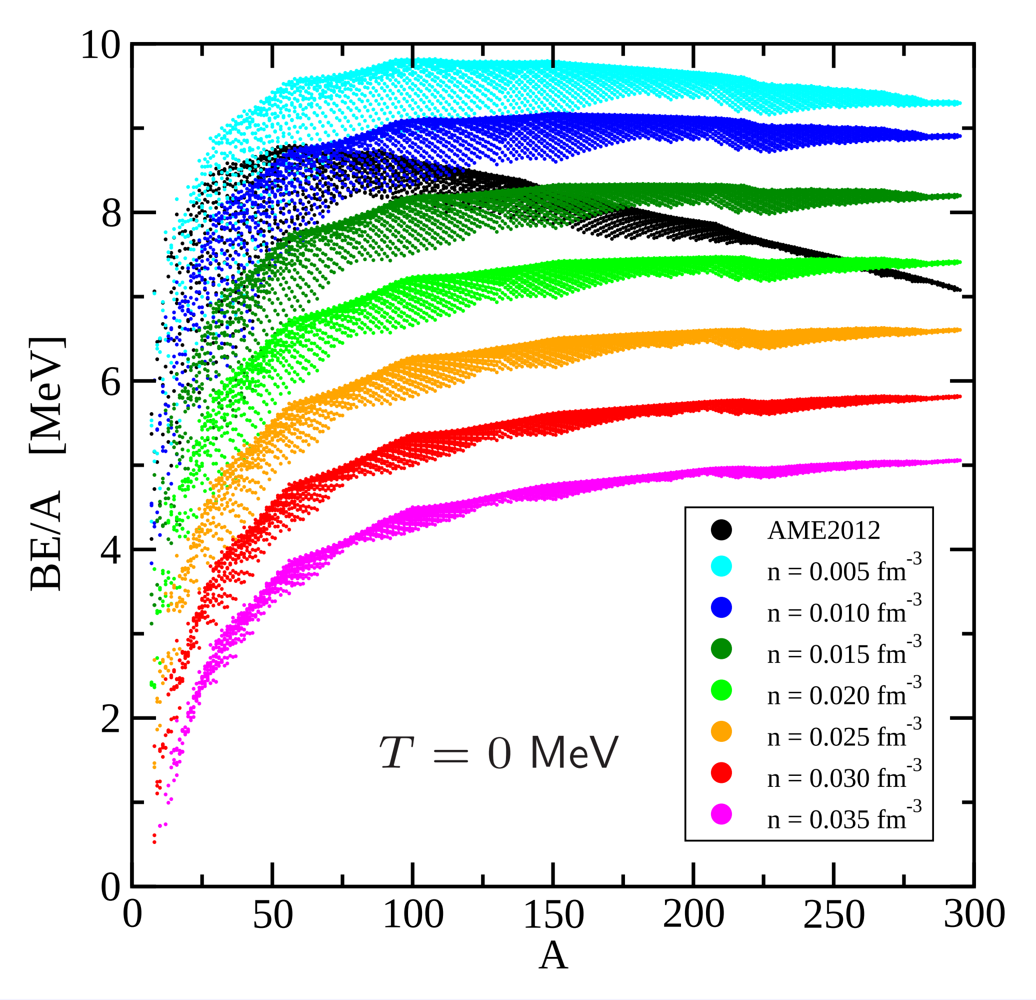
- heavy nuclei: simple parametrization
- general form: $\Delta m_i^{(\mathrm{strong})} = f_i B_i^{(0)}$ with
  vacuum binding energy $B_i^{(0)}$ and shift function

$$f_i = \begin{cases} x & \text{if} \quad x \leq 1 \\ x + \frac{(x-1)^3(y-1)}{3(y-x)} & \text{if} \quad x > 1 \text{ and } x < y \end{cases}$$

light nuclei: $x = \frac{n_i^{(\mathrm{eff})}}{n_i^{(\mathrm{diss})}(T)}, \quad y = \frac{n_{\mathrm{sat}}}{n_i^{(\mathrm{diss})}(T)}$

heavy nuclei: $x = \frac{n_i^{(\mathrm{eff})}}{n_{\mathrm{sat}}} y, \quad y = 3 + \frac{28}{A}$

effective density: $n_i^{(\mathrm{eff})} = 2\frac{Z_i Y_q + N_i(1-Y_q)}{Z_i + N_i} n_b$



Constraint from Heavy-Ion Collisions

emission of light nuclei in heavy-ion collisions at Fermi energies

- determination of density and temperature

  S. Kowalski et al. PRC 75 (2007) 014601

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$$K_c[i] = n_i/(n_p^{Z_i} n_n^{N_i})$$

L. Qin et al., PRL 108 (2012) 172701

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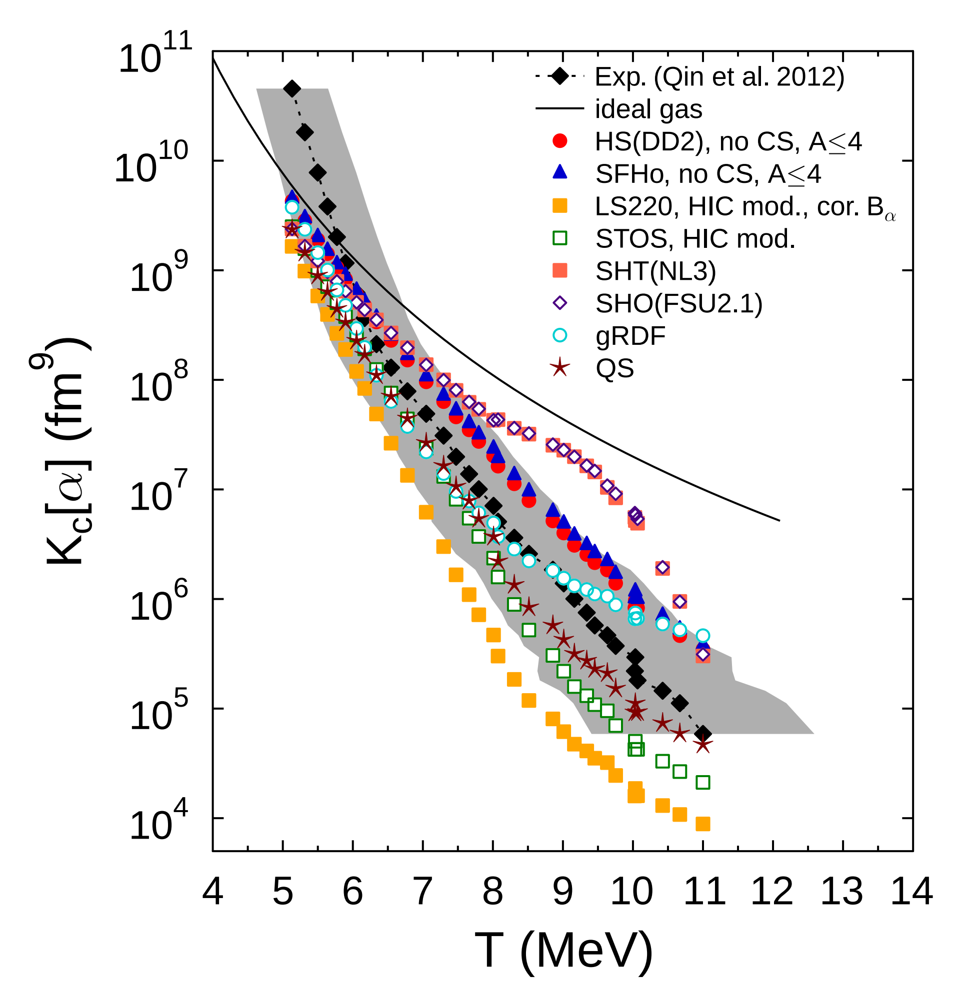

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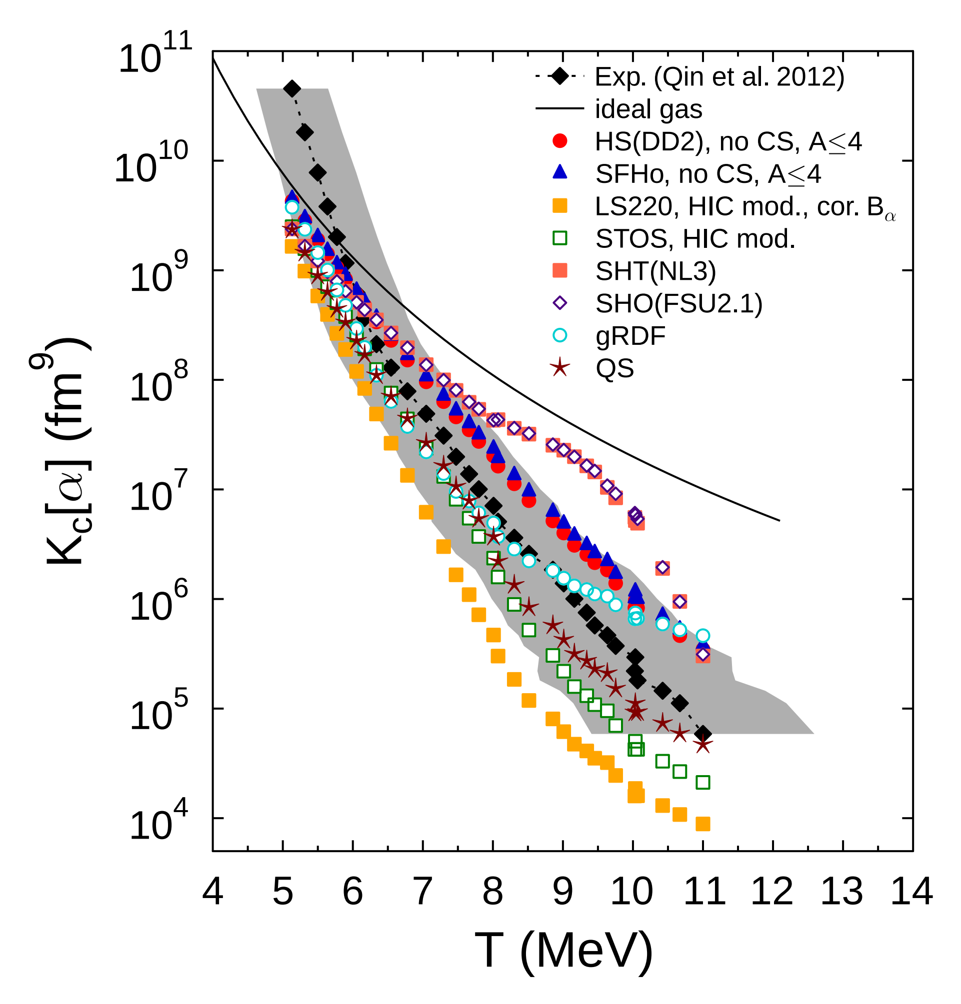
L. Qin et al., PRL 108 (2012) 172701

⇒ mixture of ideal gases/NSE description not sufficient

⇒ medium effects/correlations important

chemical equilibrium constants of α particles

from M. Hempel et al., PRC 91 (2015) 045805



Equation of State Table

range of variables

- temperature: 0.1 MeV $\leq T \leq 100$ MeV $\Rightarrow$ 76 mesh points
- baryon density: 10^{-10} fm$^{-3} \leq n_b \leq 1$ fm^{-3} $\Rightarrow$ 251 mesh points
- hadronic charge fraction: $0.01 \leq Y_q \leq 0.60 \Rightarrow$ 60 mesh points

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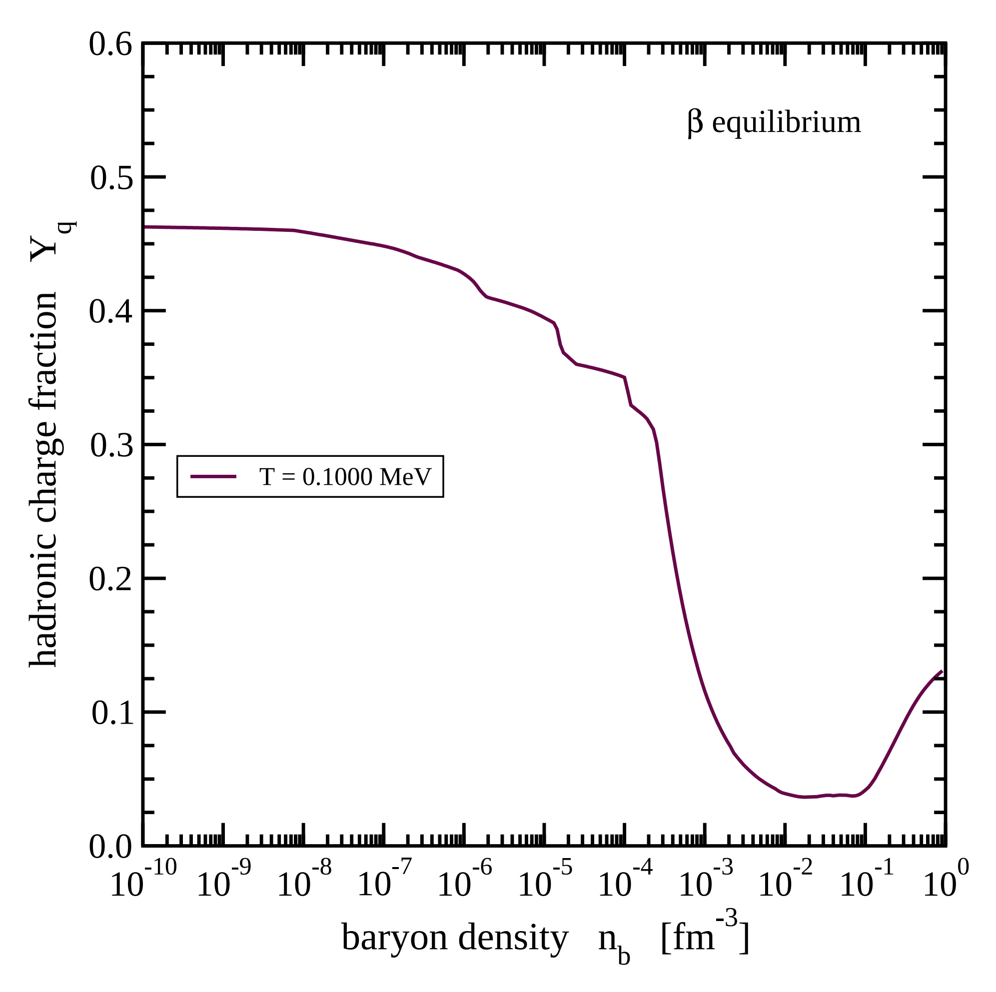
availability

- data tables will be released on CompOSE website (`http://compose.obspm.fr`)

Neutron Star Matter I

conditions: charge neutrality and β equilibrium

- hadronic charge fraction $Y_q = \sum_{i \neq e,\mu} Q_i n_i / n_b$



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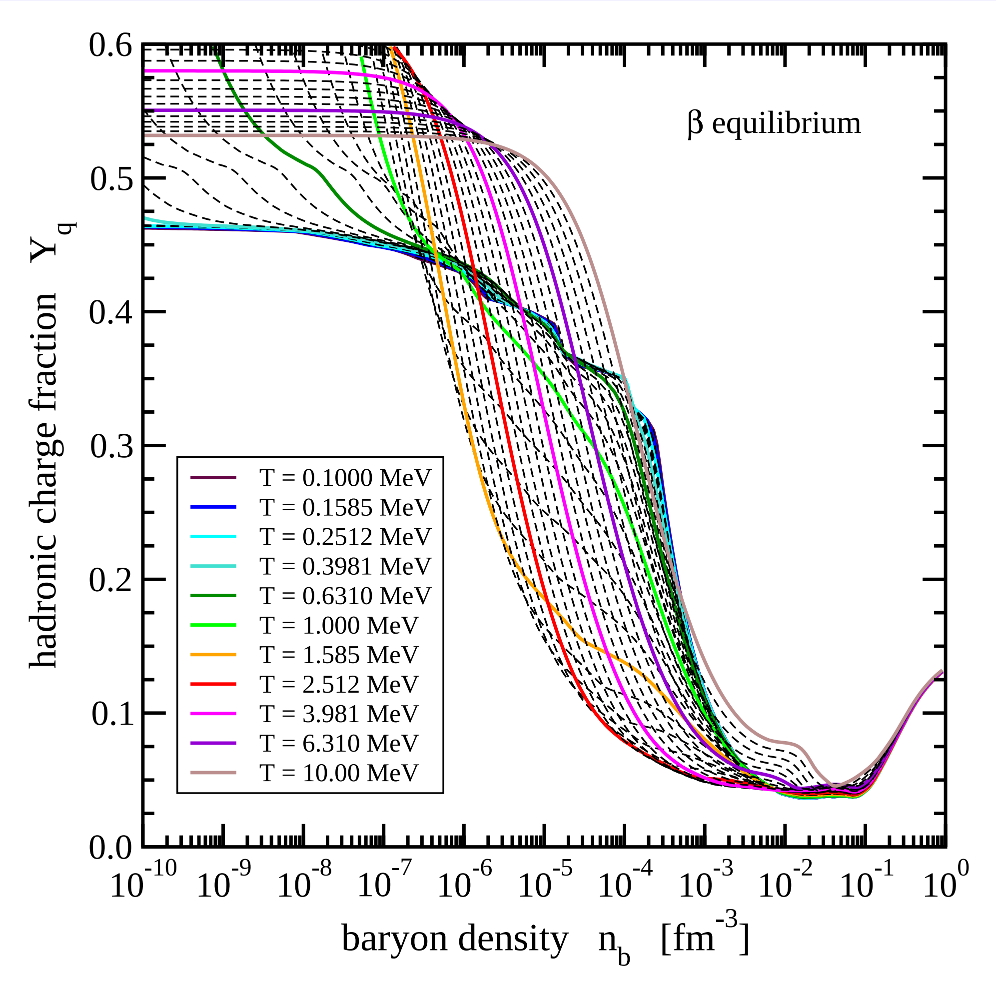

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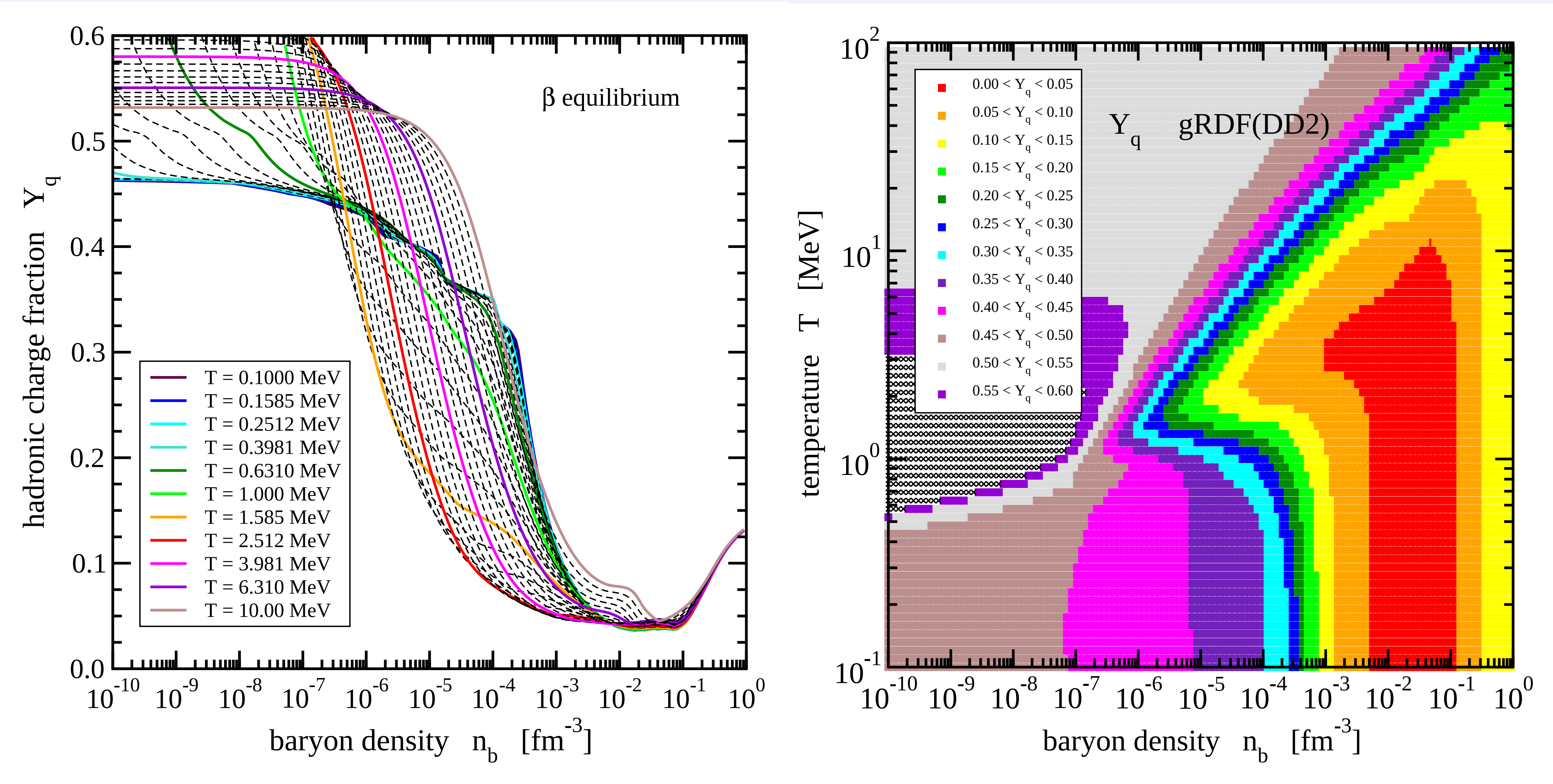

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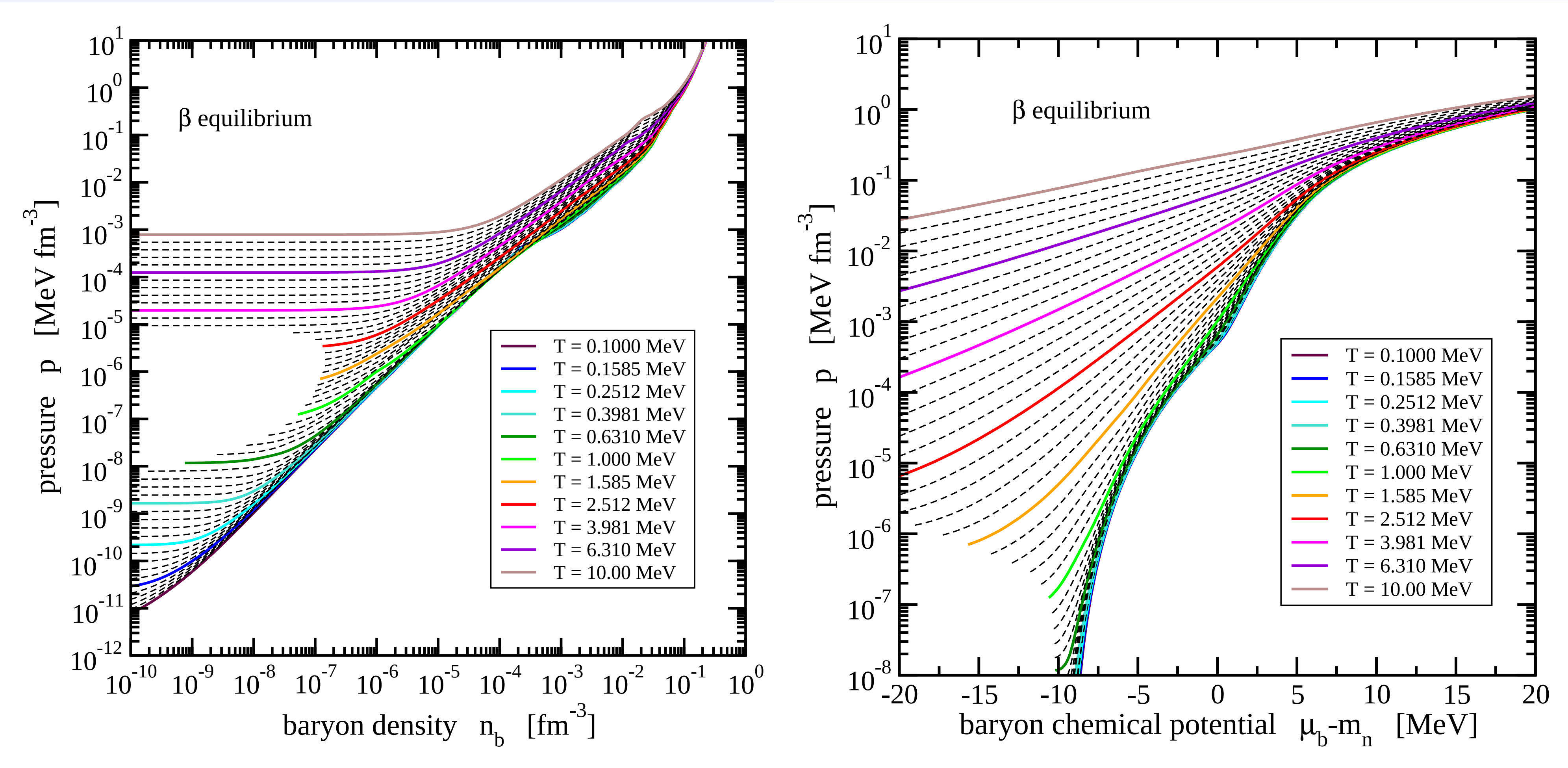
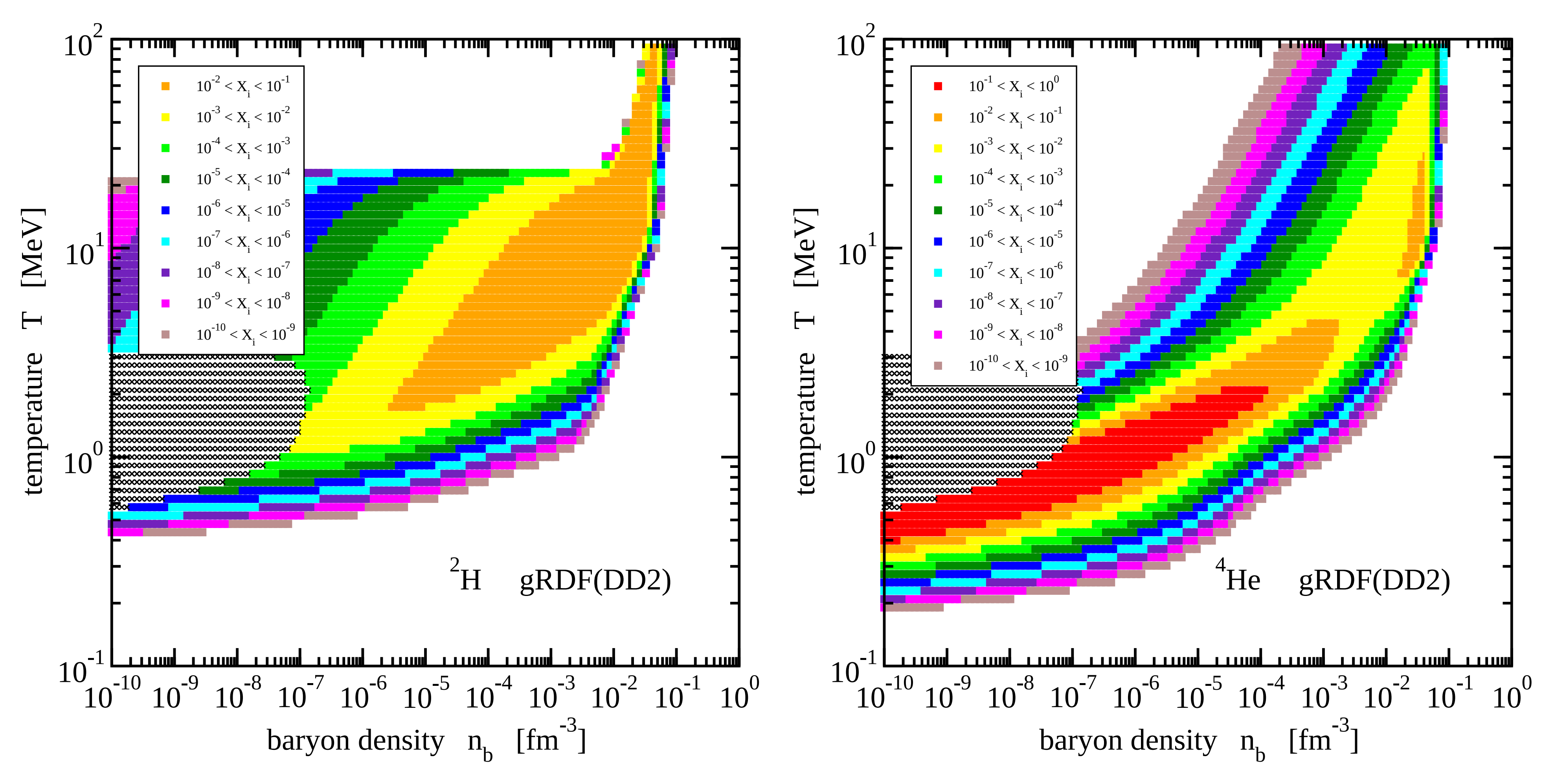


Neutron Star Matter III

conditions: charge neutrality and β equilibrium

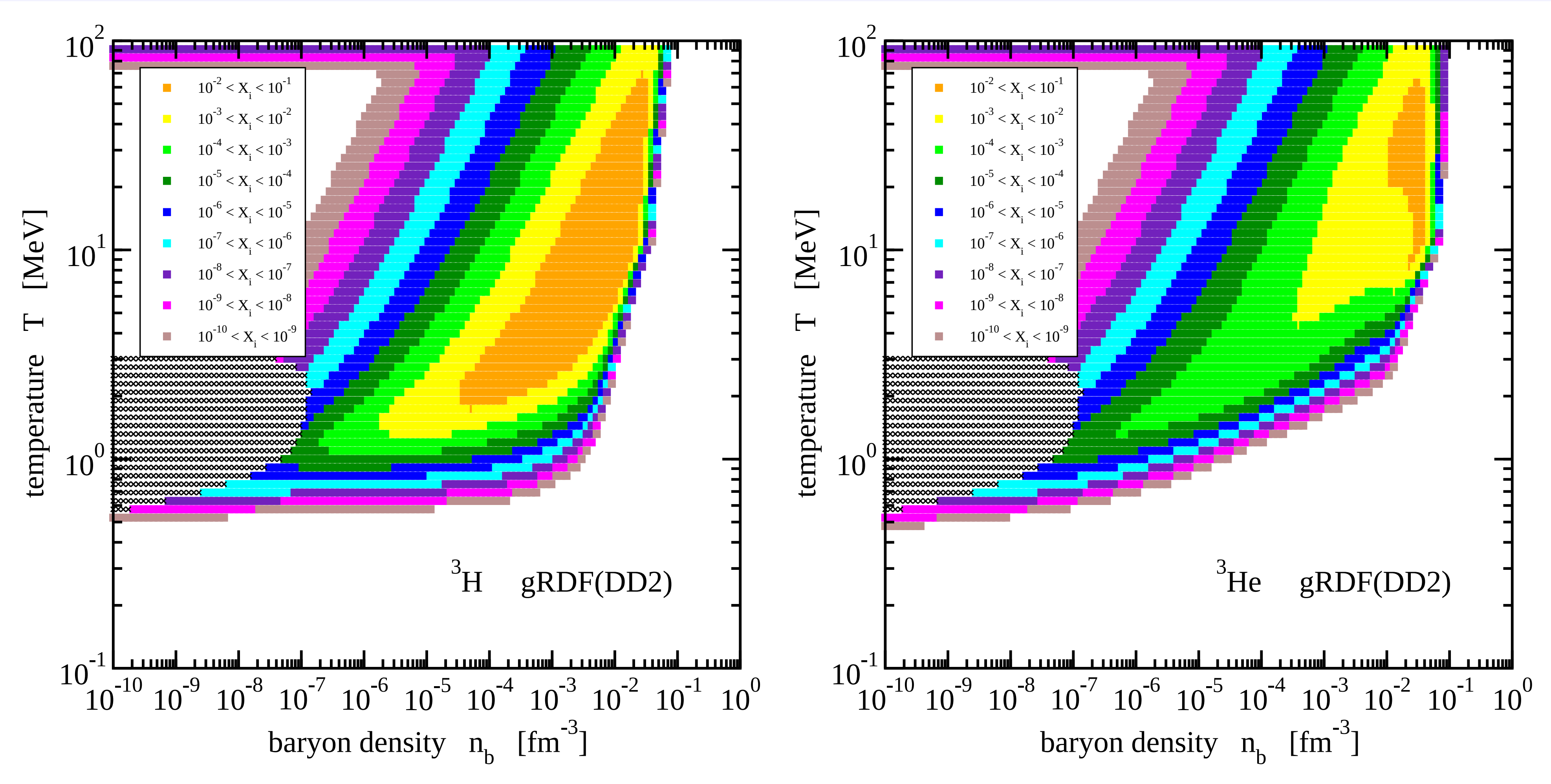
- mass fractions of deuterons $X_d = 2n_d/n_b$ and α particles $X_\alpha = 4n_\alpha/n_b$



Neutron Star Matter IV

conditions: charge neutrality and β equilibrium

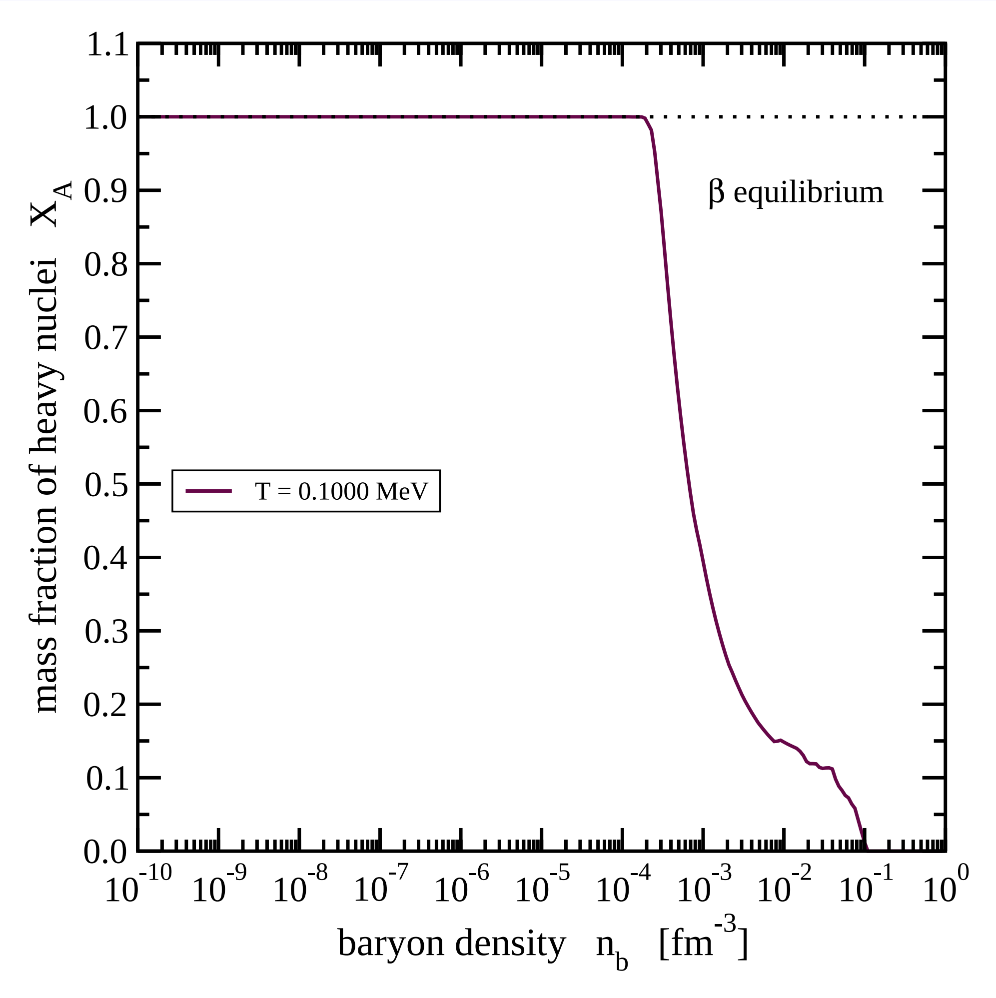
- mass fractions of tritons $X_t = 3n_t/n_b$ and helions $X_h = 3n_h/n_b$



Neutron Star Matter V

conditions: charge neutrality and β equilibrium

- mass fractions of heavy nuclei $X_A = X_{\text{heavy}} = \sum_{(A,Z),A>4} X_{(A,Z)}$



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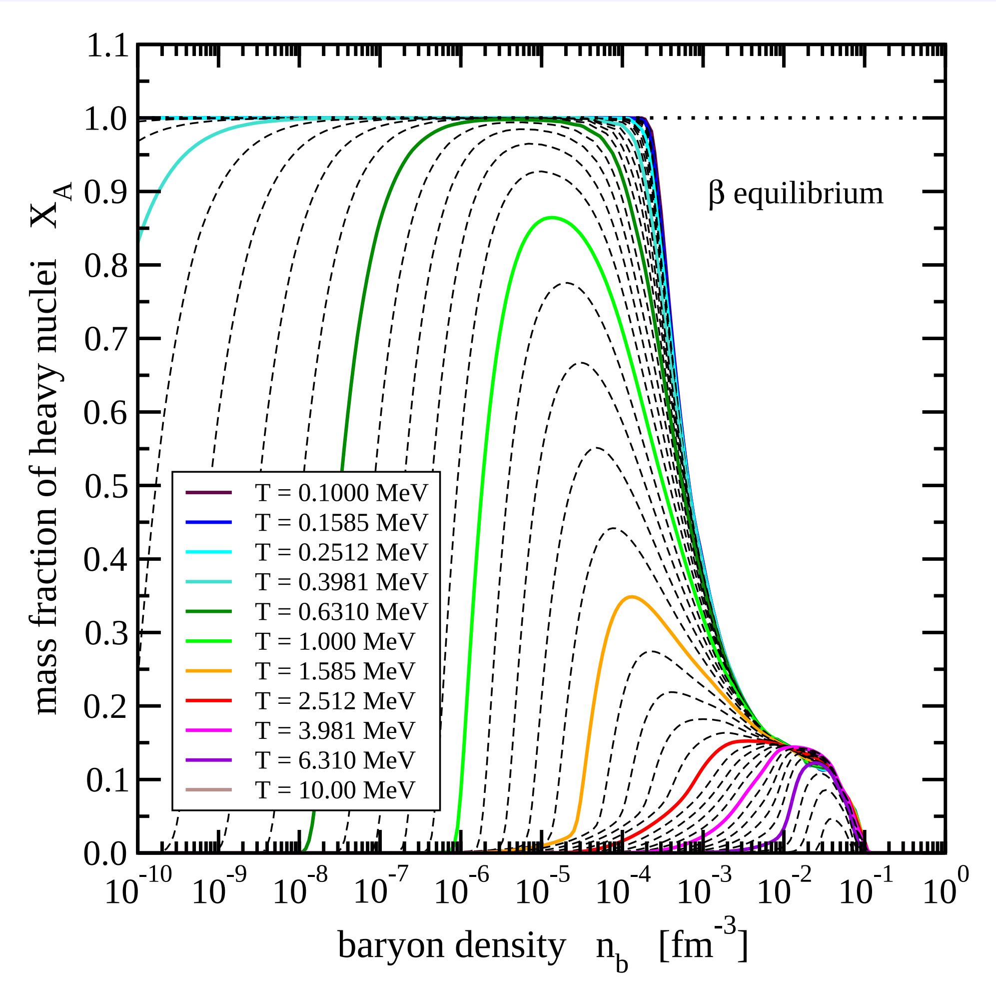

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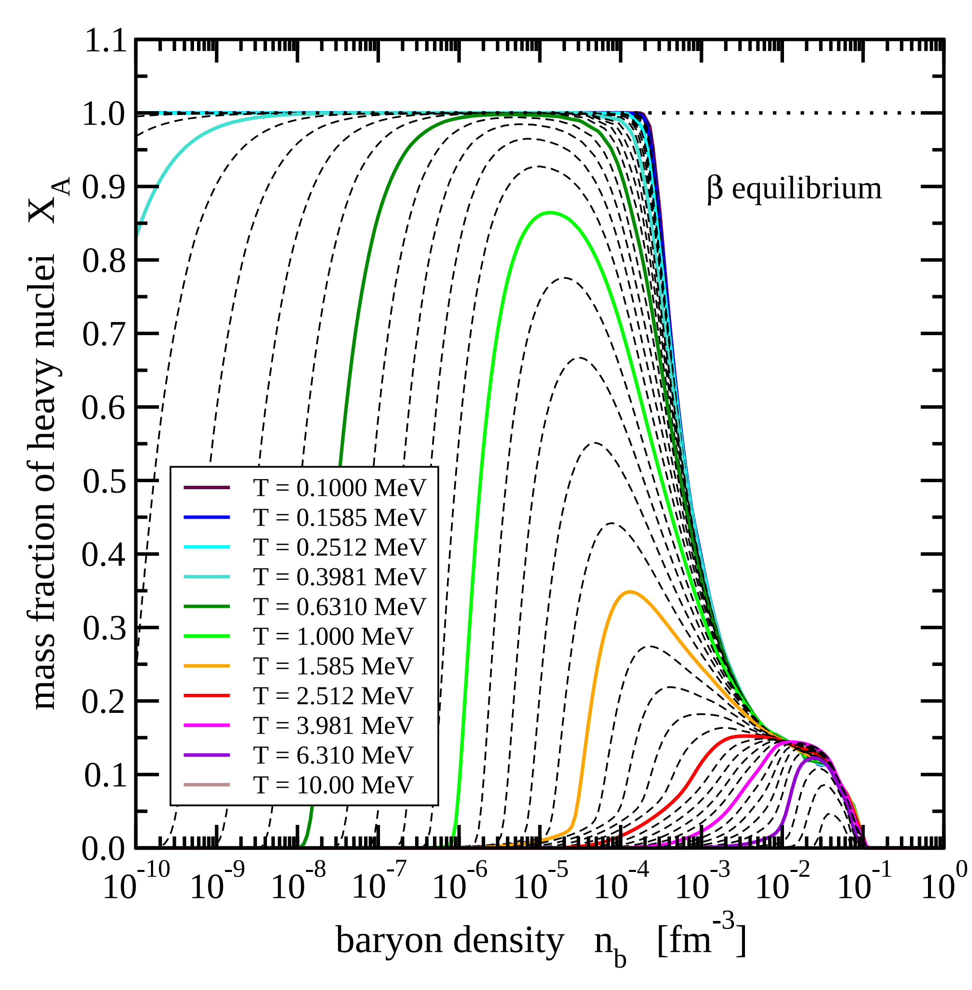
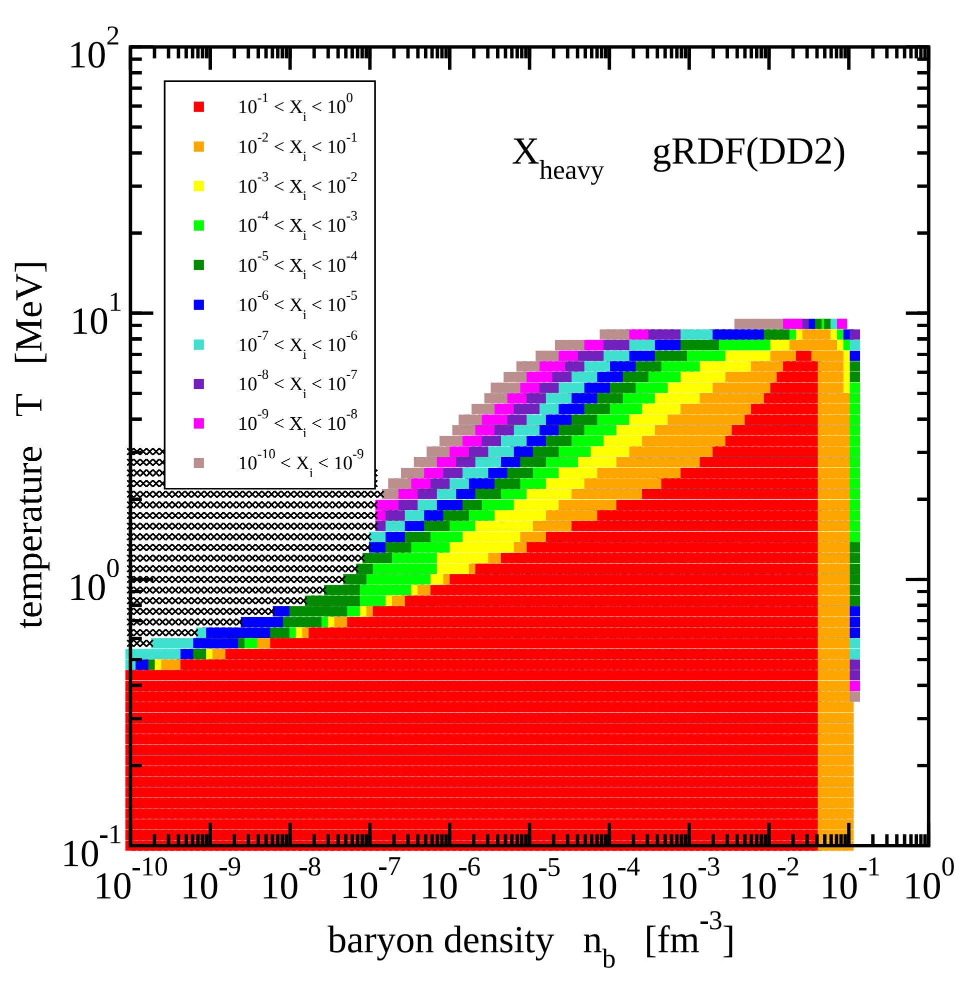
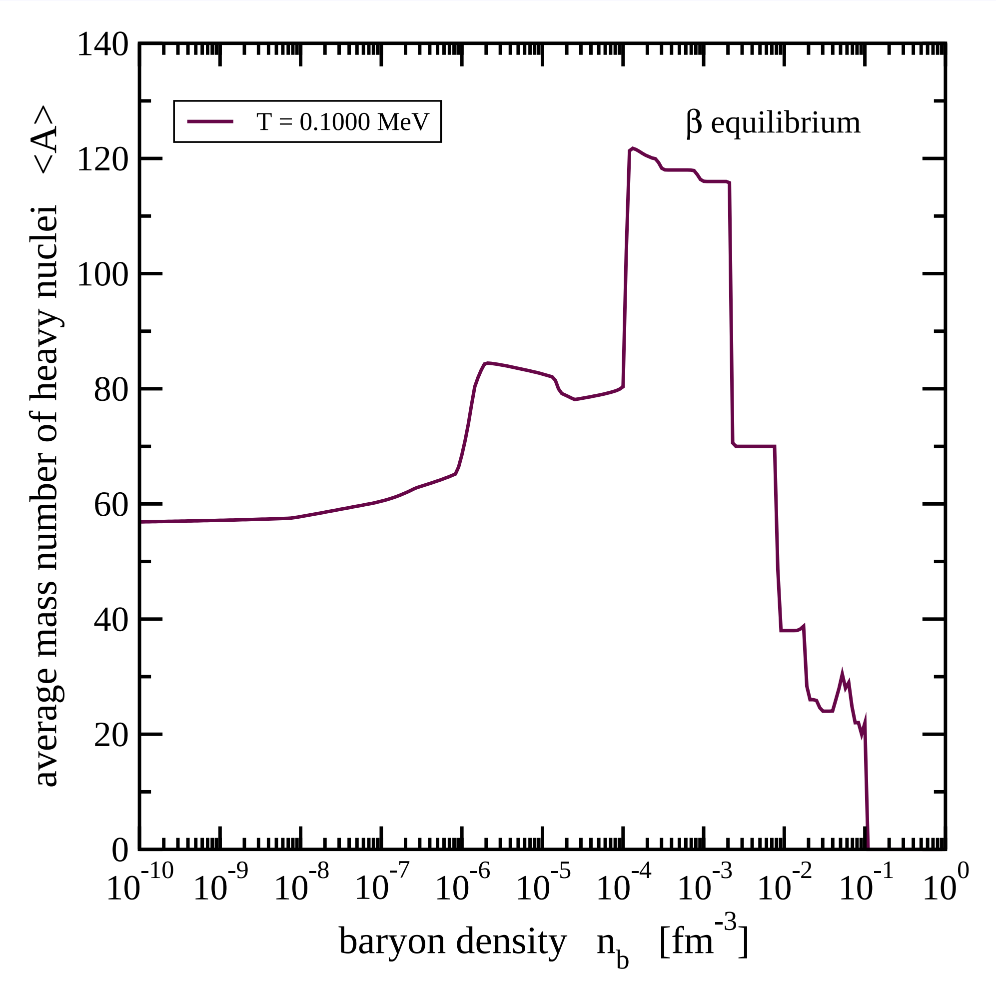




conditions: charge neutrality and β equilibrium

- average mass number of heavy nuclei $A_{\rm av} = \langle A \rangle = \sum_{(A,Z),A>4} A X_{(A,Z)} / X_{\rm heavy}$



Neutron Star Matter VI

conditions: charge neutrality and β equilibrium

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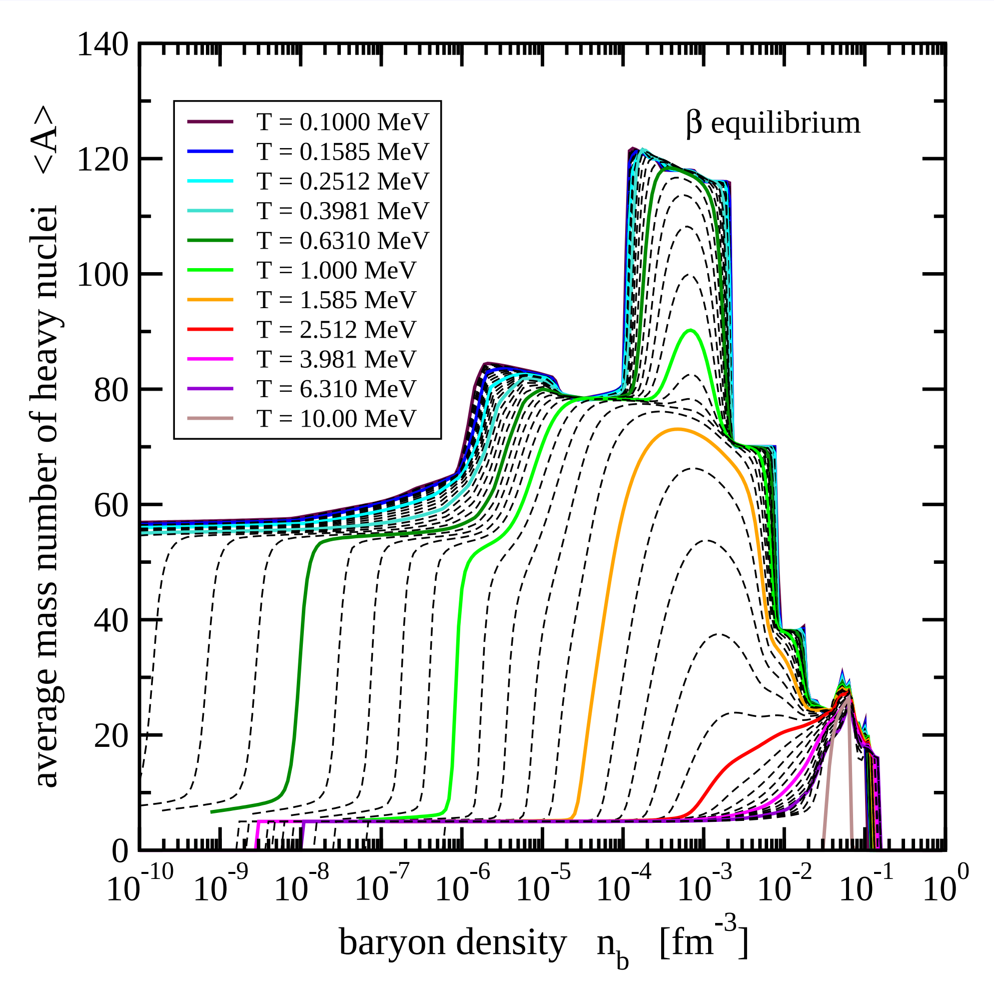
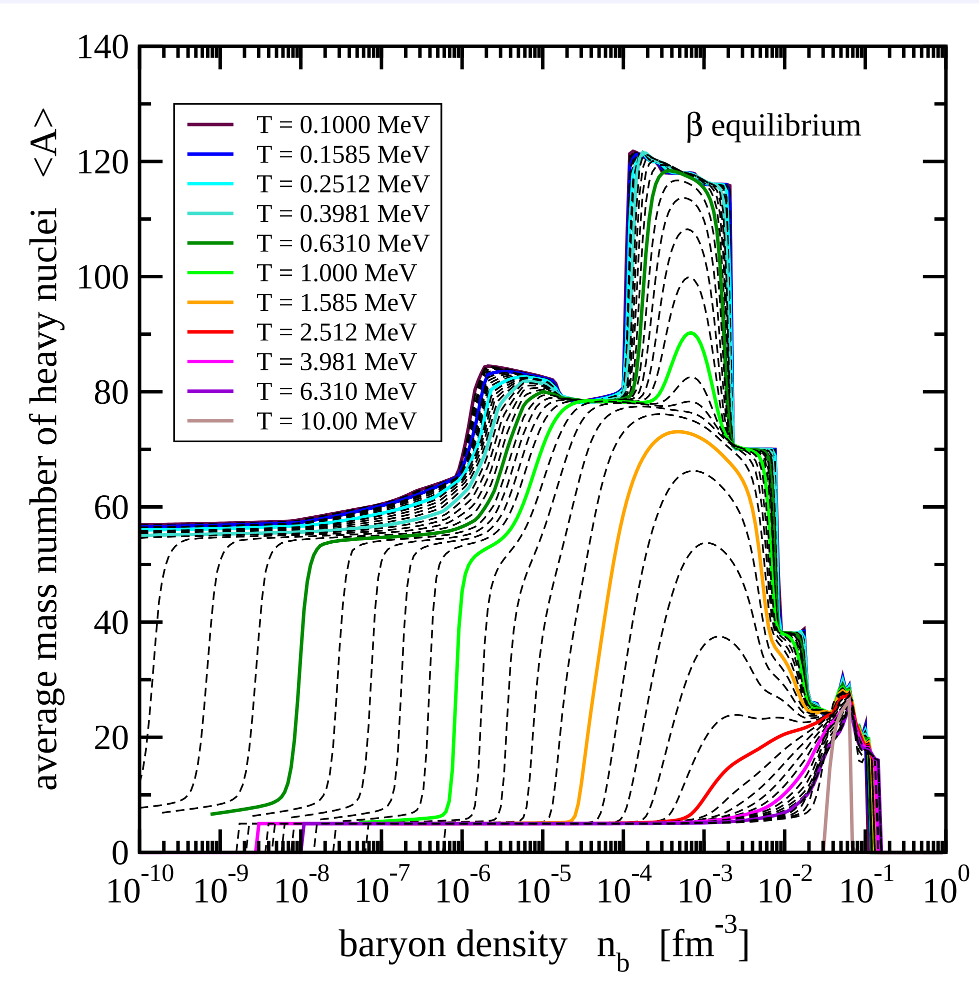
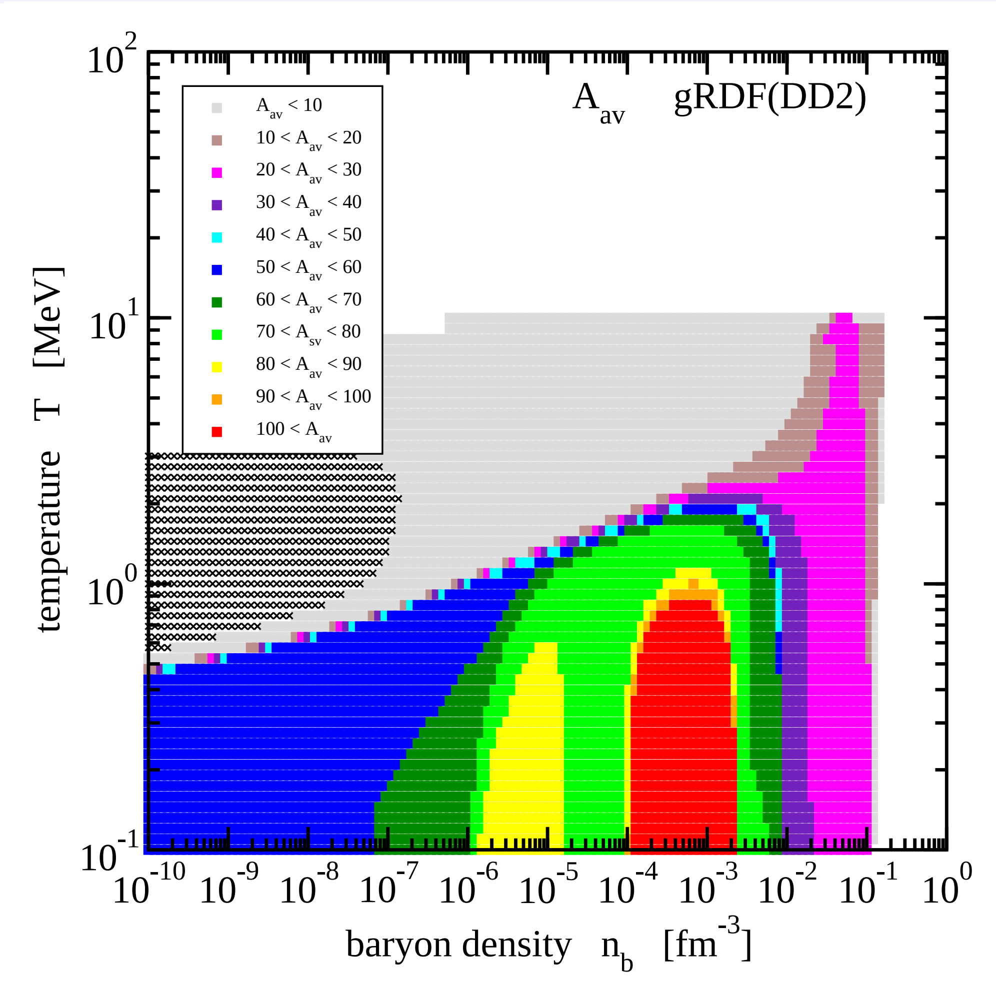


Neutron Star Matter VI

conditions: charge neutrality and β equilibrium

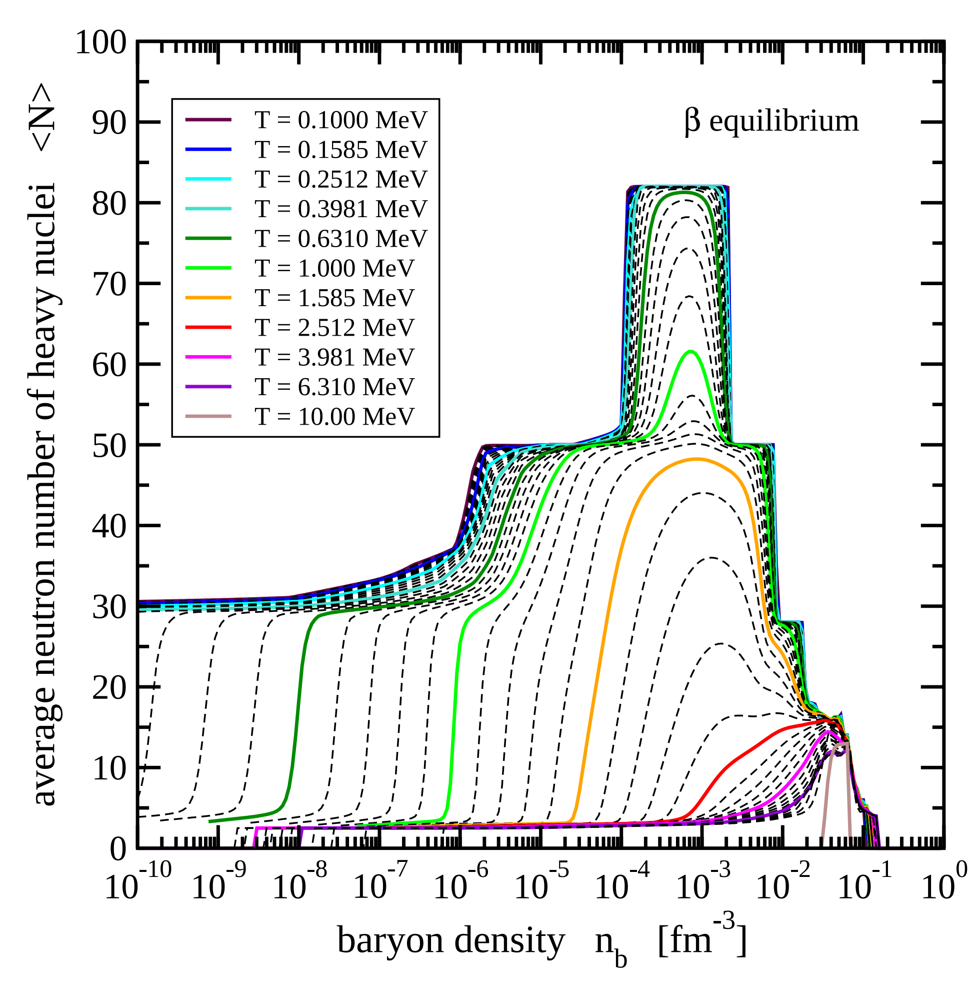
- average mass number of heavy nuclei $A_{\rm av} = \langle A \rangle = \sum_{(A,Z),A>4} A X_{(A,Z)} / X_{\rm heavy}$

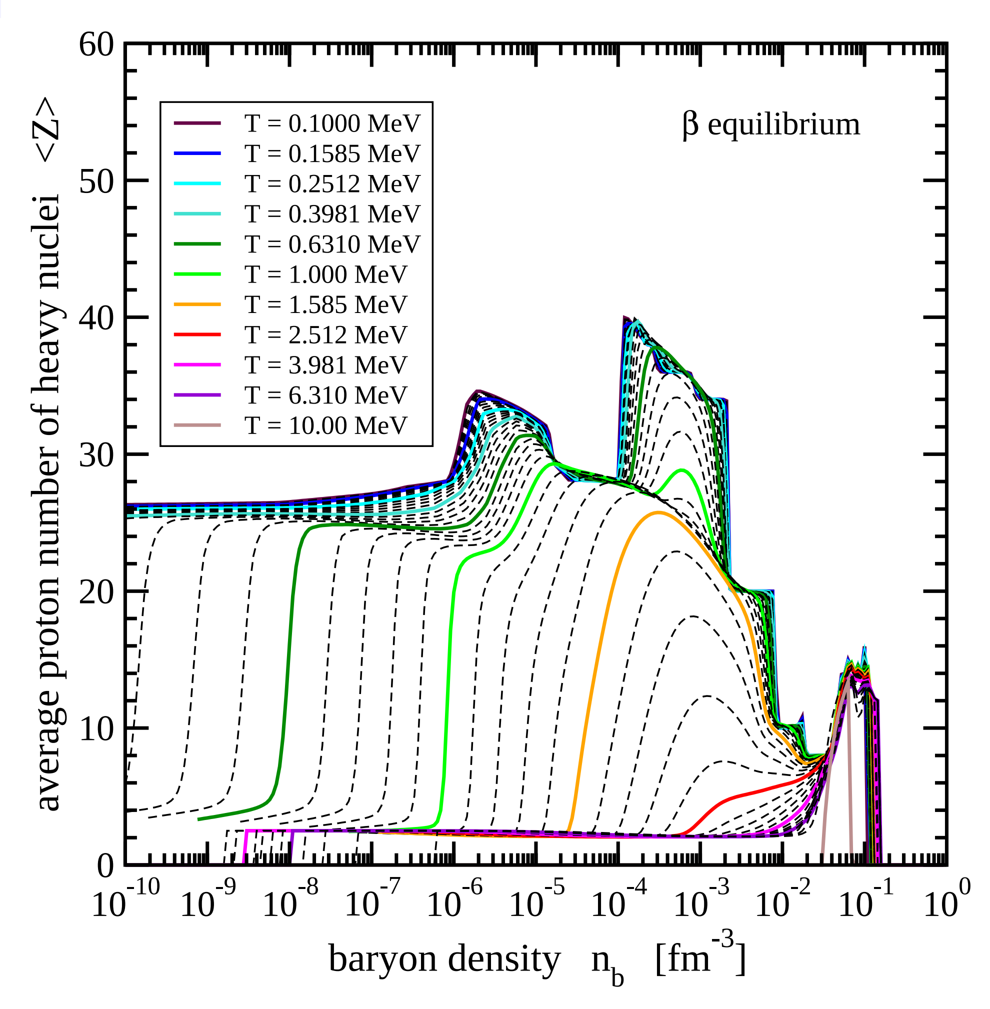




conditions: charge neutrality and β equilibrium

- average neutron and proton numbers of heavy nuclei, $\langle N \rangle$ and $\langle Z \rangle$





Low-Temperature Limit

- gap in EoS tables between $T = 0$ and $T_{\mathrm{min}} > 0$
- phase transition from gas/liquid phase to solid phase
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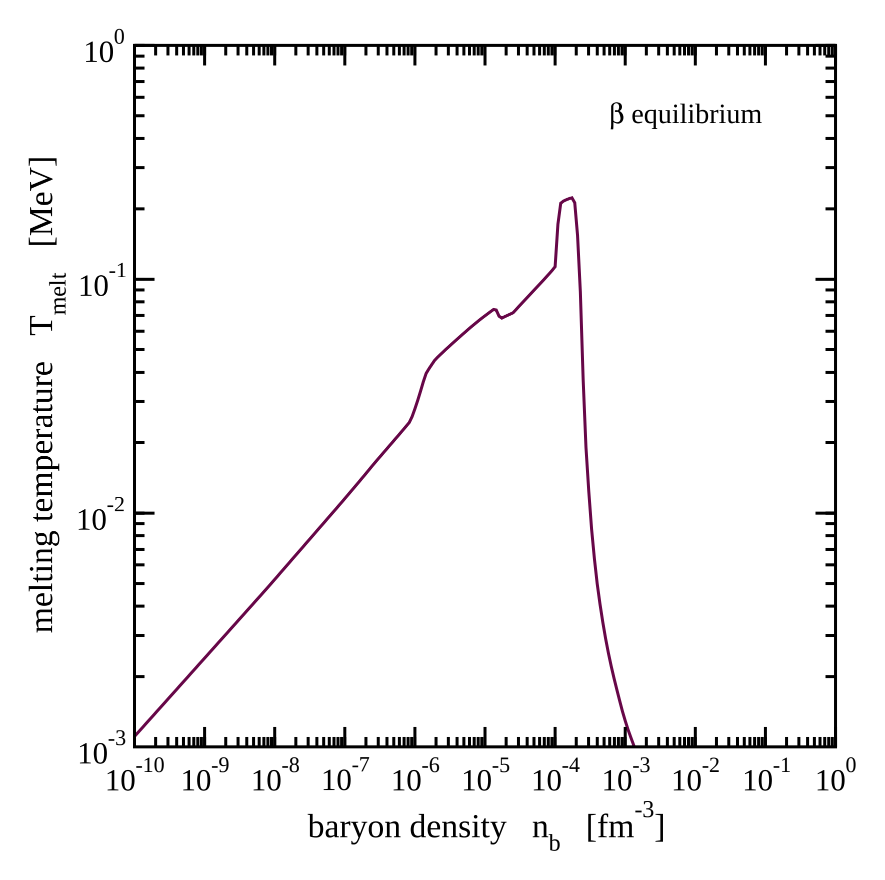
- Wigner-Seitz approximation not sufficient
- better: effective Coulomb contribution from Monte Carlo simulation (one-component plasma, OCP)
  $\Rightarrow$ phase transition for plasma parameter

$$\Gamma = \frac{Z^{5/3}e^2}{a_e T} \approx 175 \qquad a_e = \left(\frac{3n_e}{4\pi}\right)^{1/3}$$

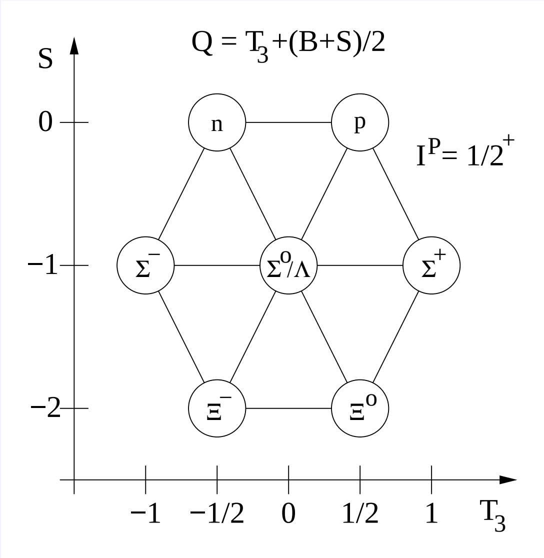
- improved description with model for crystal (to be studied)

DD2 (preliminary)



Hyperon Puzzle

- high densities
  ⇒ large baryon chemical potentials
  ⇒ heavy baryons appear (hyperons, . . .)



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  - couplings to mesons:
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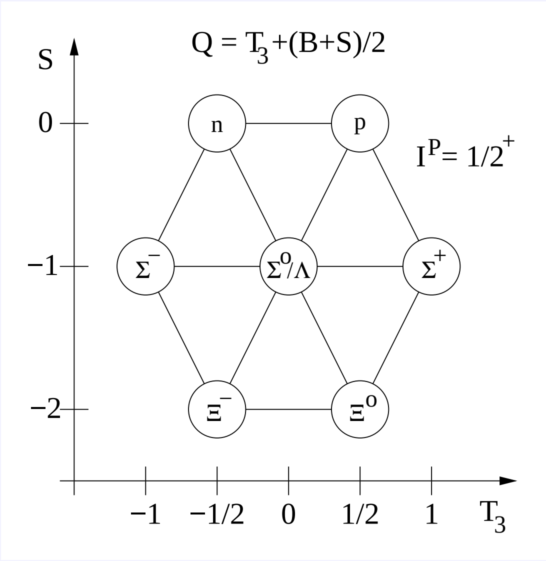

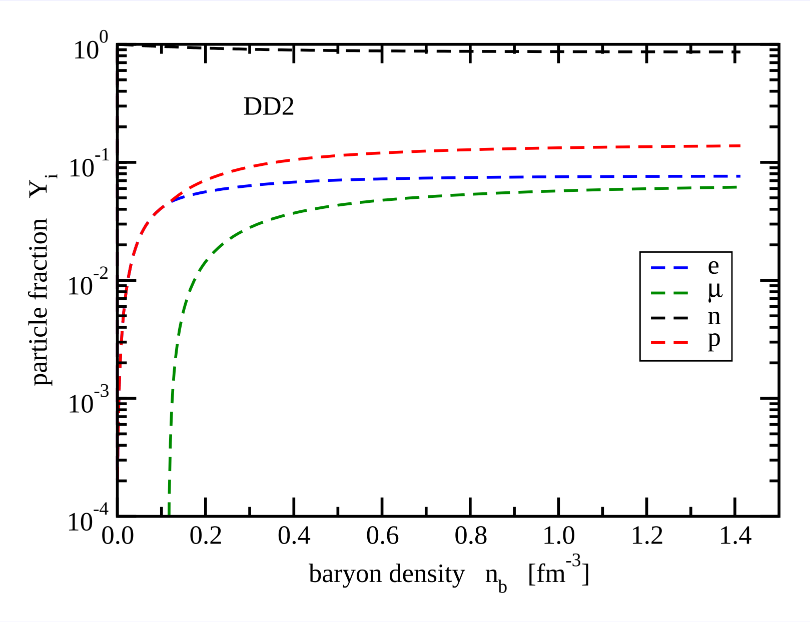

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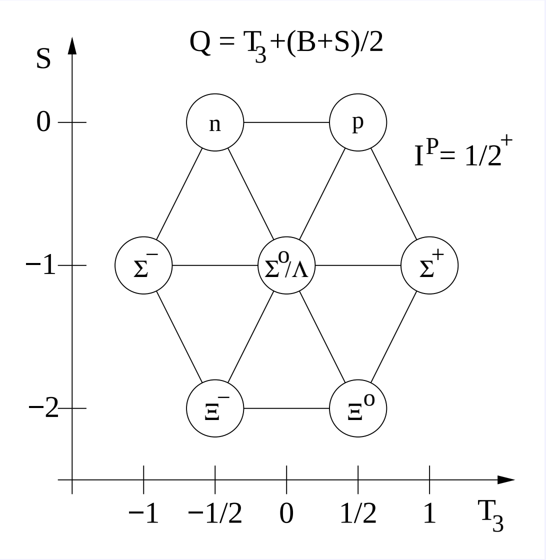

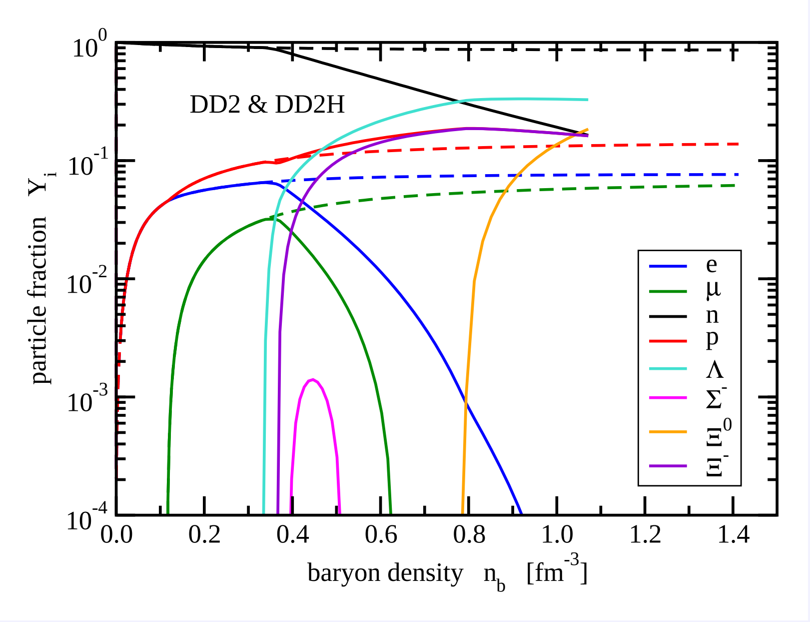

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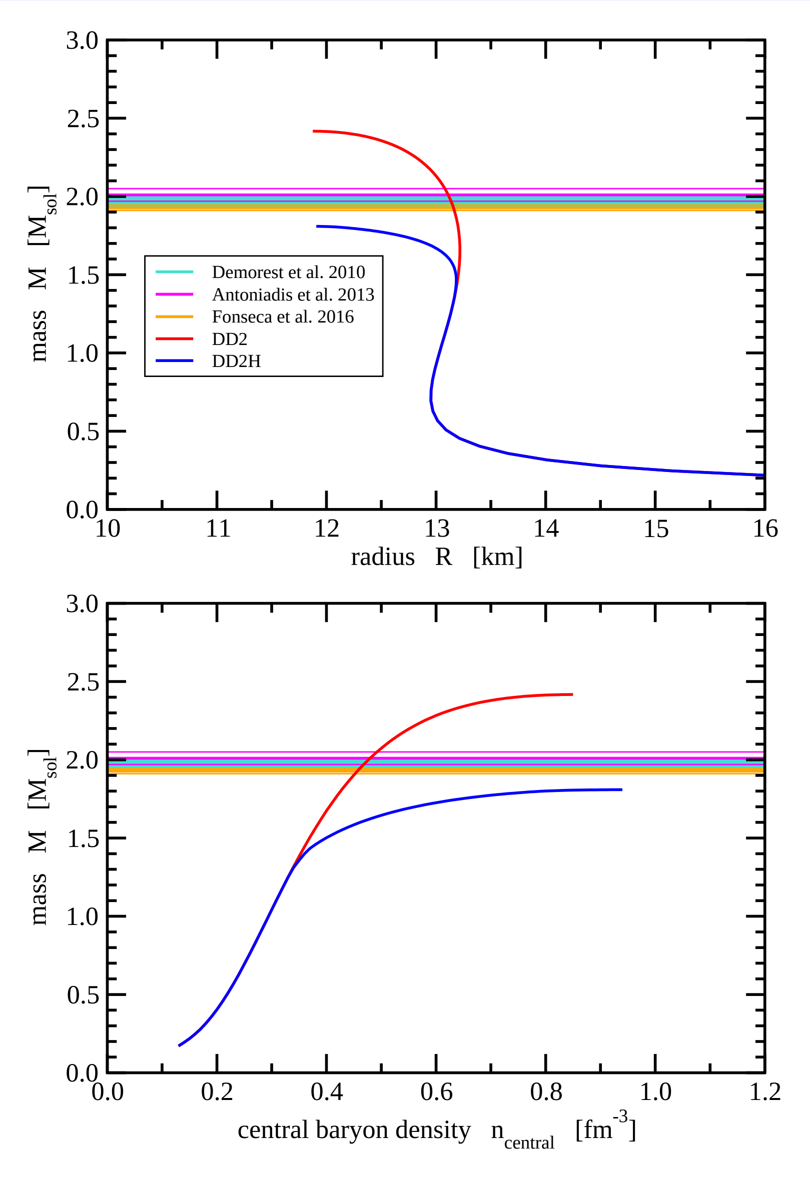

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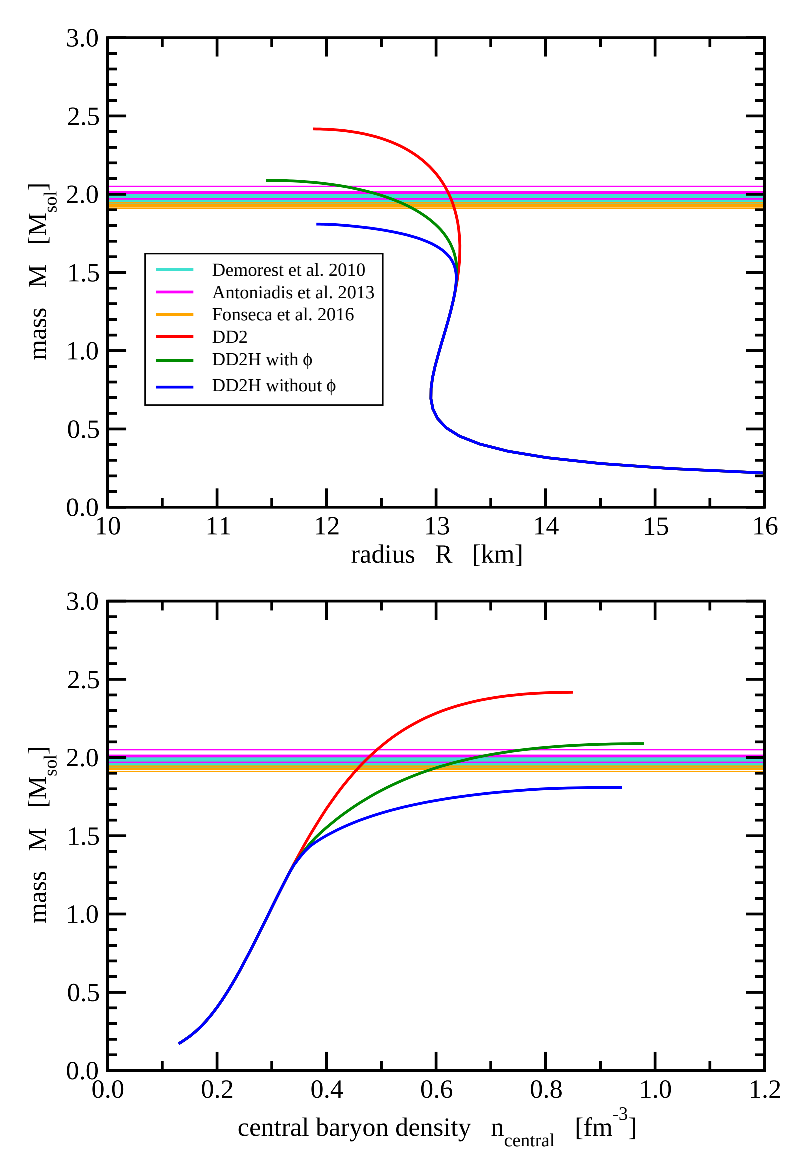

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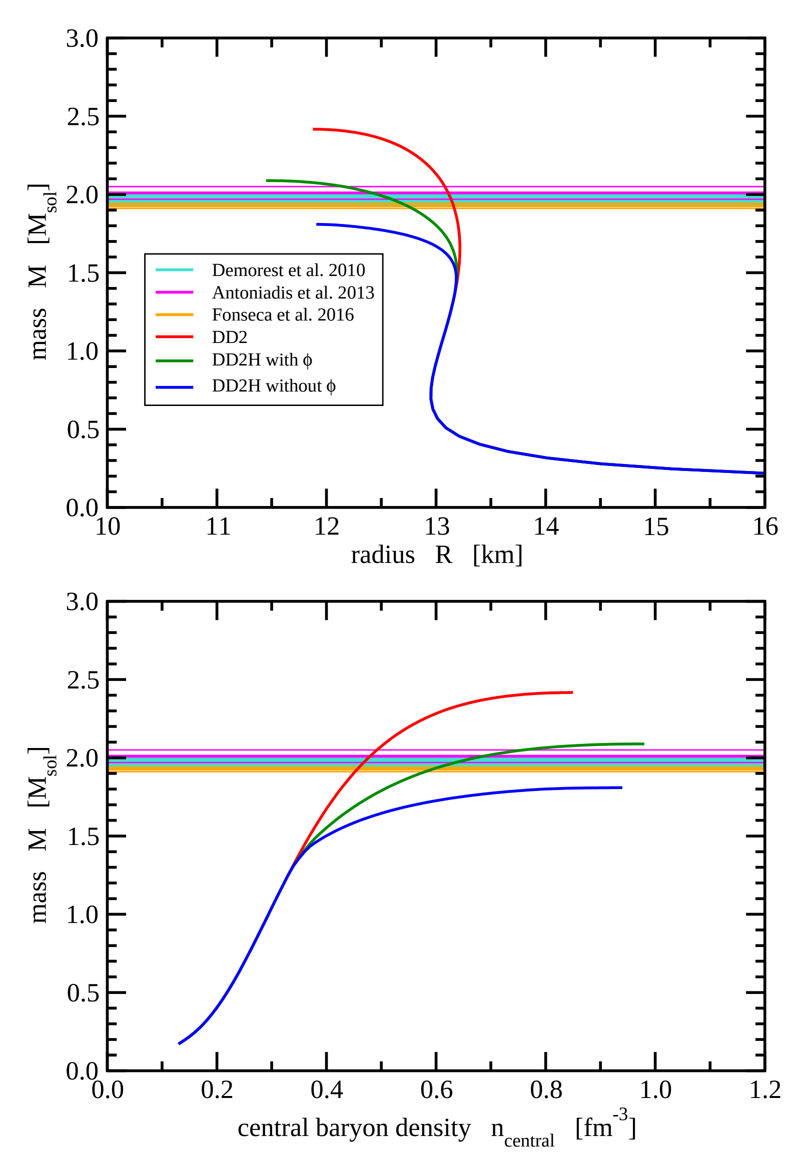
- alternative solutions?
  - transition to quark matter at low densities?
  - . . .



Optical Potential Constraint

- nucleon optical potential from p-A scattering

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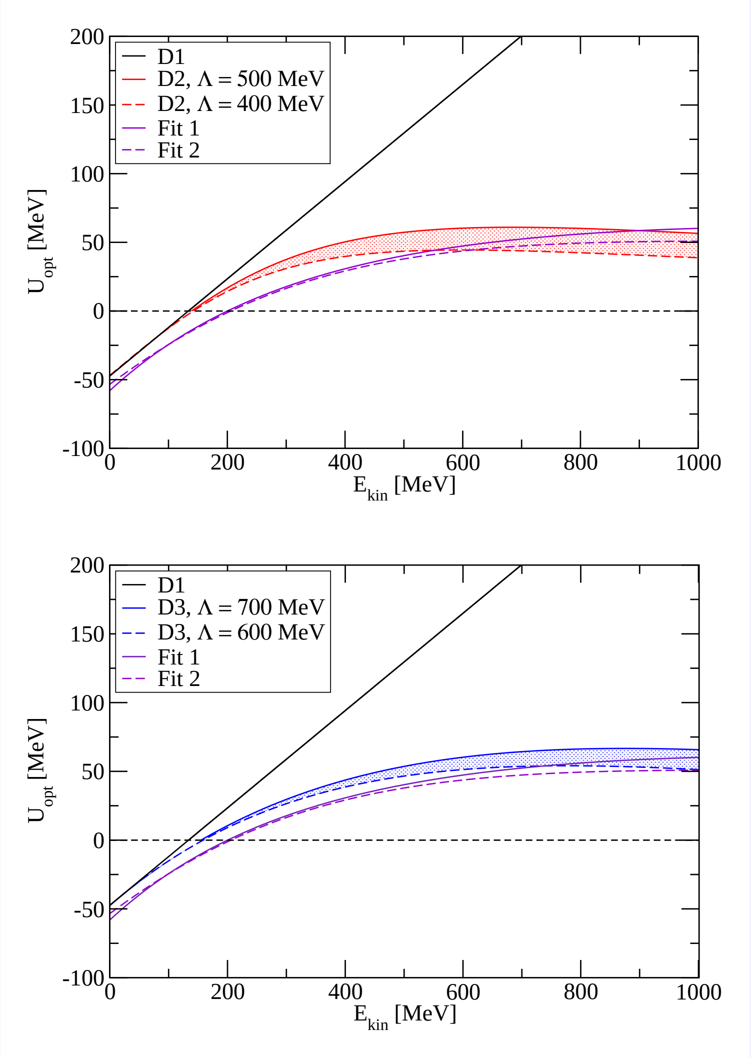

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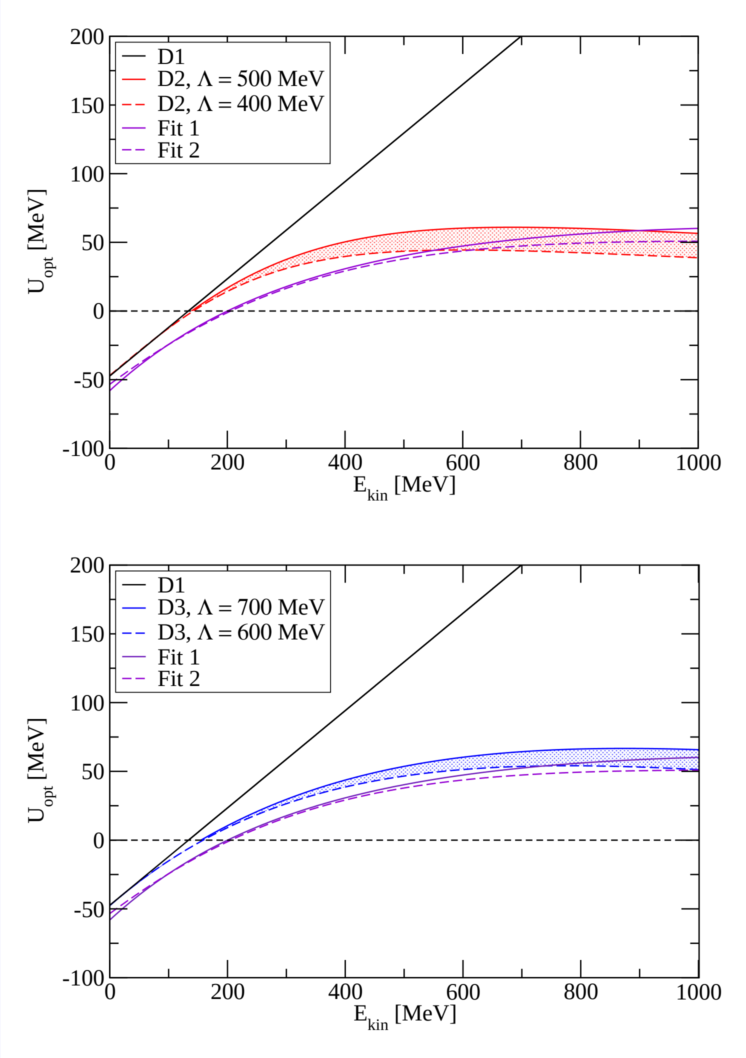

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 Stefan Typel

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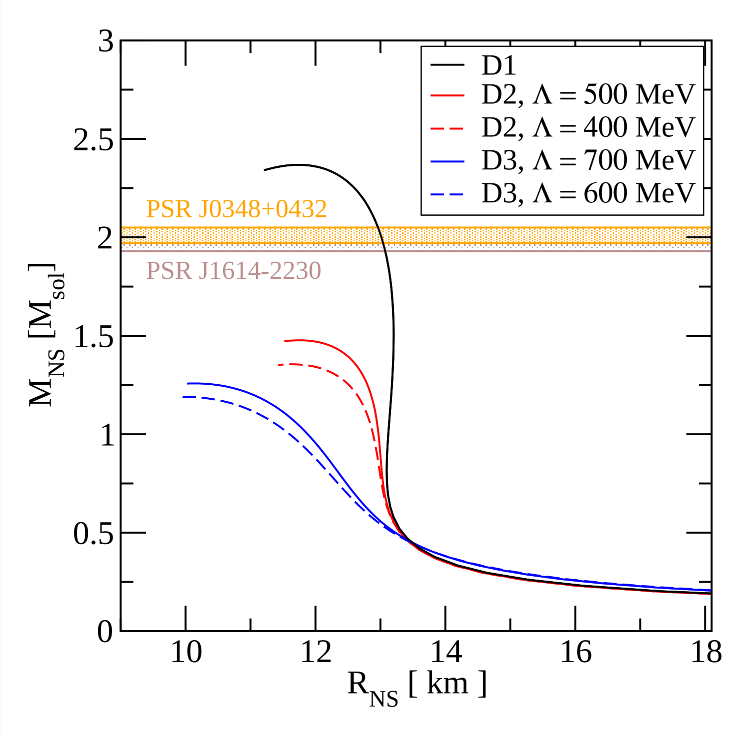

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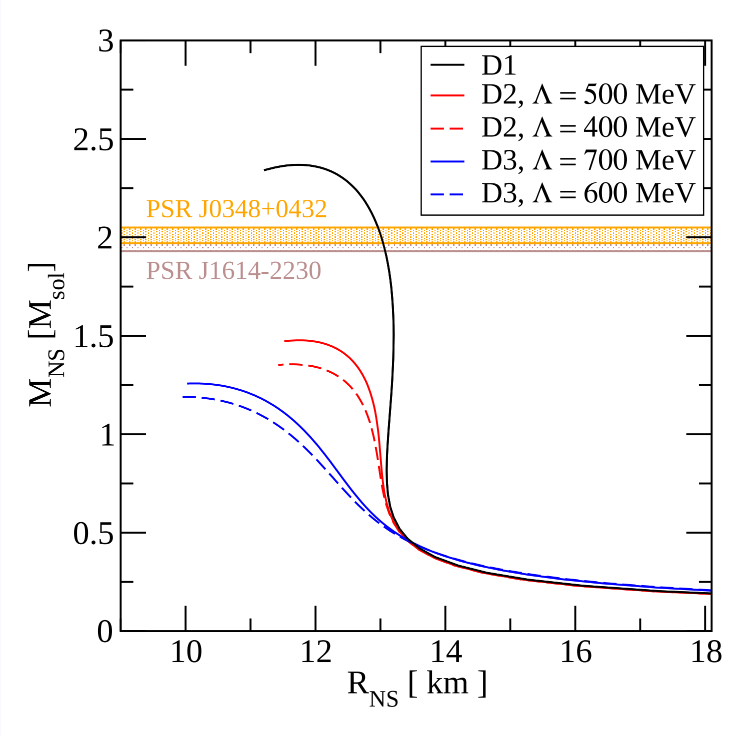
- fit of model parameters to properties of finite nuclei needed (in progress)



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- outlook:
  - preparation of global EoS table ($\Rightarrow$ CompOSE)
  - extension of model with quarks ($\Rightarrow$ hadron-quark phase transition)
  - treatment of liquid/gas - solid phase transition ($\Rightarrow$ crystallization)