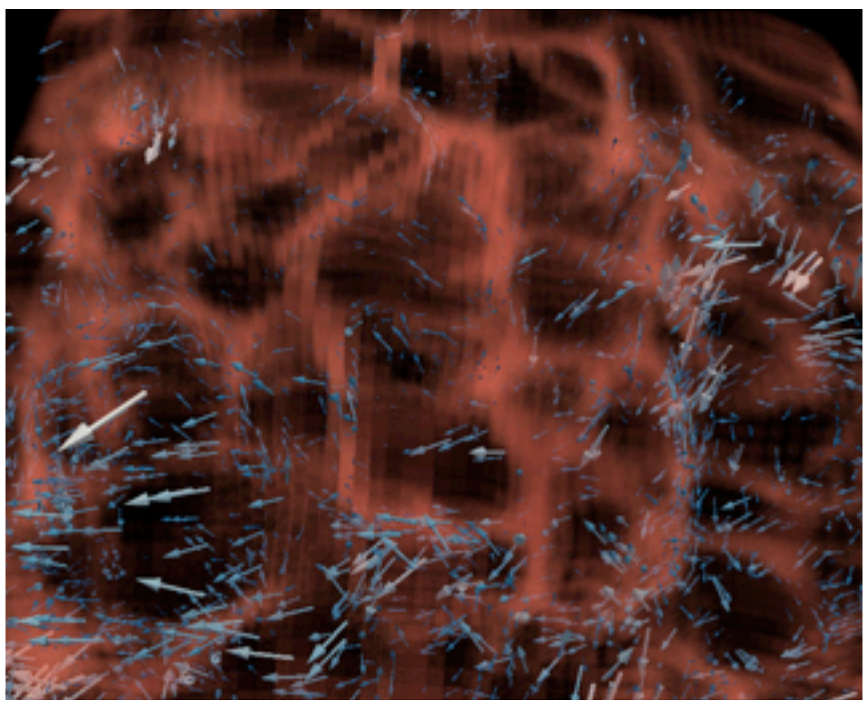
Modelling of PWN: Magnetohydrodynamics and particle transport (in three dimensions)



Oliver Porth

with Serguei Komissarov,
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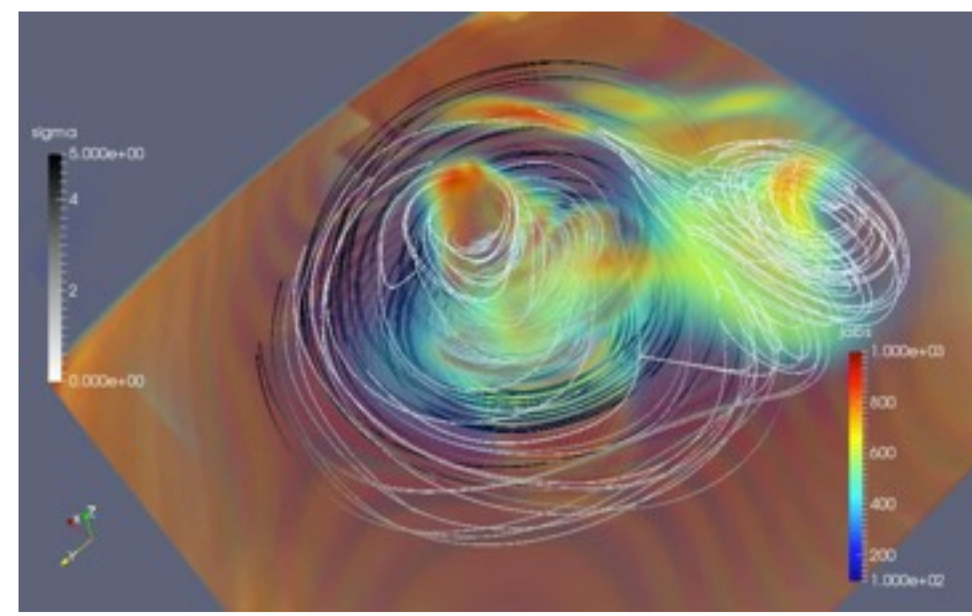


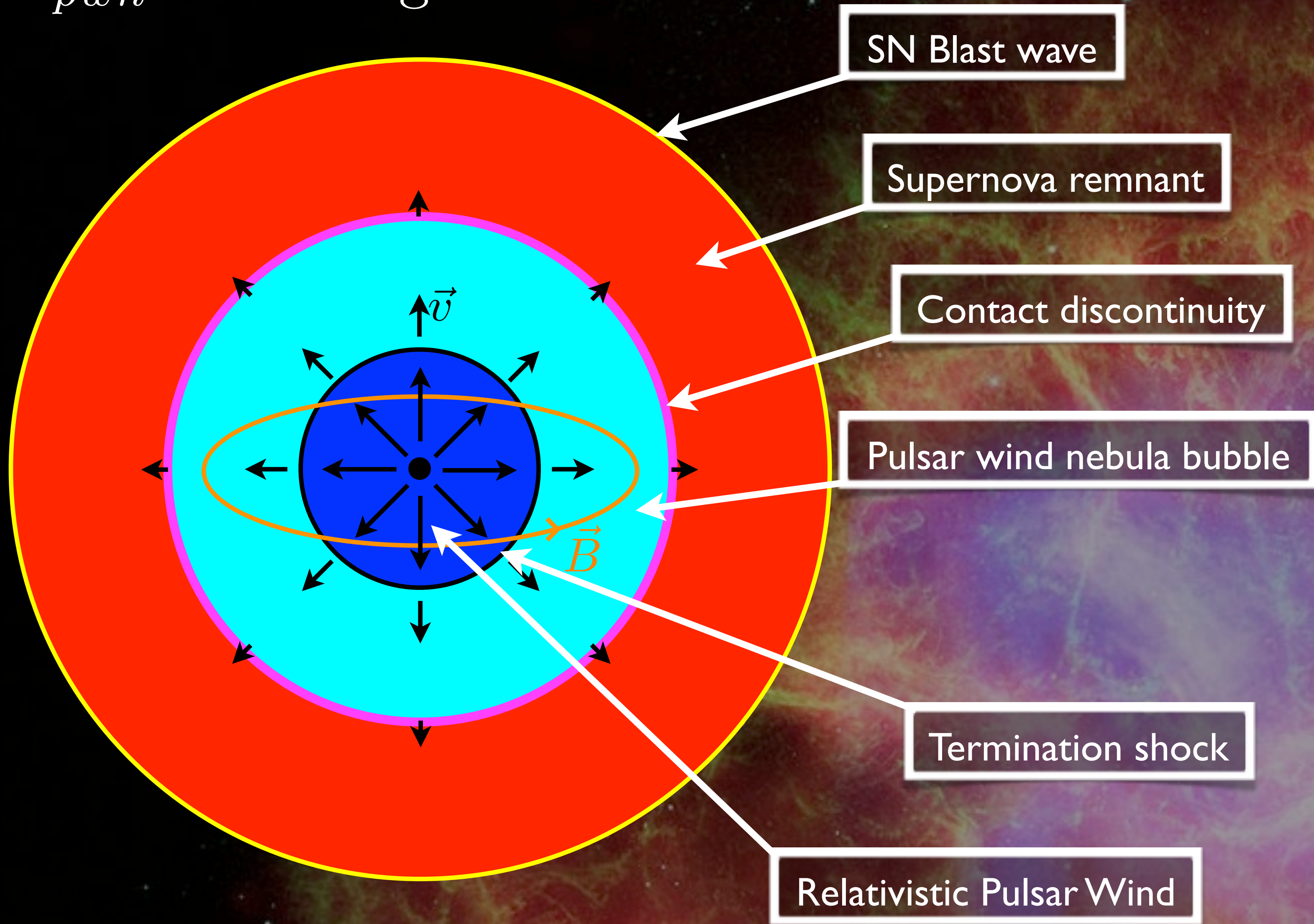
Overview

- PWN: Our favorite relativistic plasma laboratory in the sky
  - From the spherical cow to 3D dynamical models
- Electron transport in turbulent PWN
  - Test particle simulations and Fokker-Planck
  - Back to the spherical cow: Fitting of observations

$$E_{snr} = 10^{51}\mathrm{erg}$$

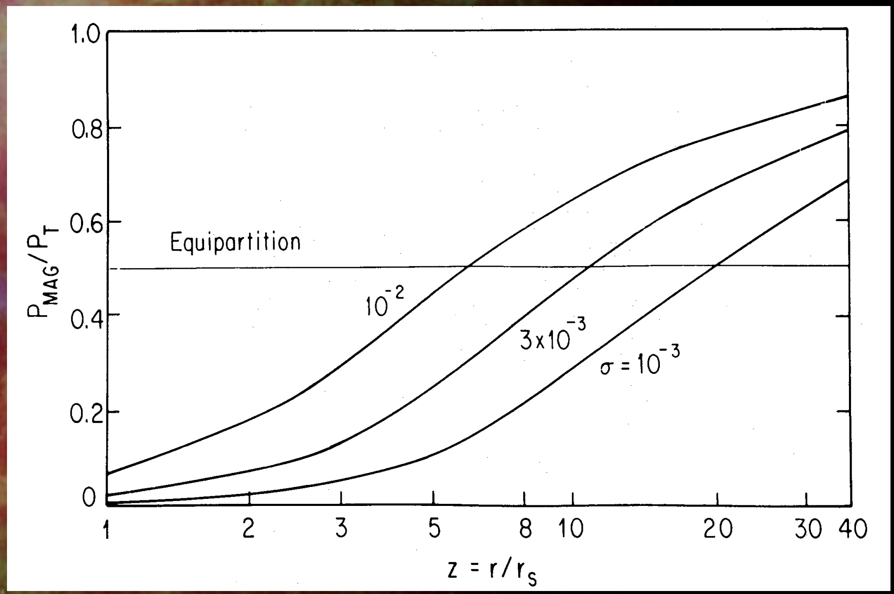
$$E_{pwn} = 10^{49}\mathrm{erg}$$

1D Model



Rees & Gunn (1974), Kennel & Coroniti (1984)

- particle dominated relativistic pulsar wind with purely azimuthal magnetic field terminates at shock
- sub-sonic nebula flow velocity decreases to match speed of remnant
- magnetic field increases towards the outer boundary of the nebula



- electrons are accelerated at the termination shock to relativistic energies according to $n \propto E^{-2.2}$
- loose energy due to synchrotron and inverse Compton emission. => Successful to model spectrum from visible to γ-rays

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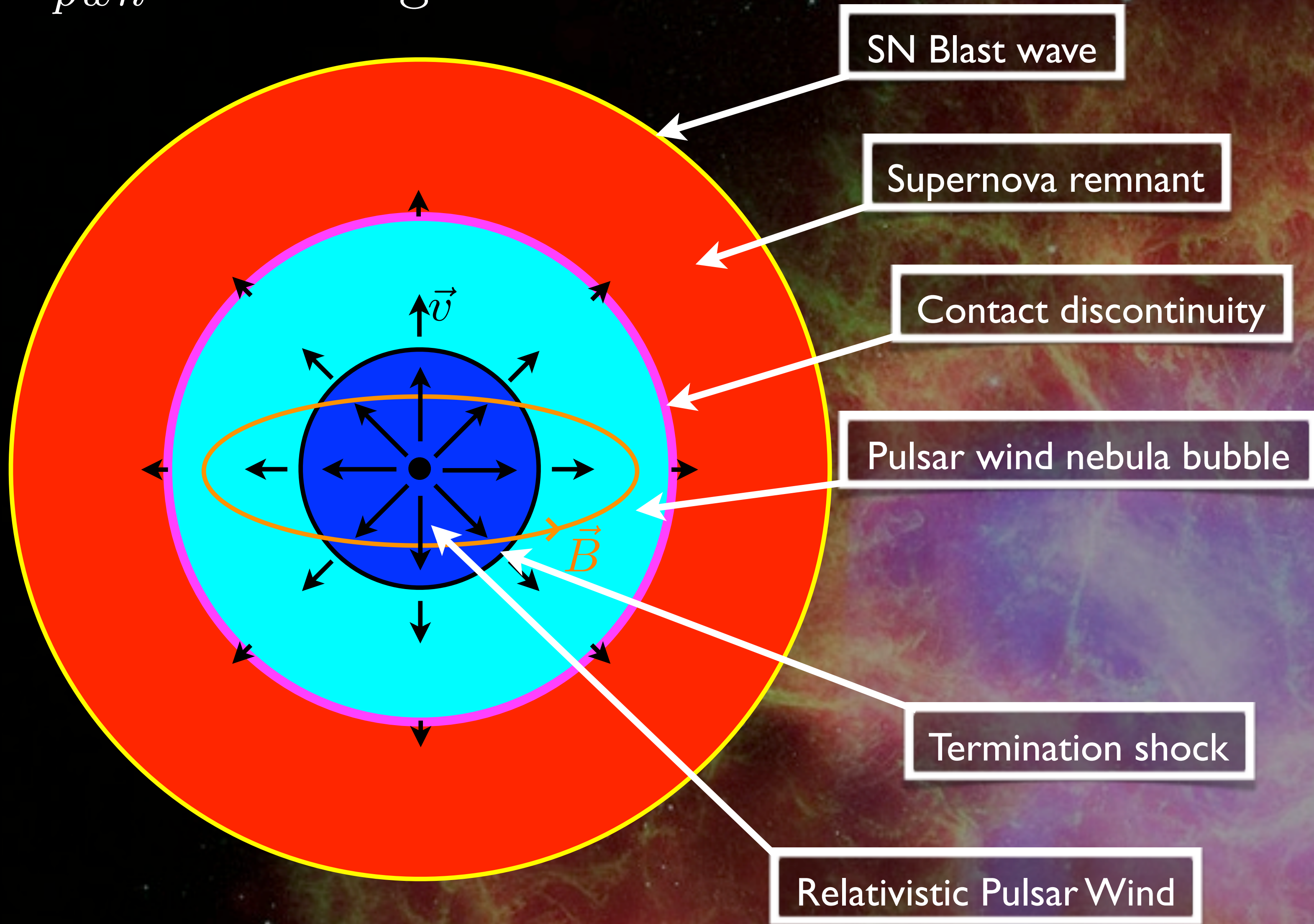

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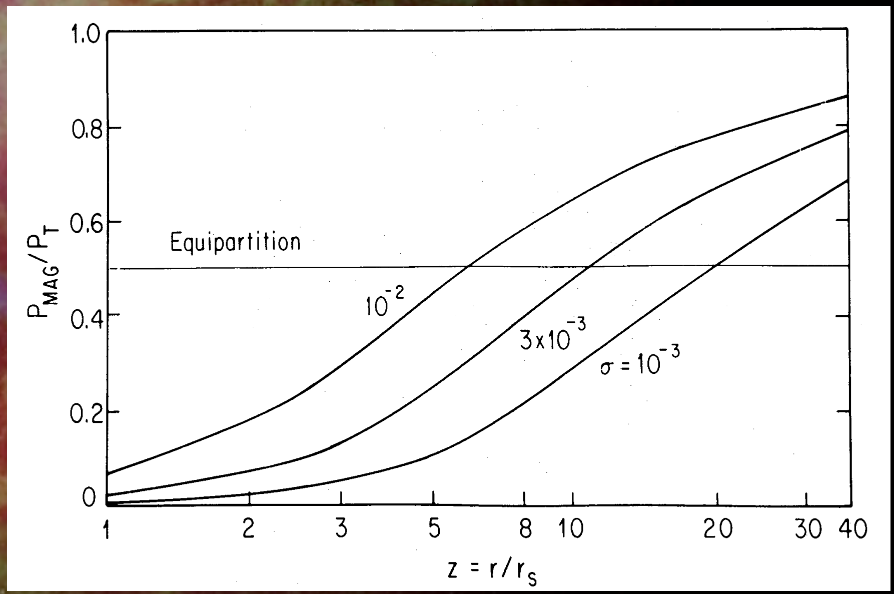

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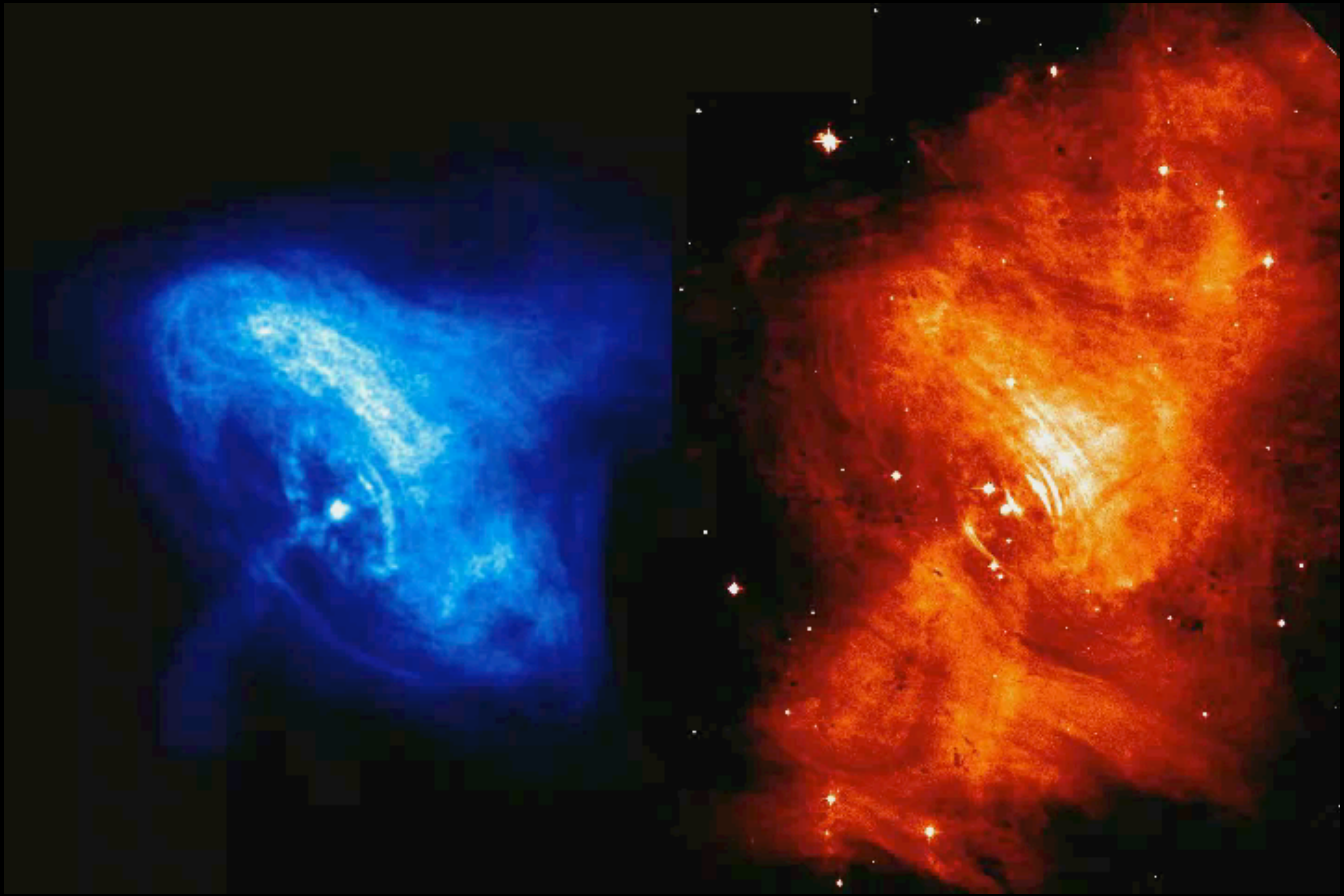
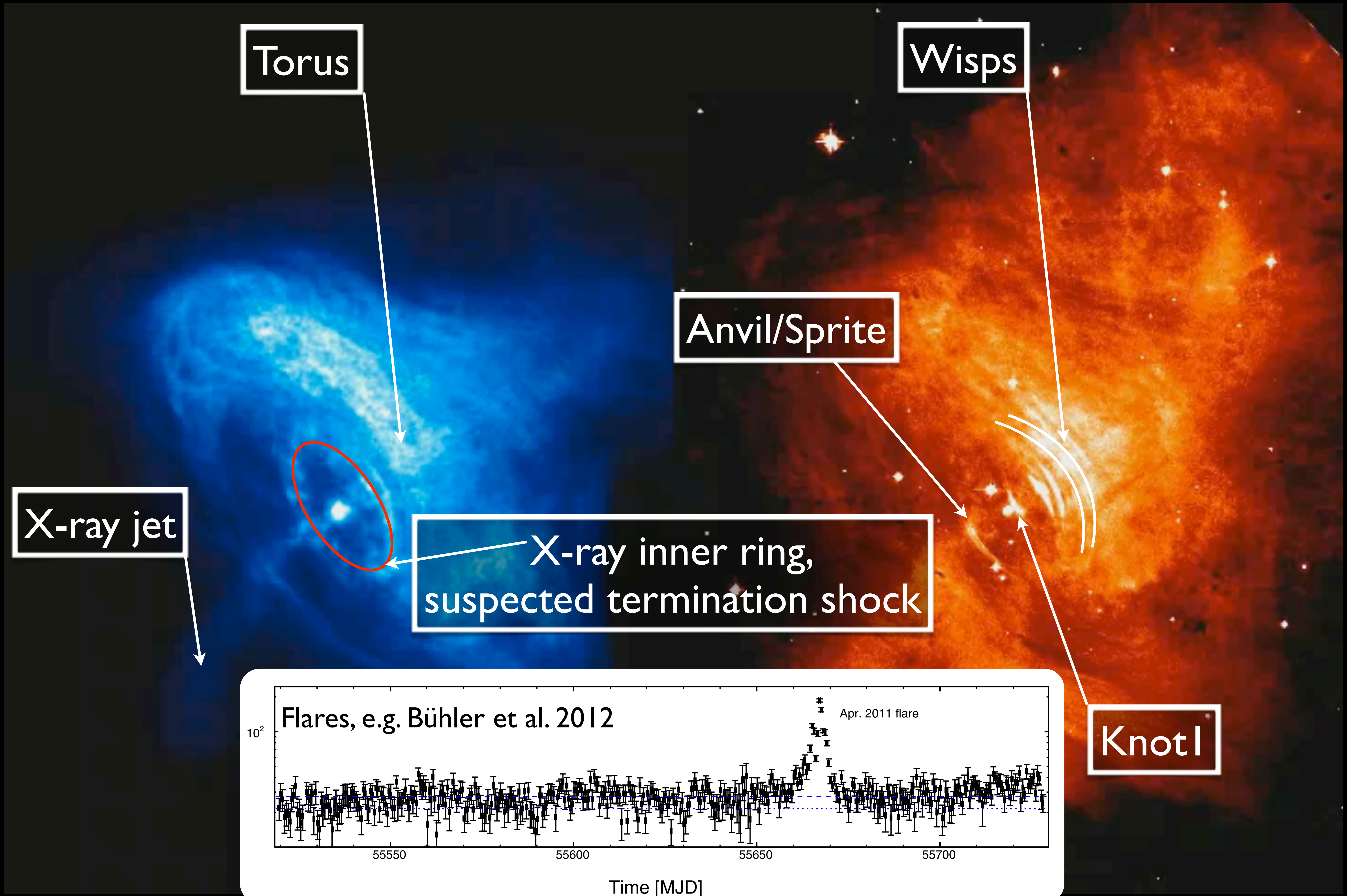
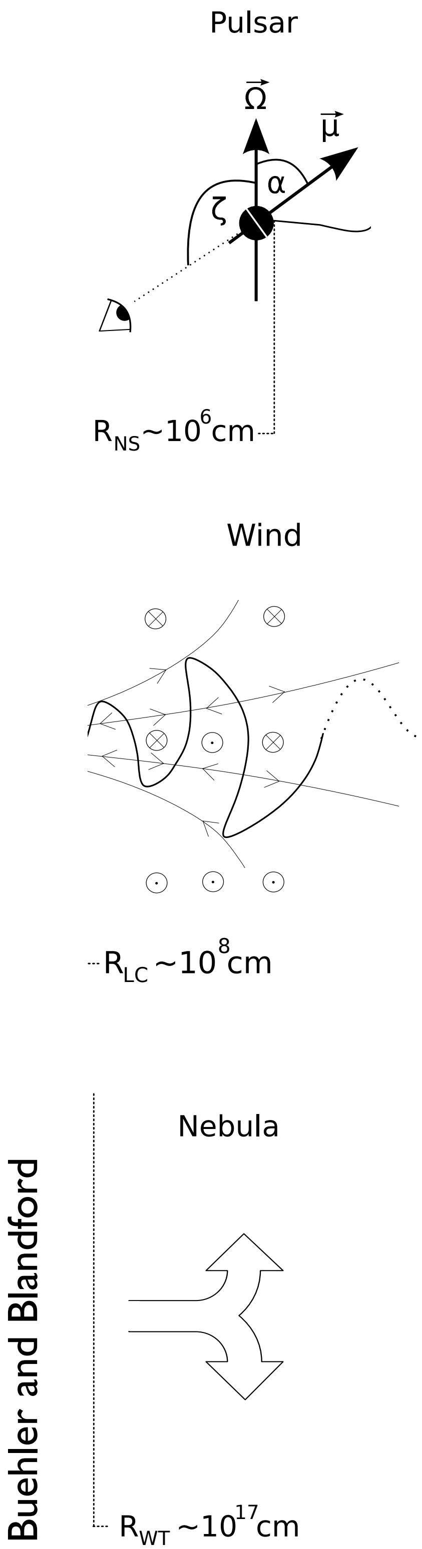


Credits: X-ray: NASA/CXC/ASU/J.Hester et al.; Optical: NASA/HST/ASU/J.Hester et al.



The general morphology could already be modeled in 2D: Del Zanna, Komissarov...

$$\sigma \equiv \frac{f_{\rm m}}{f_{\rm k}} = \frac{c}{4\pi}\frac{|\mathbf{E}\times\mathbf{B}|}{|\Gamma^2 \rho h \mathbf{v} c^2|} \simeq \frac{1}{4\pi}\frac{B'^2_\phi}{\rho c^2}$$



- Pulsar magnetospheres: Poynting dominated wind at light cylinder $\sigma \sim 10^4$ (high sigma)
  e.g. Michel 1982, Arons 2007
- Uncollimated ideal RMHD wind is inefficient at bulk flow acceleration and remains Poynting dominated $\sigma \gtrsim 1$ (high sigma)
  Tomimatsu 1994, Beskin et al 1998, Komissarov 2009, Lyubarski 2010
- Yet 1D and 2D dynamical RMHD of the nebula models require particle dominated winds at the termination shock $\sigma \sim 10^{-3}$ - 10^{-2} (low sigma)
  Rees & Gunn (1974), Kennel & Coroniti 1984, Komissarov+ 2004, Del Zanna+ 2004, Volpi+ 2008, Camus+ 2009, Olmi+ 2014

(Dynamical) Modelling problem:

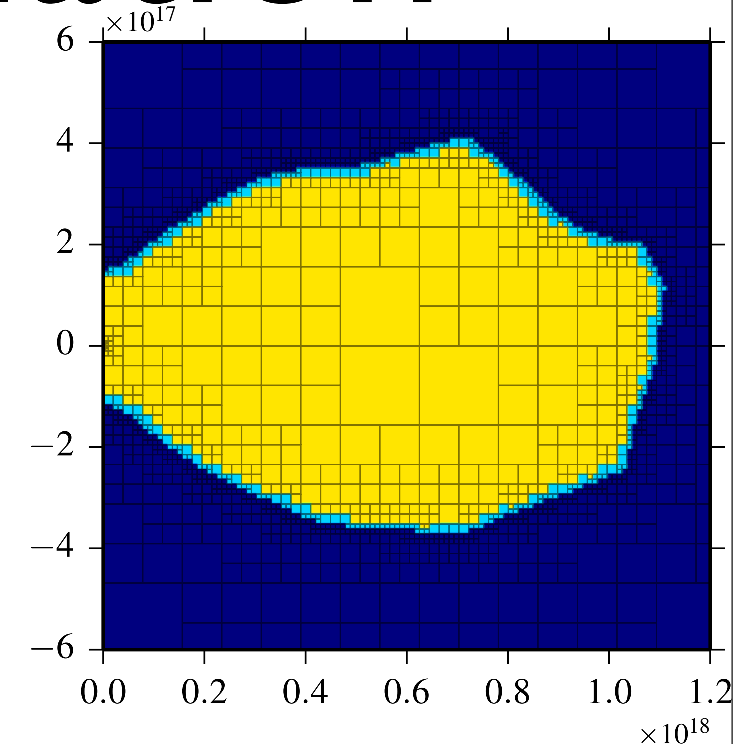
Can't match shock size and expansion speed at same time with high sigma!

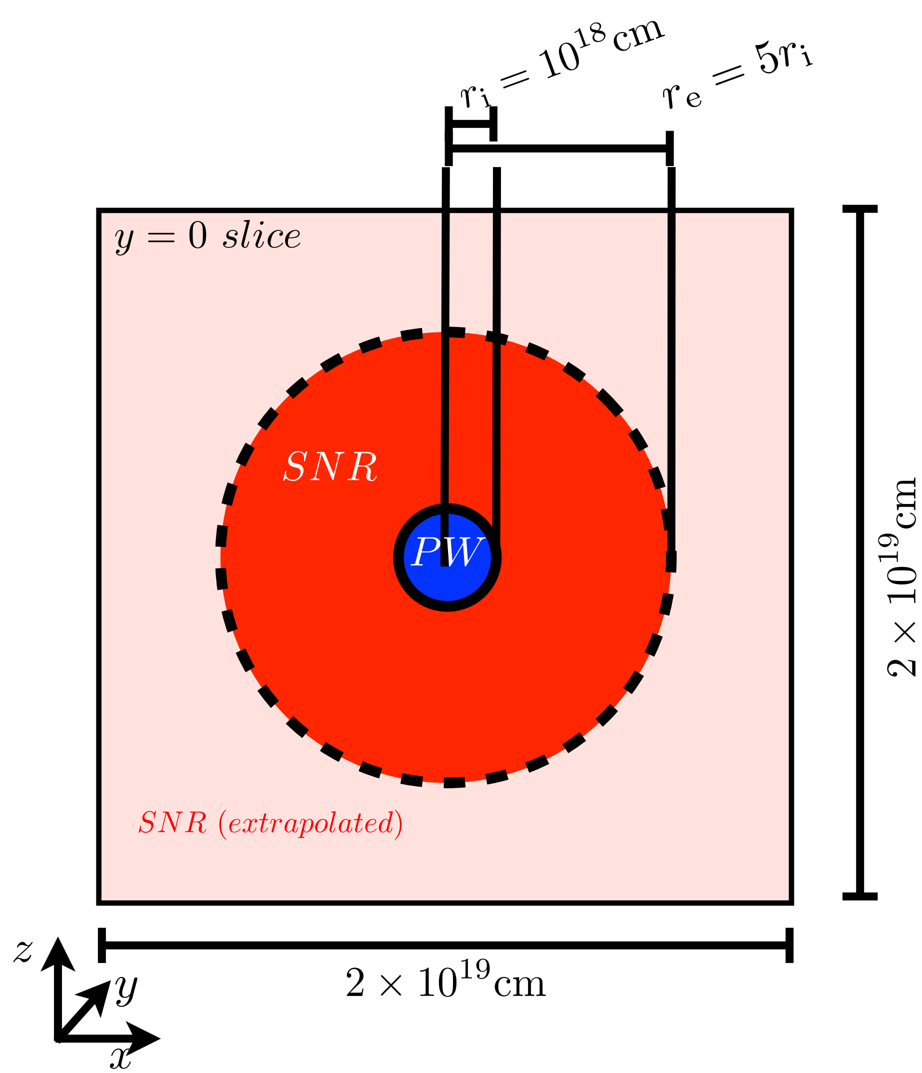
Setup of PWN simulation

Numerical details:

Domain:
3D Cartesian box, 20 lightyears^3
MPI-AMRVAC[1]
ideal RMHD,
Minkowski spacetime,
ideal gas EOS with γ=4/3

Adaptive mesh refinement:
Base resolution 64^3
PWN on level 5-6; hllc lim03
Termination shock on level 8-10; tvdlf minmod



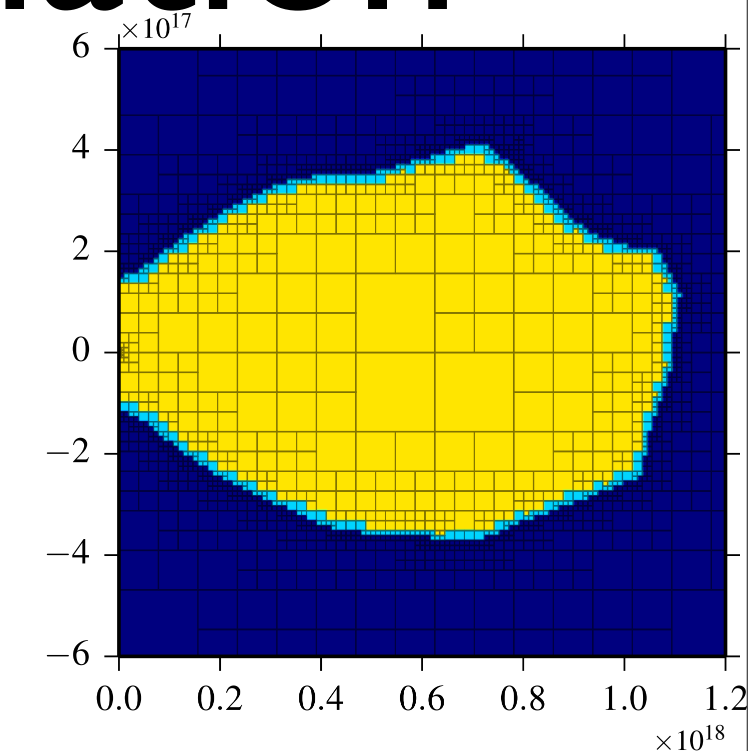


- Initialize relativistic Pulsar Wind inside expanding Supernova remnant
- PW much lighter than SNR => Shock structure (termination shock, contact and forward shock) forms
- We study the dynamics downstream of the termination shock

Coroniti 1990, Lyubarski 2003, Sironi & Spitkovsky 2011, Amano & Kirk 2013, Tchekhovskoy et al. (2016)

[1] https://gitlab.com/mpi-amrvac/amrvac

Setup of PWN simulation



- Pulsar wind setup

$L_{\mathrm{tot}} = 5 \times 10^{38}\,\mathrm{erg\,s^{-1}}$ $\Gamma = 10$ Lorentz factor in PW

Anisotropic total energy flux

$$f_{\mathrm{tot}}(r,\theta) = \frac{1}{r^2}(\sin^2\theta + b) \quad , b = 0.03$$

$$f_{\mathrm{m}}(r,\theta) = \sigma(\theta)\frac{f_{\mathrm{tot}}(\theta, r)}{1+\sigma(\theta)}$$

$$f_{\mathrm{k}}(r,\theta) = \frac{f_{\mathrm{tot}}(r,\theta)}{1+\sigma(\theta)}$$

Assume: stripes are entirely dissipated

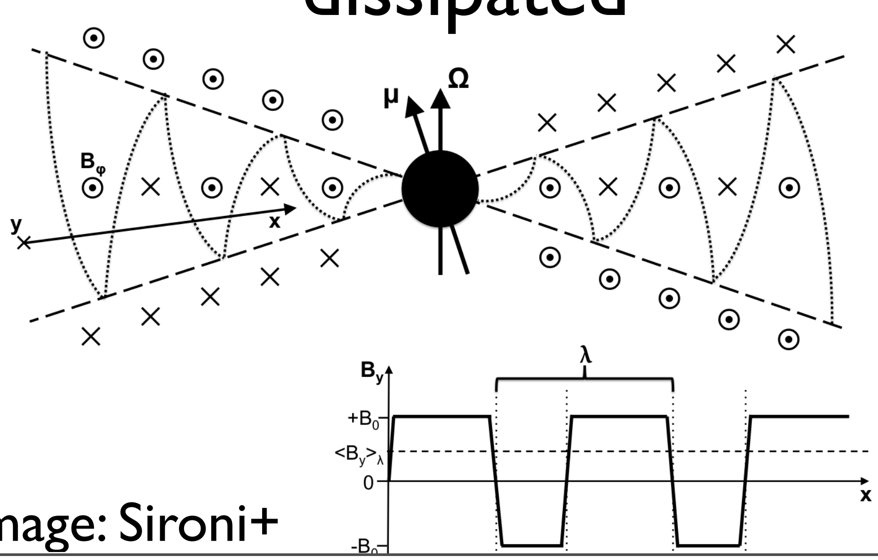
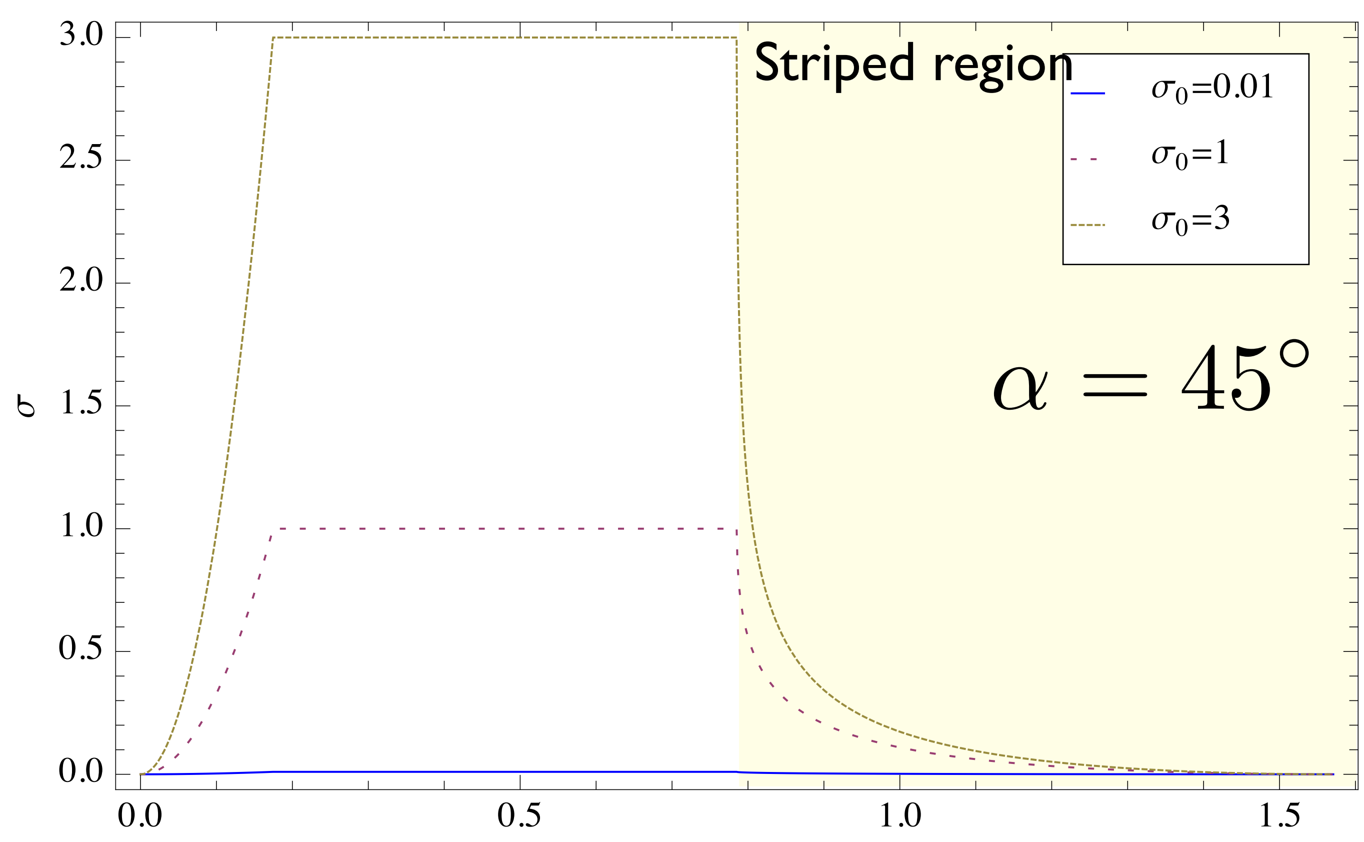


Image: Sironi+



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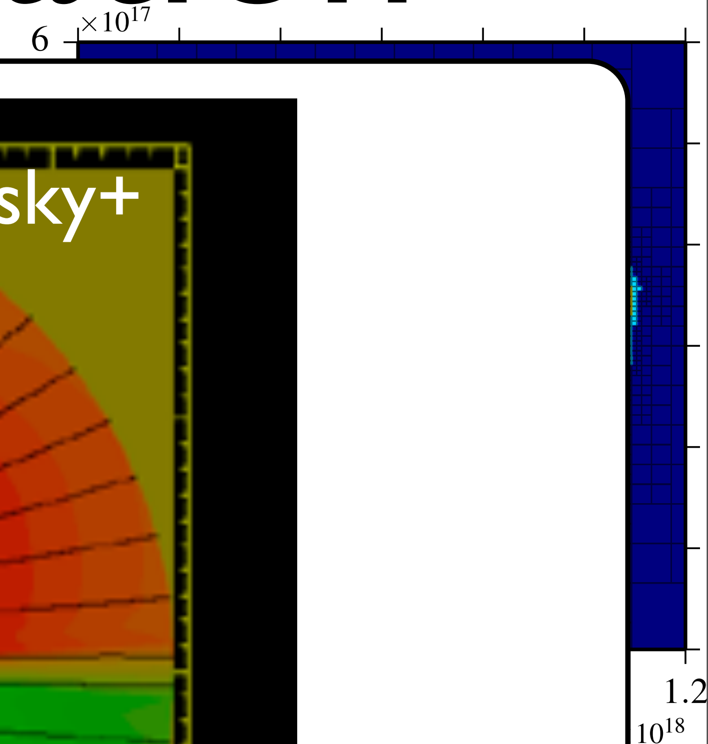
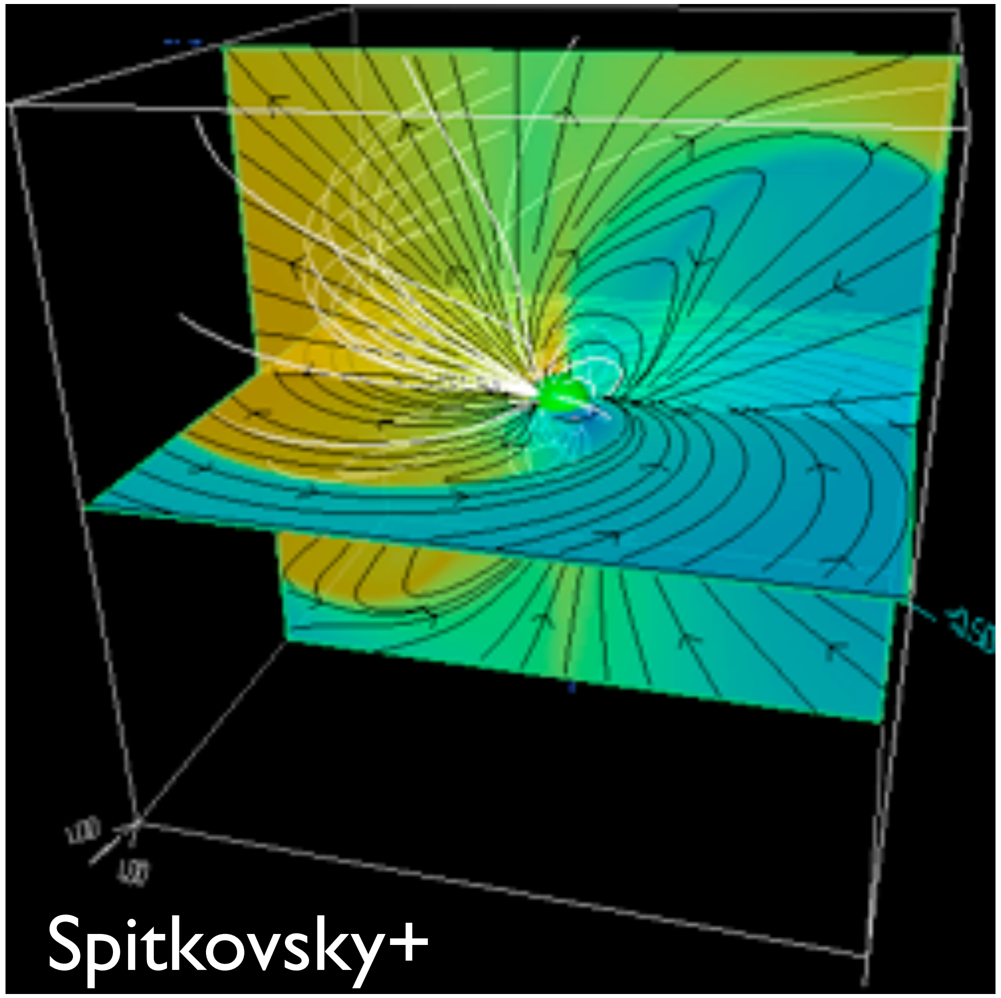
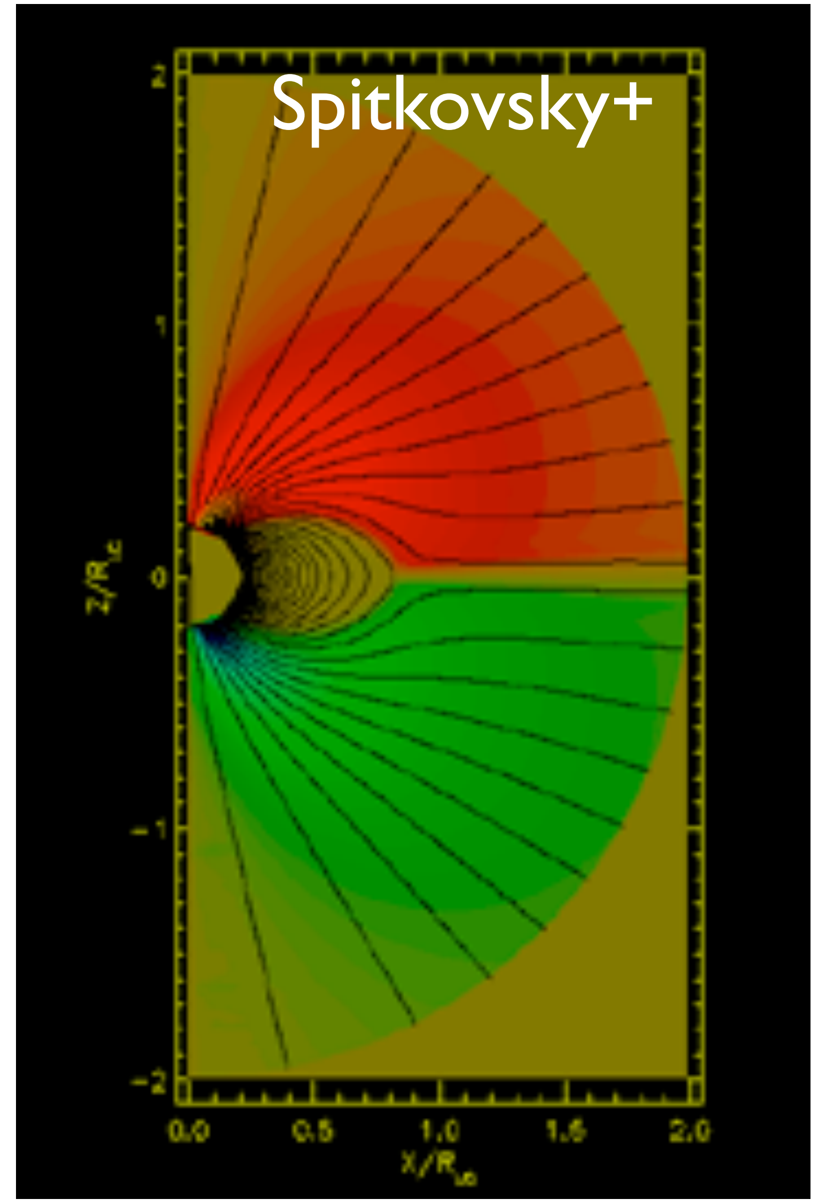
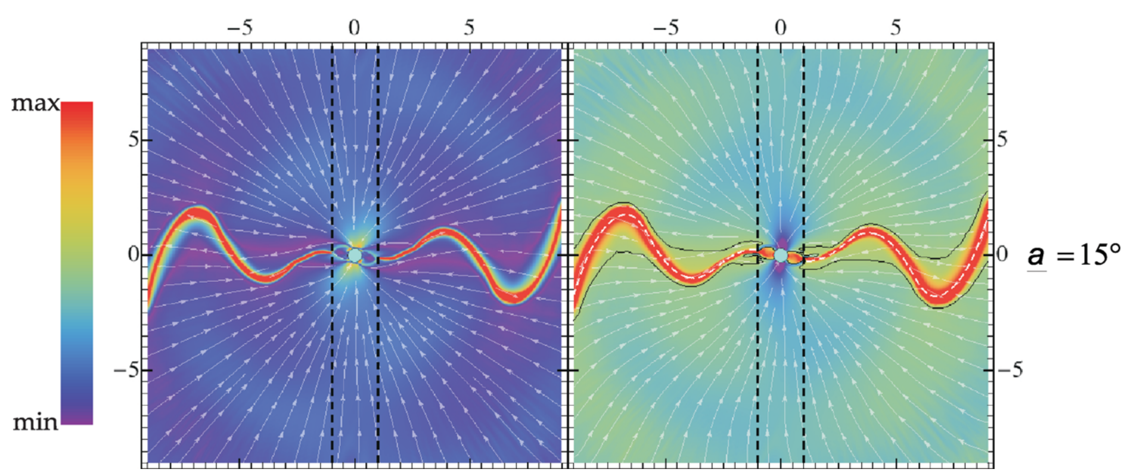


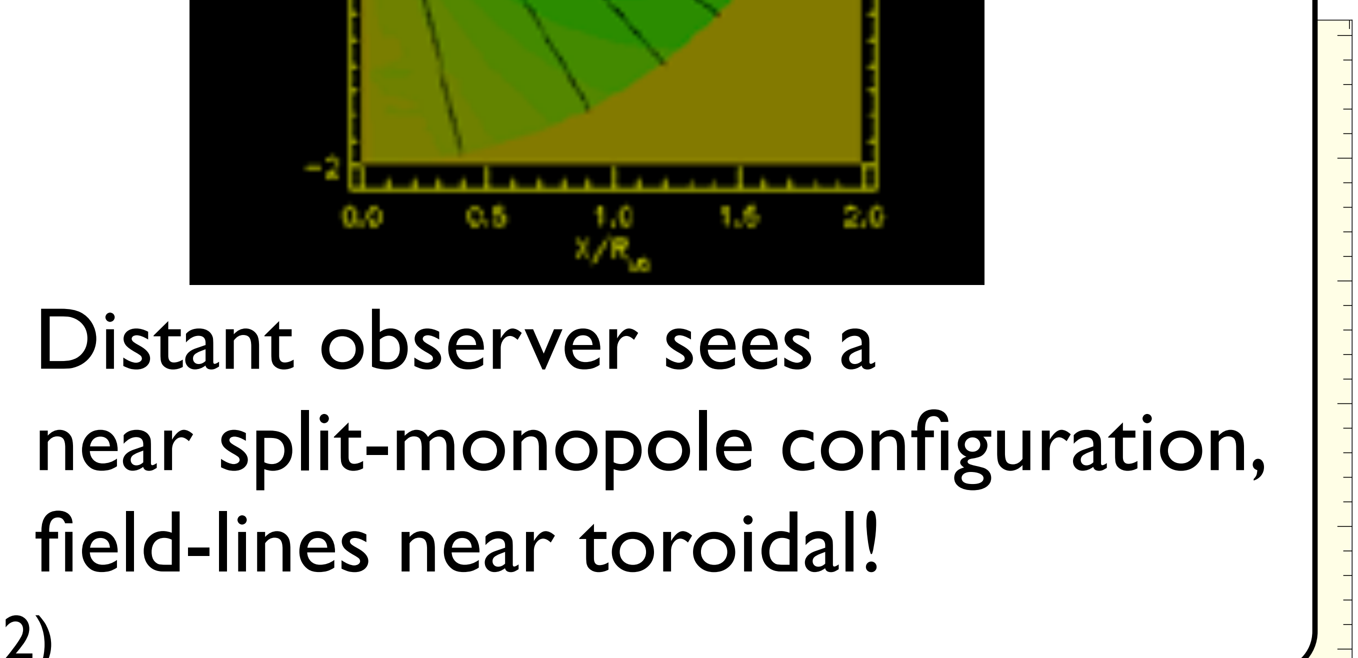


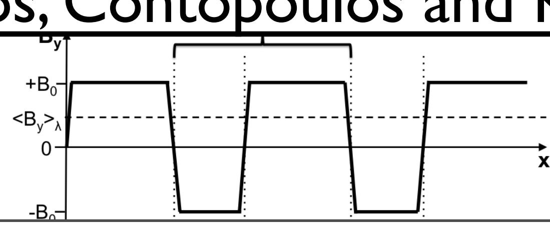




Distant observer sees a near split-monopole configuration, field-lines near toroidal!

Kalapotharakos, Contopoulos and Kazanas (2012)





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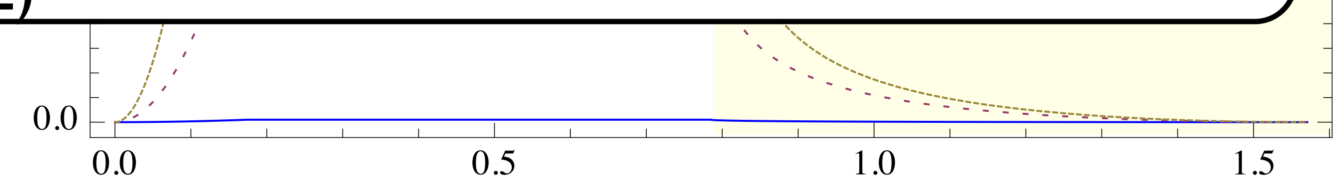

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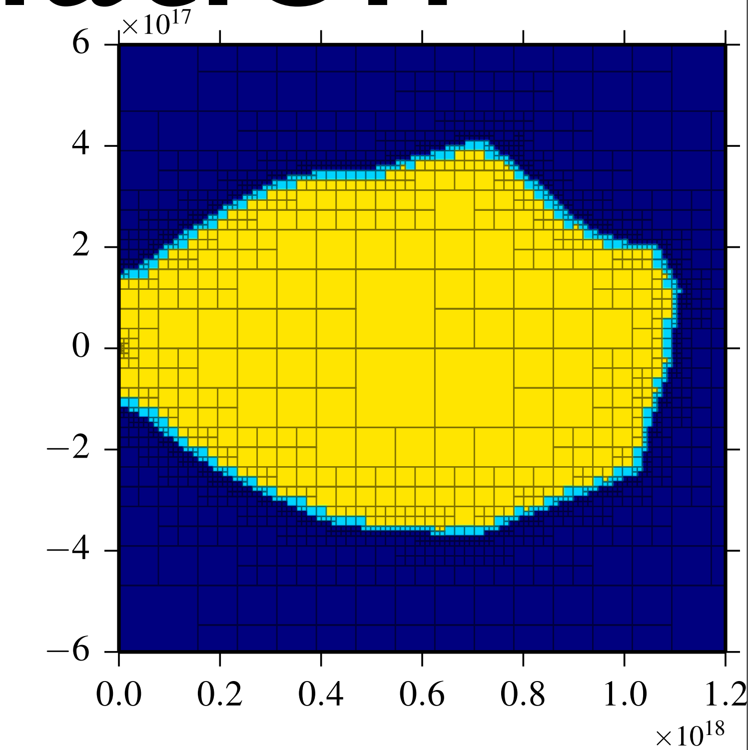

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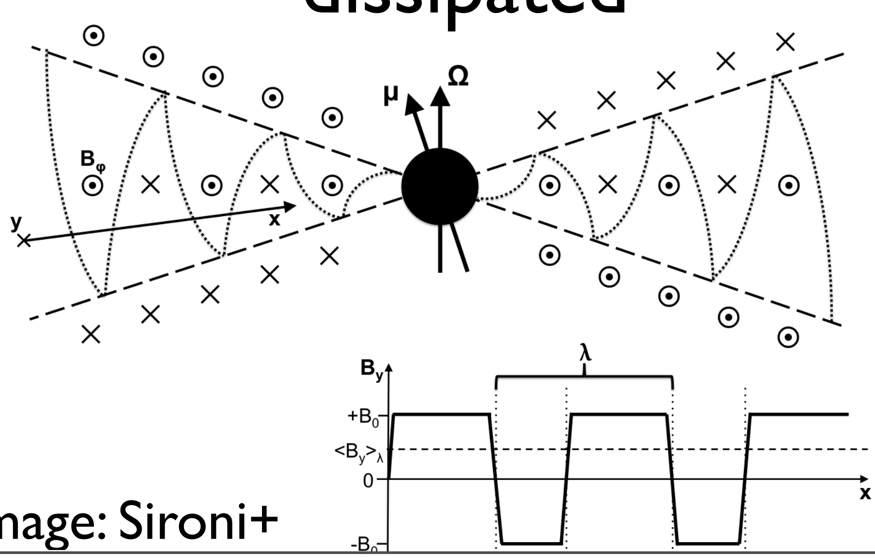

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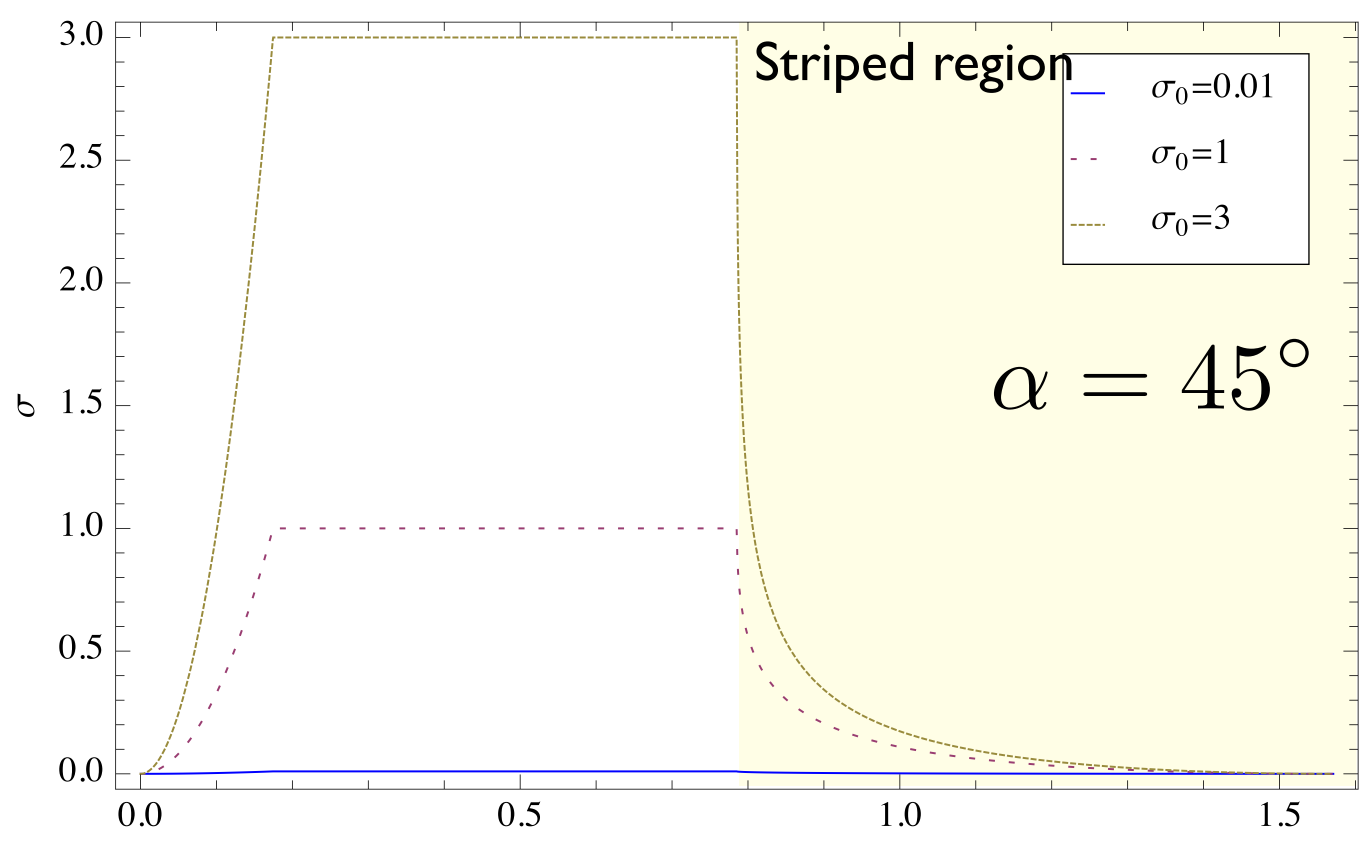

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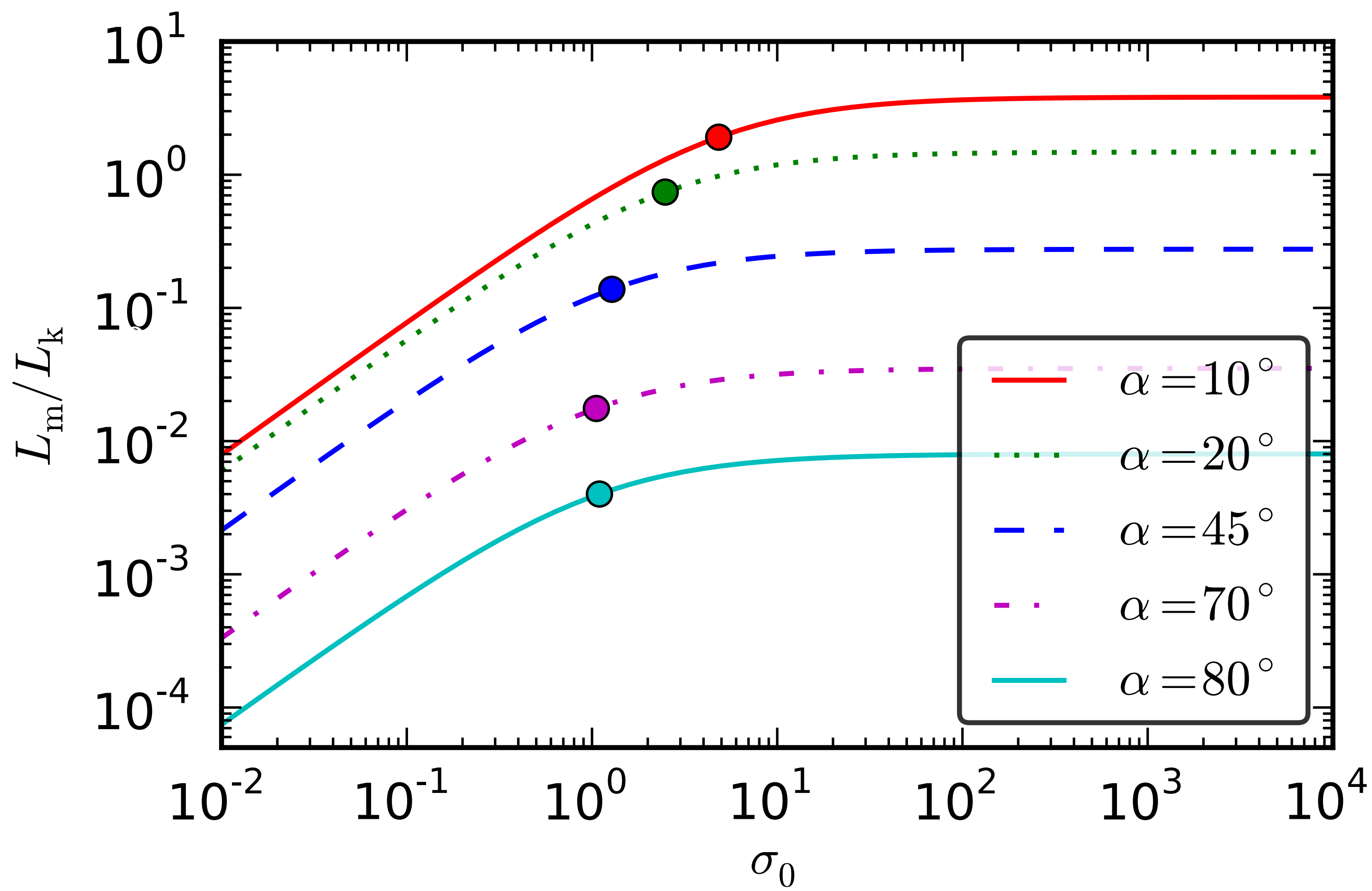
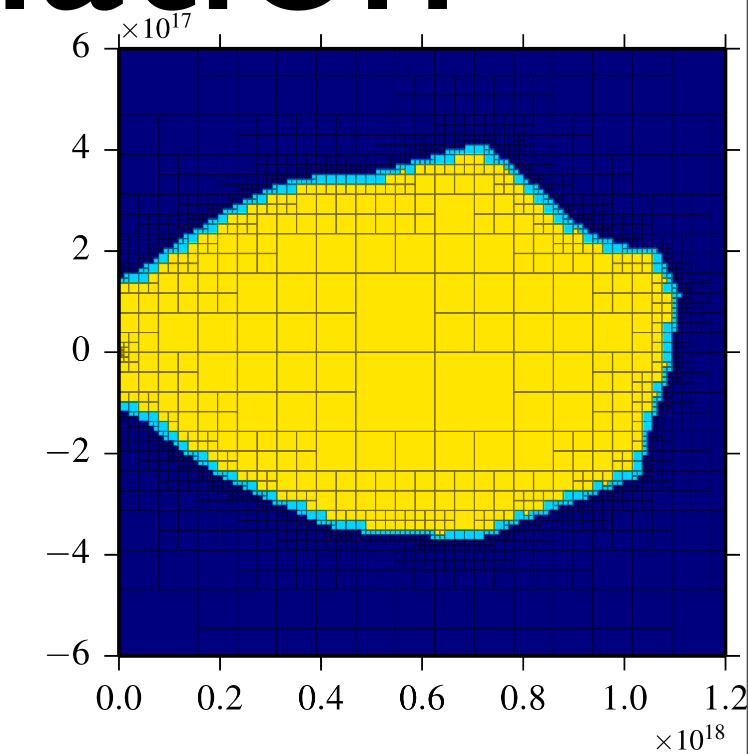




Magnetization after annihilation:

$$\sigma(\theta) = \frac{\tilde{\sigma}_0(\theta)\chi_\alpha(\theta)}{1 + \tilde{\sigma}_0(\theta)(1 - \chi_\alpha(\theta))}$$

Limits:

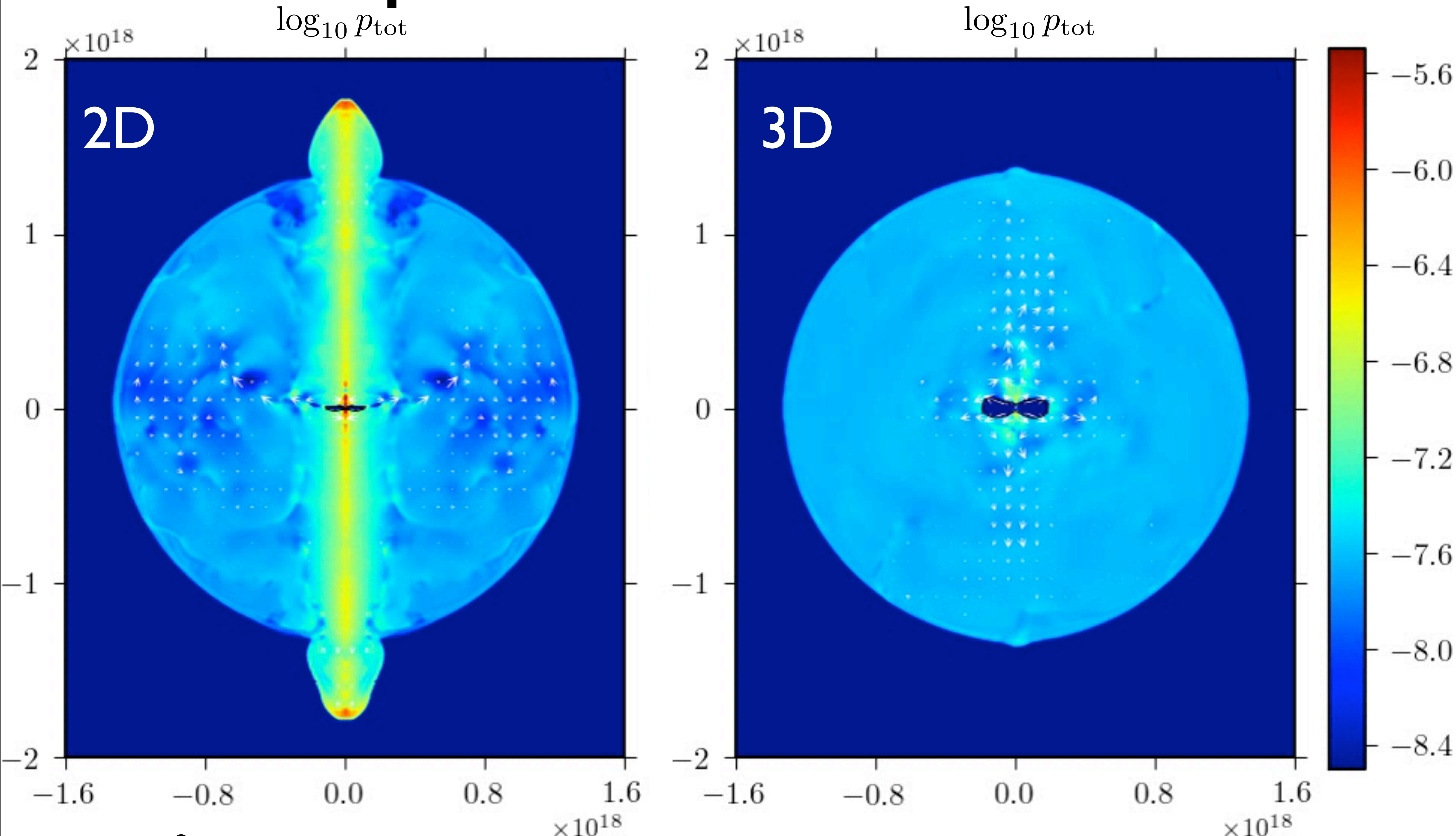
$$\sigma_1 \to \chi_\alpha/(1 - \chi_\alpha) \; (\tilde{\sigma}_0 \to \infty)$$

$$\sigma_1 \to \tilde{\sigma}_0 \chi_\alpha \; (\tilde{\sigma}_0 \to 0)$$

Coroniti 1990, Lyubarski 2003, Sironi & Spitkovsky 2011

Simulation results

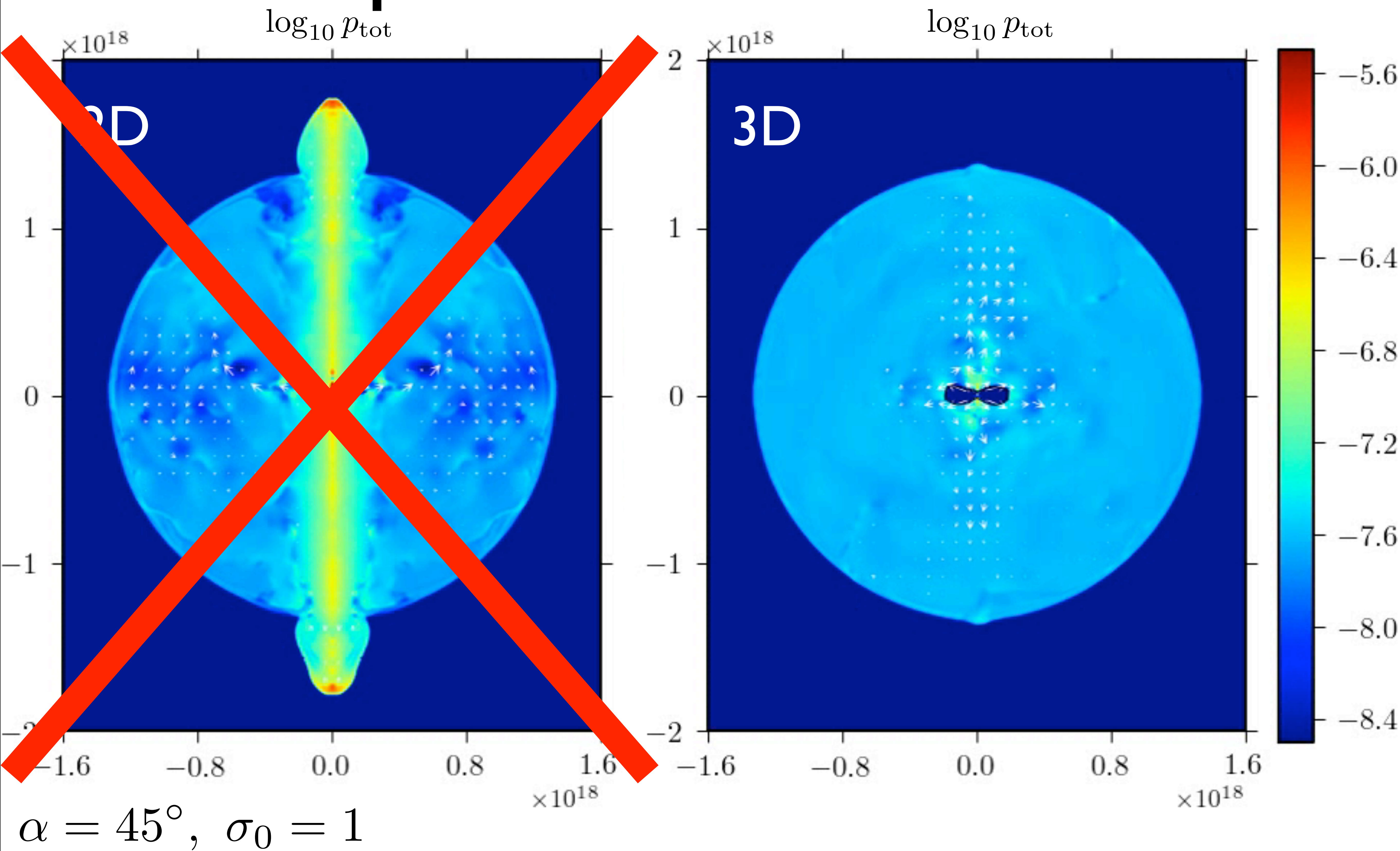
Total pressure in 2D and 3D



$\alpha = 45^\circ, \ \sigma_0 = 1$

Total pressure slices for consecutive simulation snapshots 51 years after start of simulation

Total pressure in 2D and 3D



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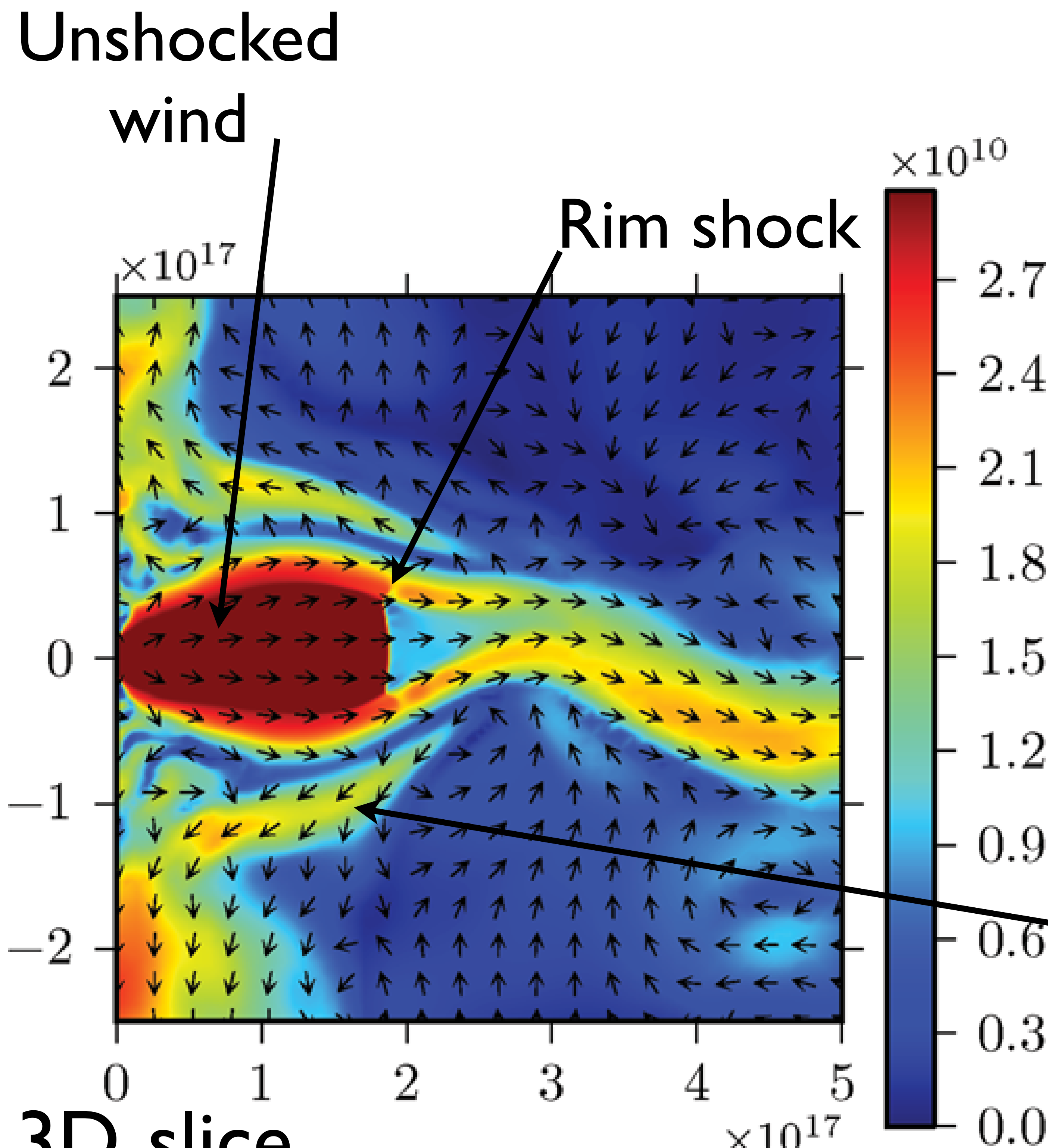
Shock structure, closeup

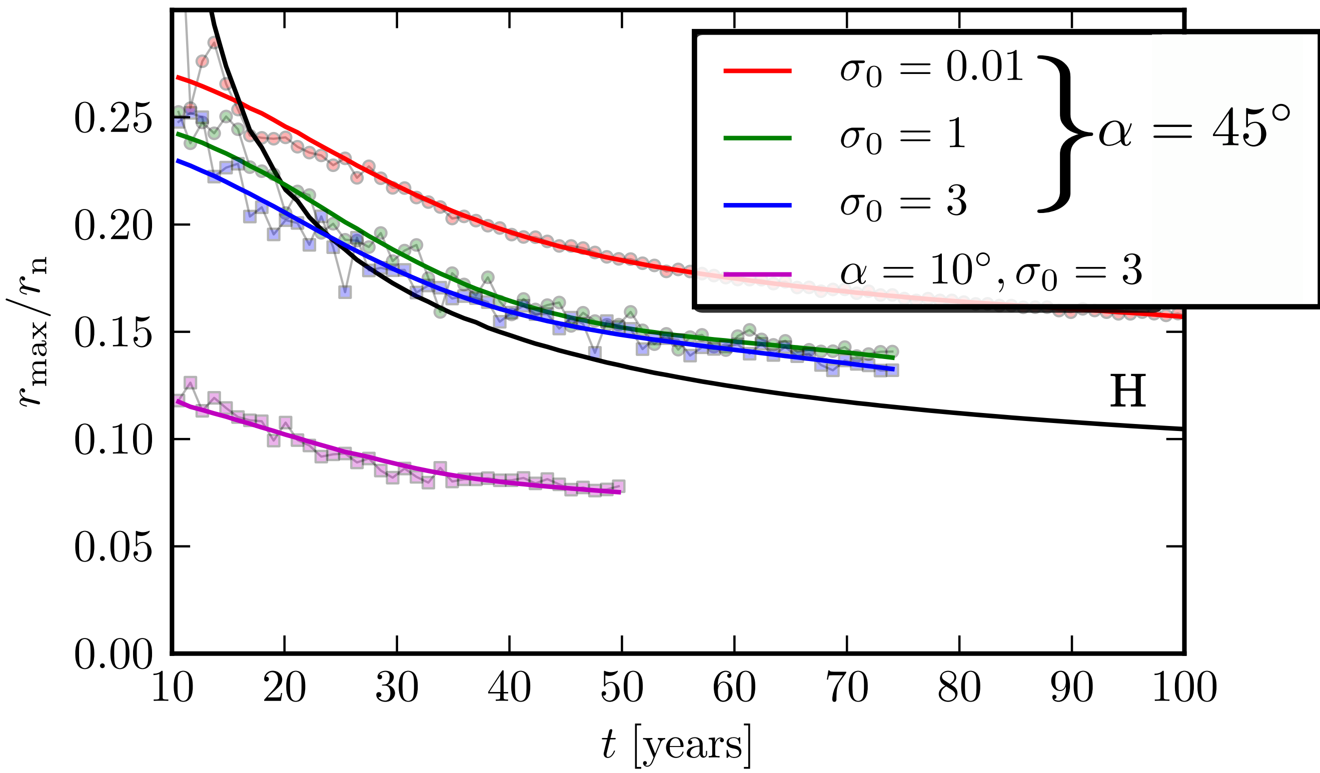


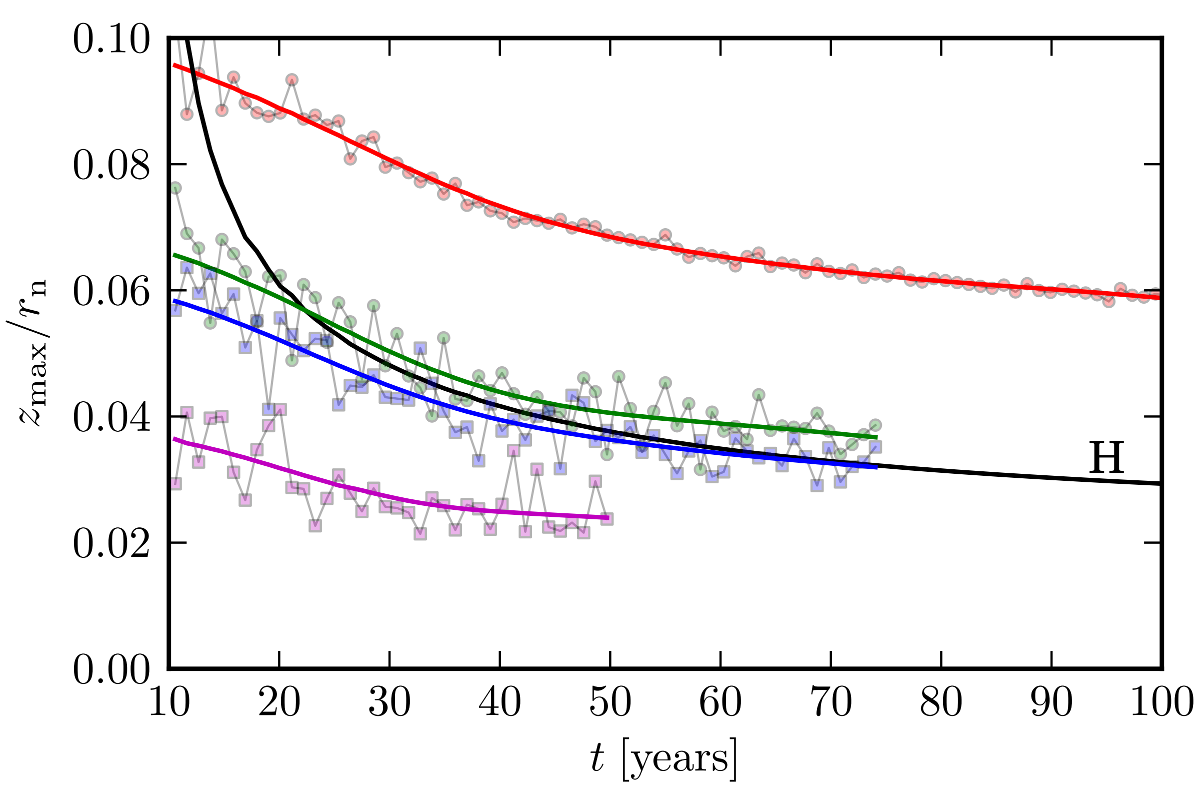
3D, slice

- Closeup of the termination shock, velocity magnitude
  - Shock "Torus" due to anisotropic wind $\propto \sin^2\Theta$
  - Oblique shock gives rise to fast equatorial flow
  - Plasma gets diverted into poloidal jet

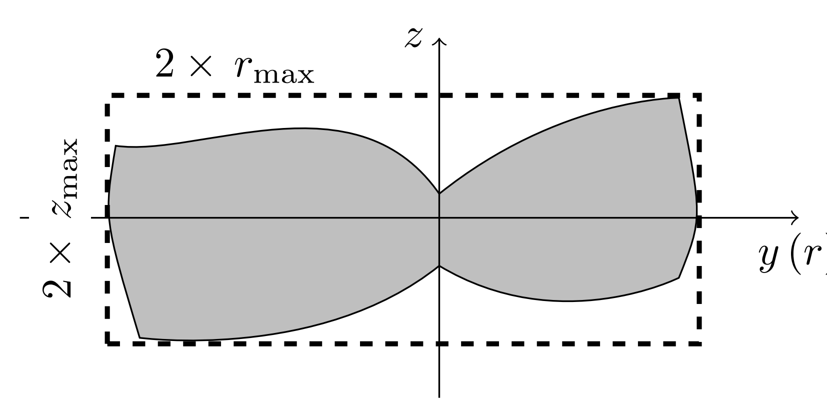
Plasma re-focussed due to hoop-stress

Shock radii

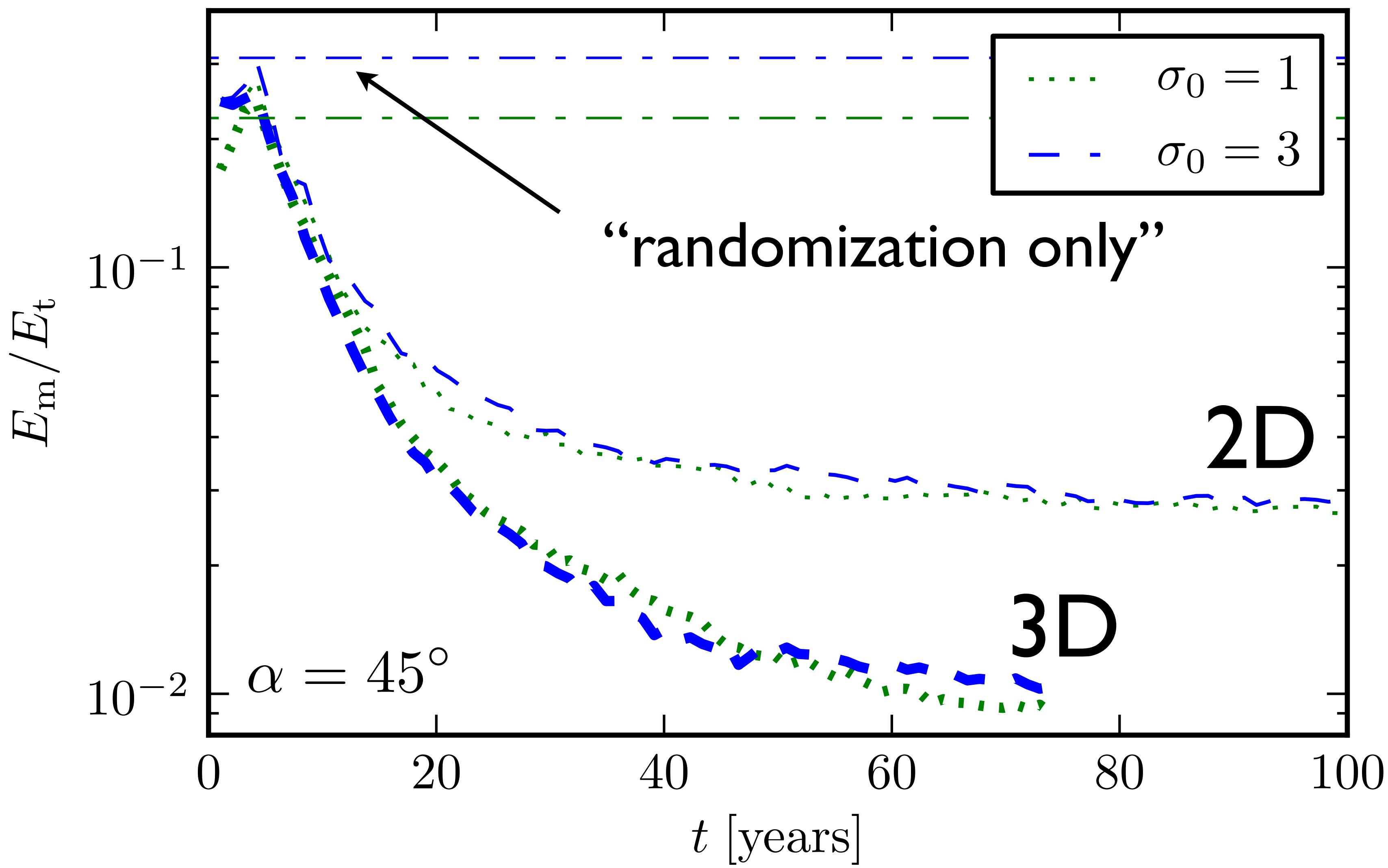




- H is the non-magnetic theory, in self-similar phase: r_{max}/r_n=0.095
- self-similar regime after t~200
- Observations provide r_{max}/r_n=0.085
- Shock sizes in 3D:
  - Don't collapse for high σ_0
  - little dependence on σ_0



What happened to the sigma problem?



Dissipation in the nebula
α=45°
2D: thin lines
3D: thick lines

Observed value from fitting Synchrotron and i.Compton emission:
$E_e \cong 30 E_m$

- 2D cases are also fairly dissipative!
- Dynamics dominated by gas pressure

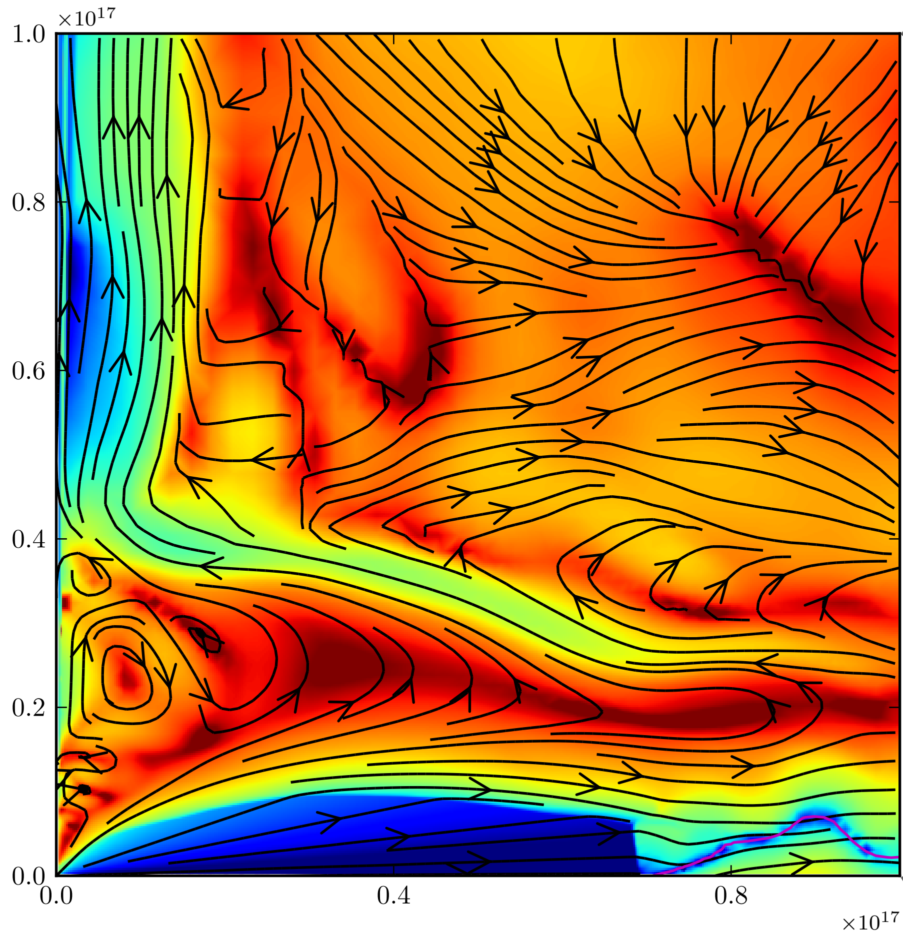
Disclaimer: Dissipation entirely numerical!

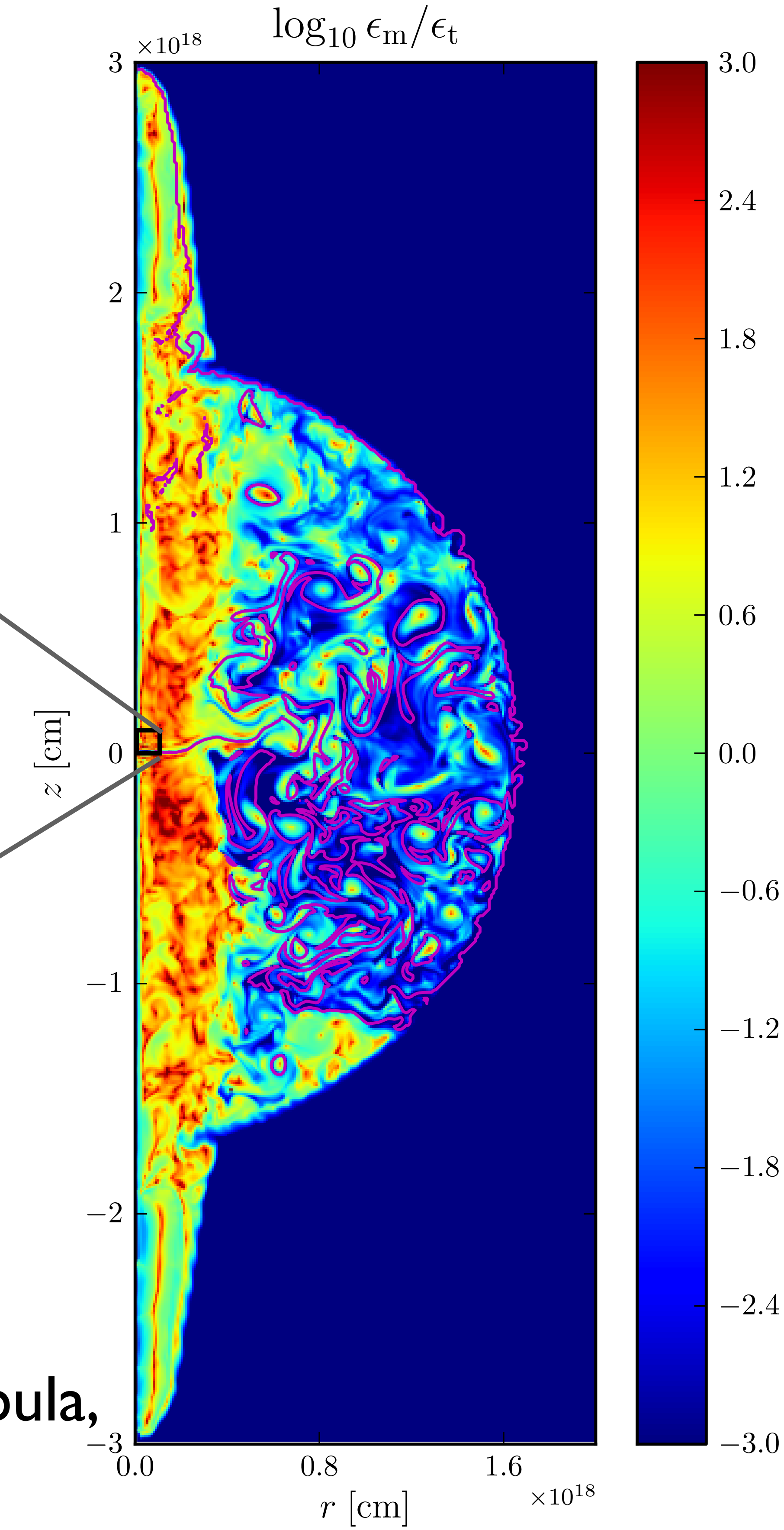
Lyutikov (2010), Komissarov (2012)

What remains of the sigma problem

- Dissipation in the Nebula

$\alpha = 45°, \ \sigma_0 = 1$





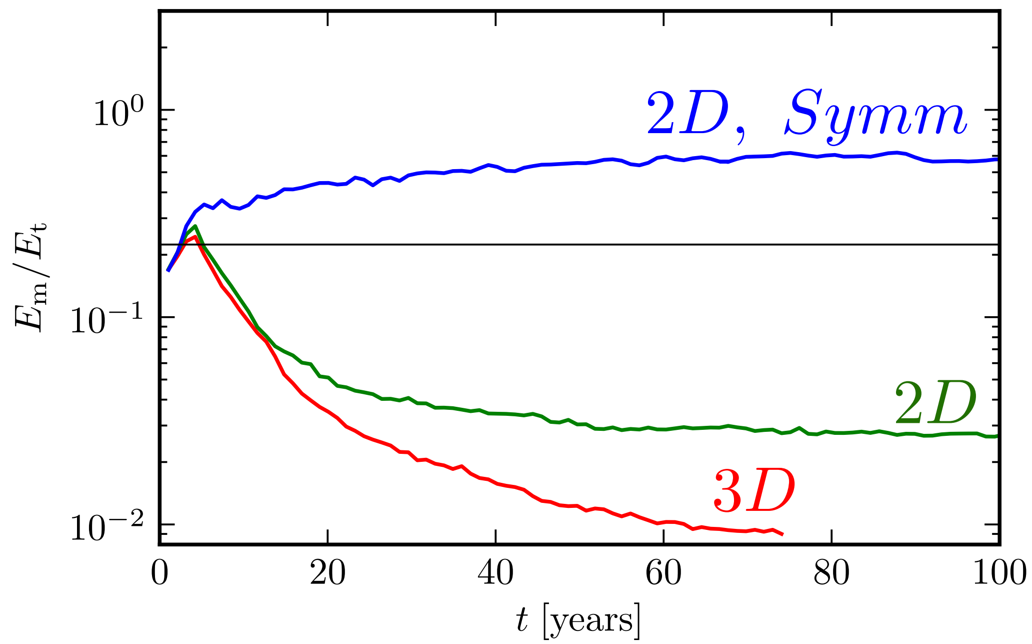
Dissipation region in 2D run.
magenta line: magnetic null

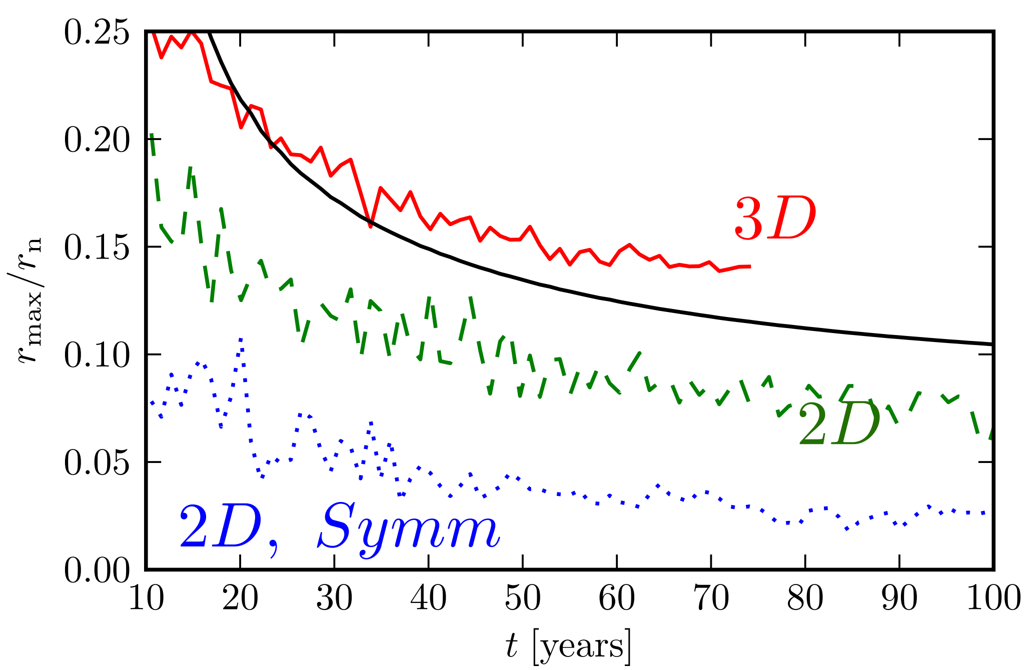
- Polarities from opposing hemispheres can mix in Nebula, e.g. Camus+ 2009; Olmi+2014

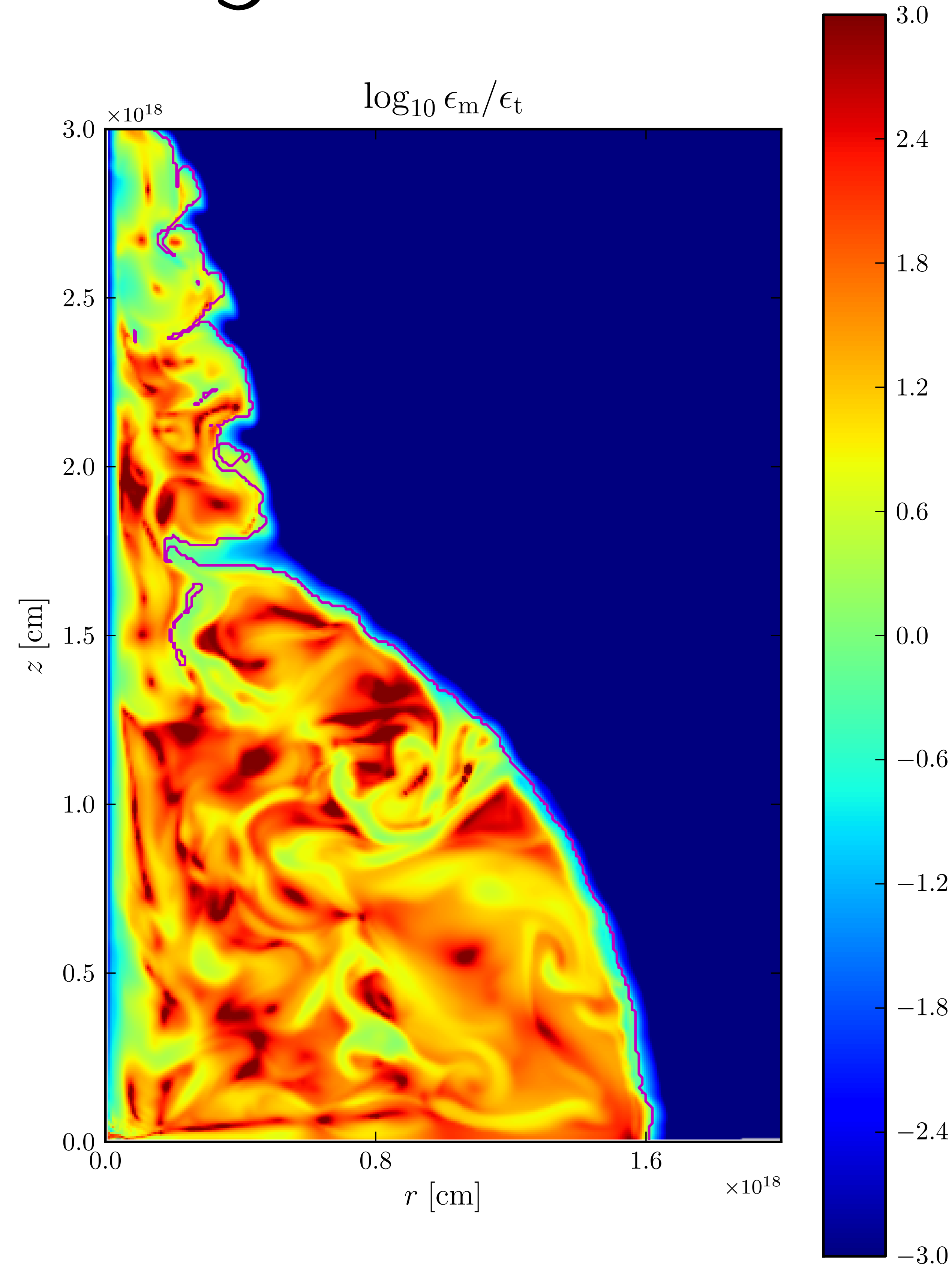
What happened to the sigma problem?

- **Dissipation in the Nebula**

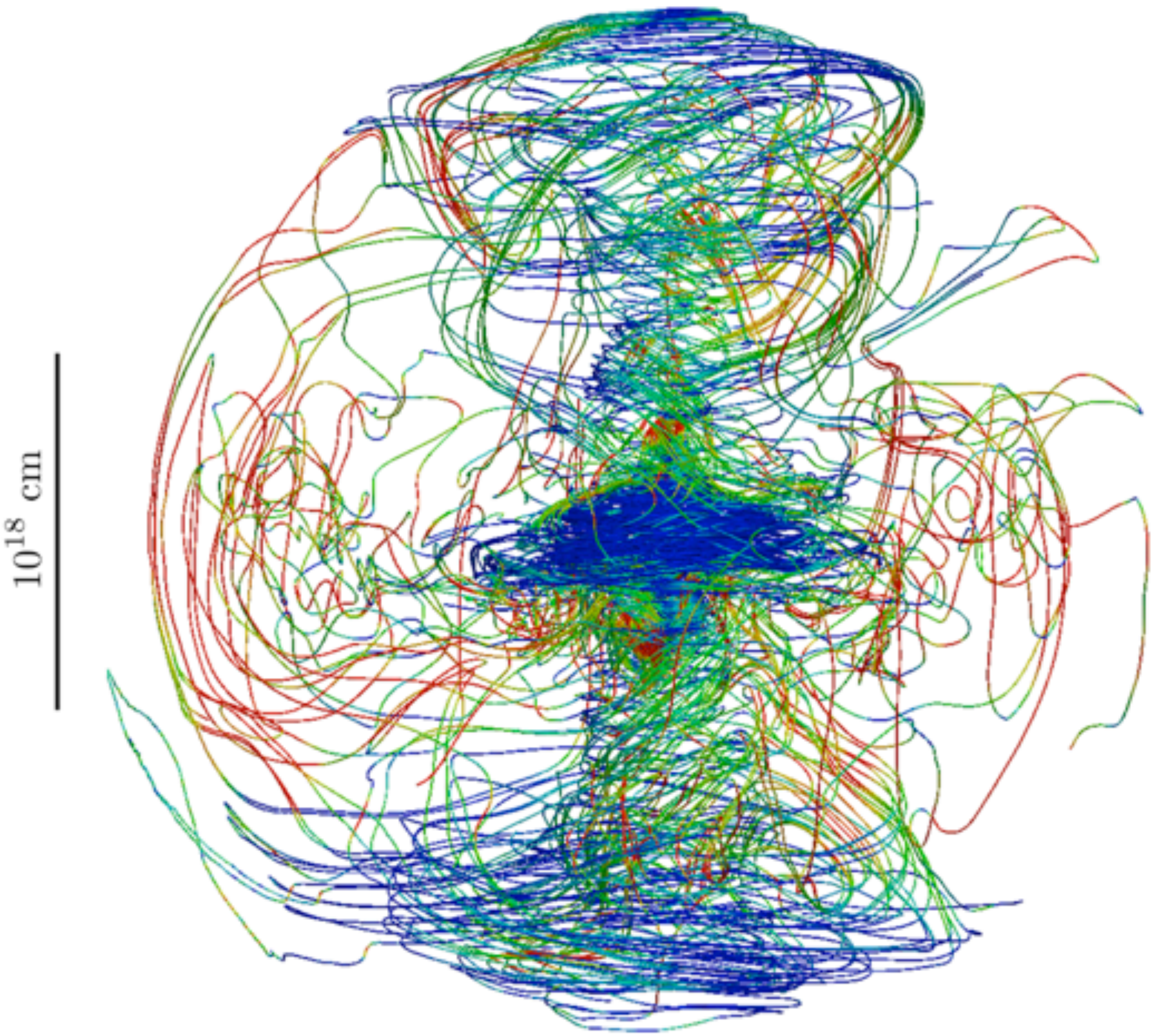
$\alpha = 45^\circ, \ \sigma_0 = 1$

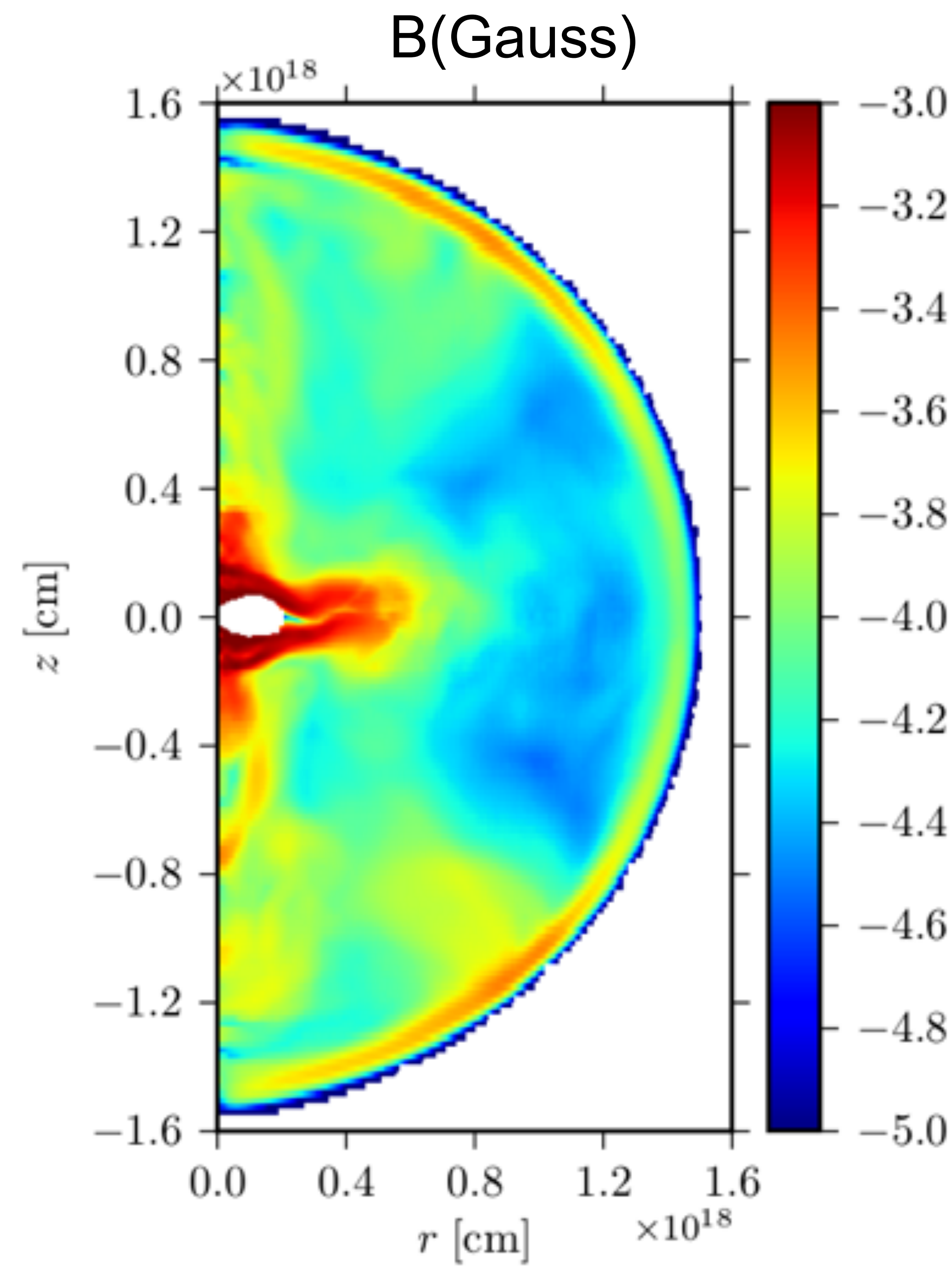






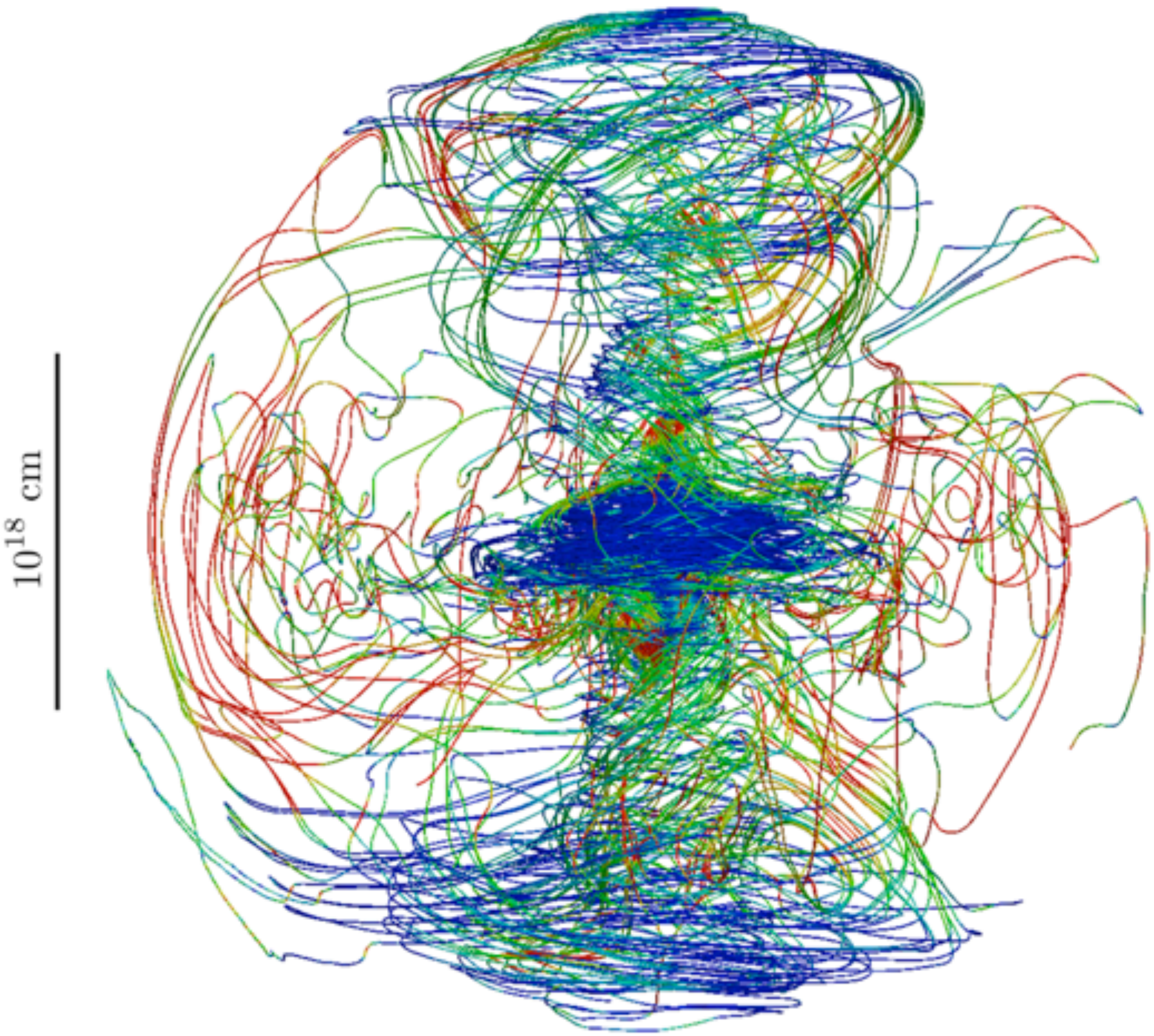
Magnetic field in the nebula



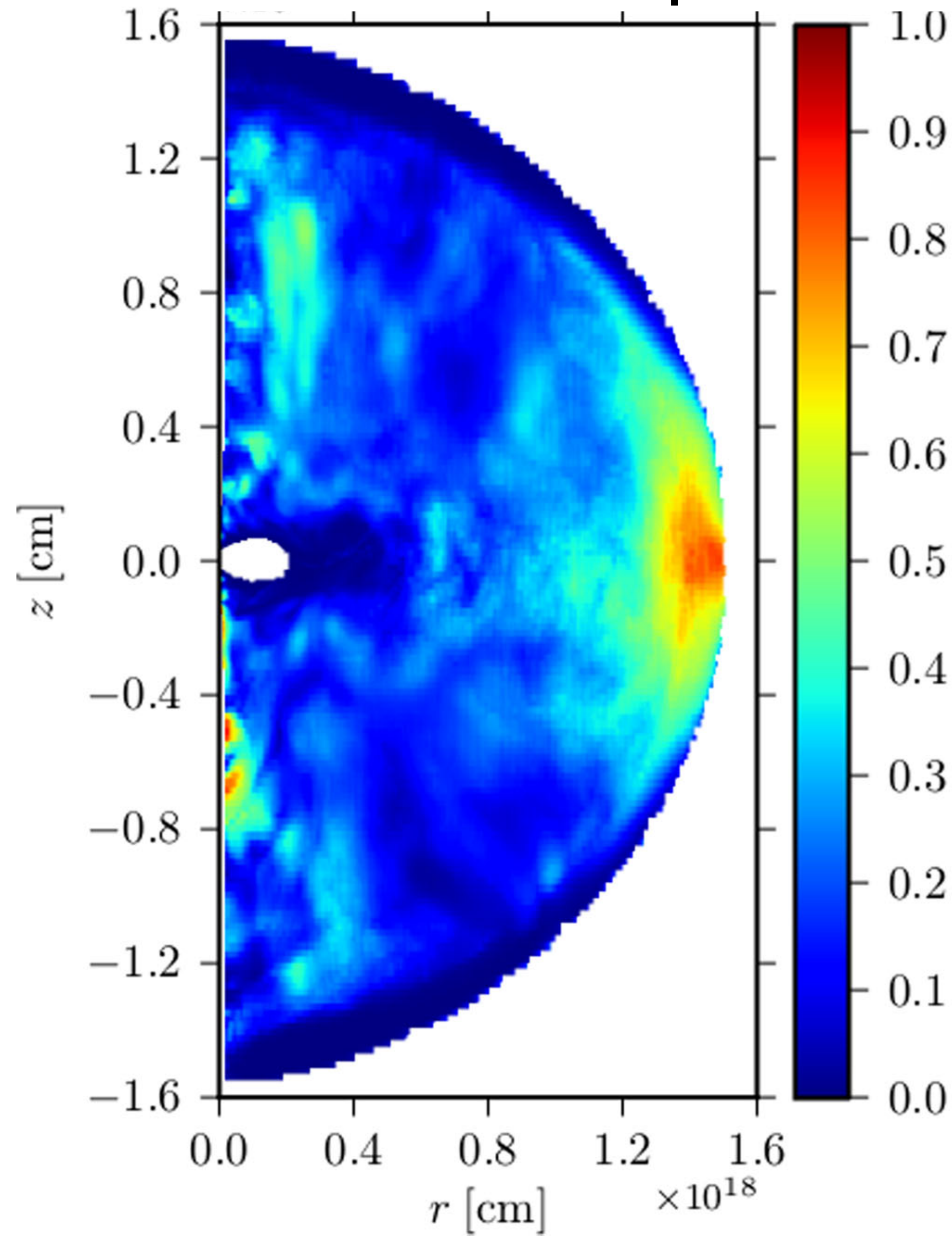


The magnetic field is strongest in the vicinity of the termination shock (in contrast to classical models), where it is still predominantly azimuthal. It is disordered further away from the shock.

Magnetic field in the nebula



Field orientation: Blue toroidal, red poloidal



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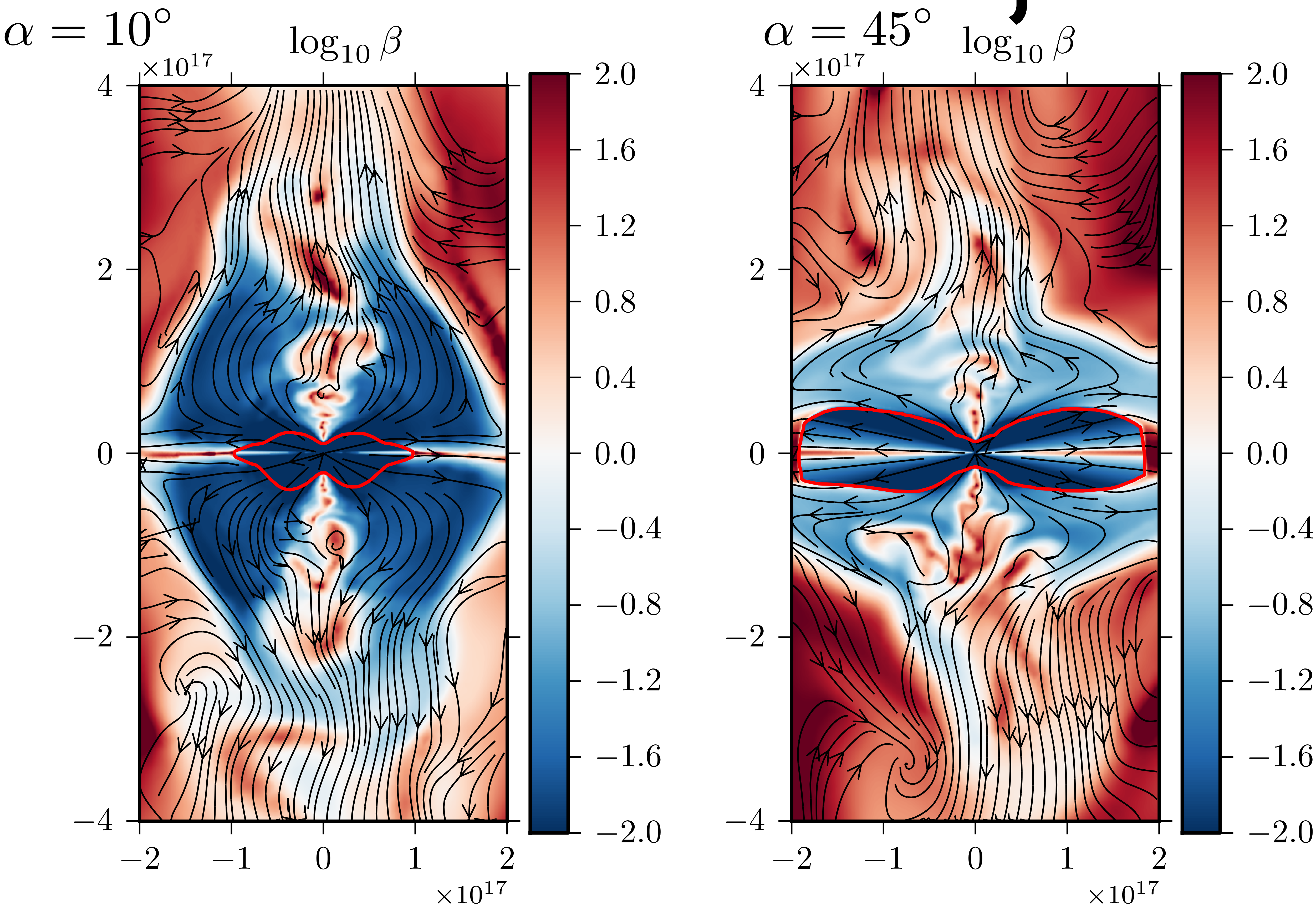
Polar beam and jet



$\sigma_0 = 3$

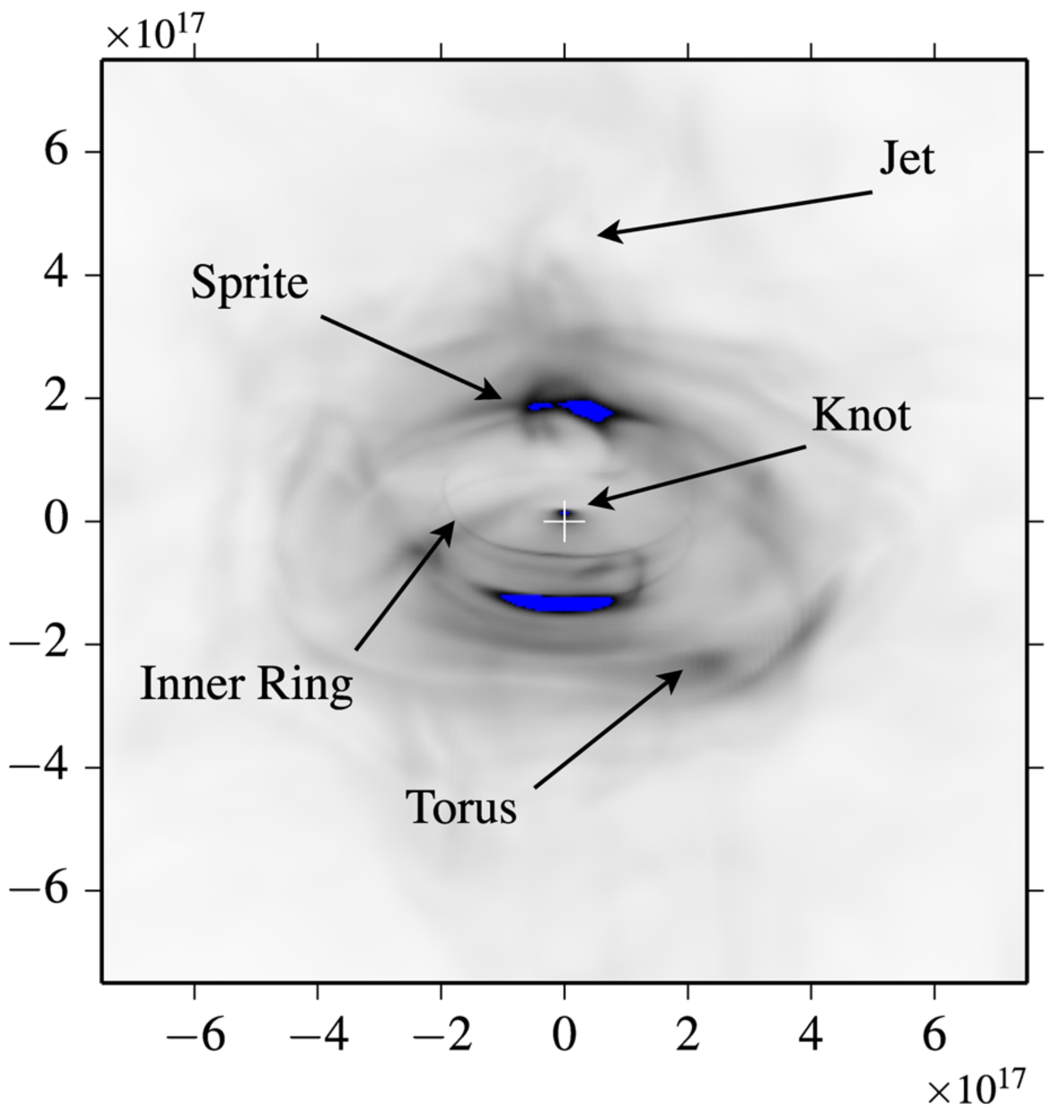
Radiation modeling - Particle injection at termination shock

as Kennel & Coroniti (1984)

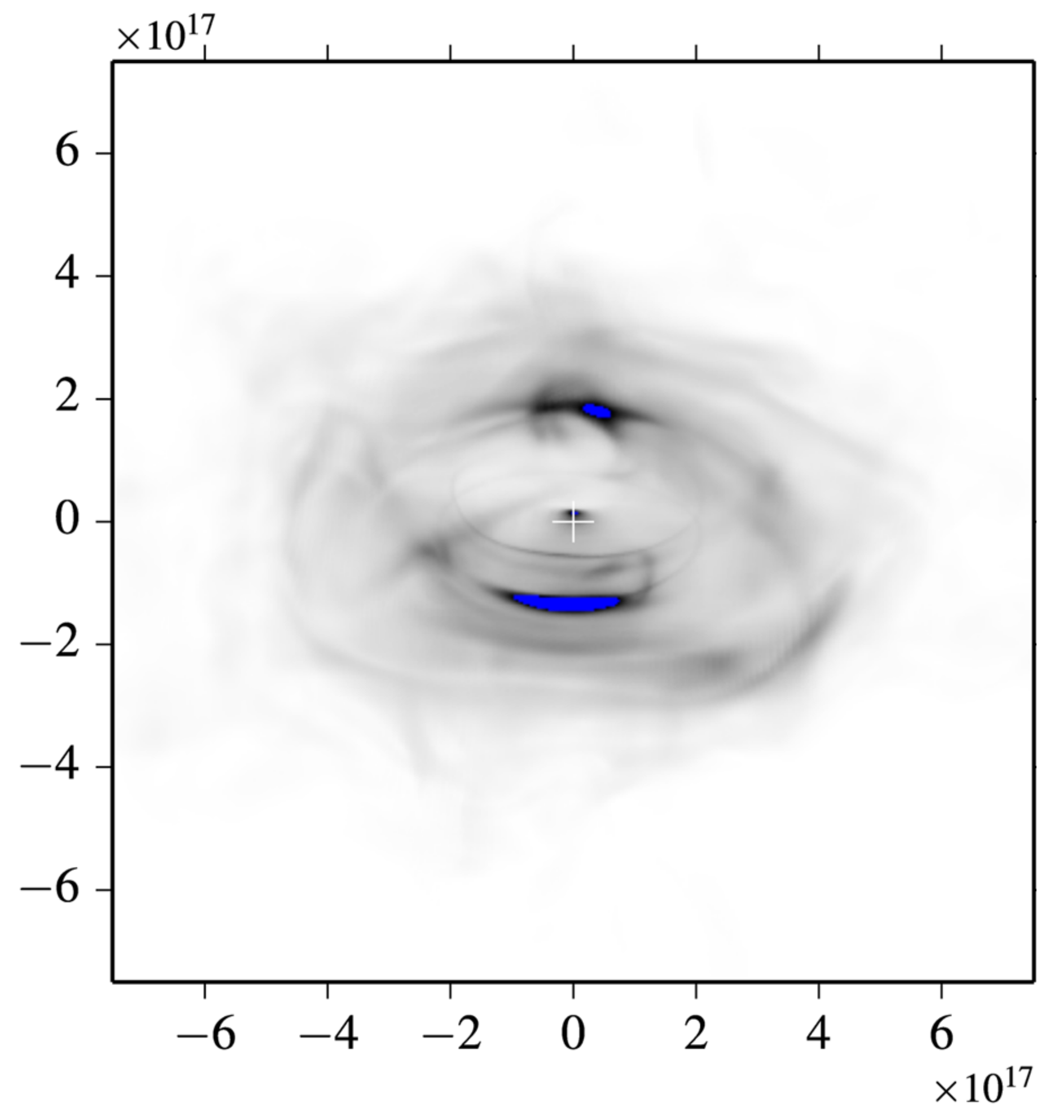
Injection at striped region of TS, follow adiabatic and radiative losses of cutoff energy:

$$f(\epsilon) = A\, n_0 \epsilon^{-p} \qquad \text{for} \qquad \epsilon < \epsilon_{\infty,0},$$

Optical $\nu = 10^{15}$Hz



hard Xray $\nu = 10^{19}$Hz

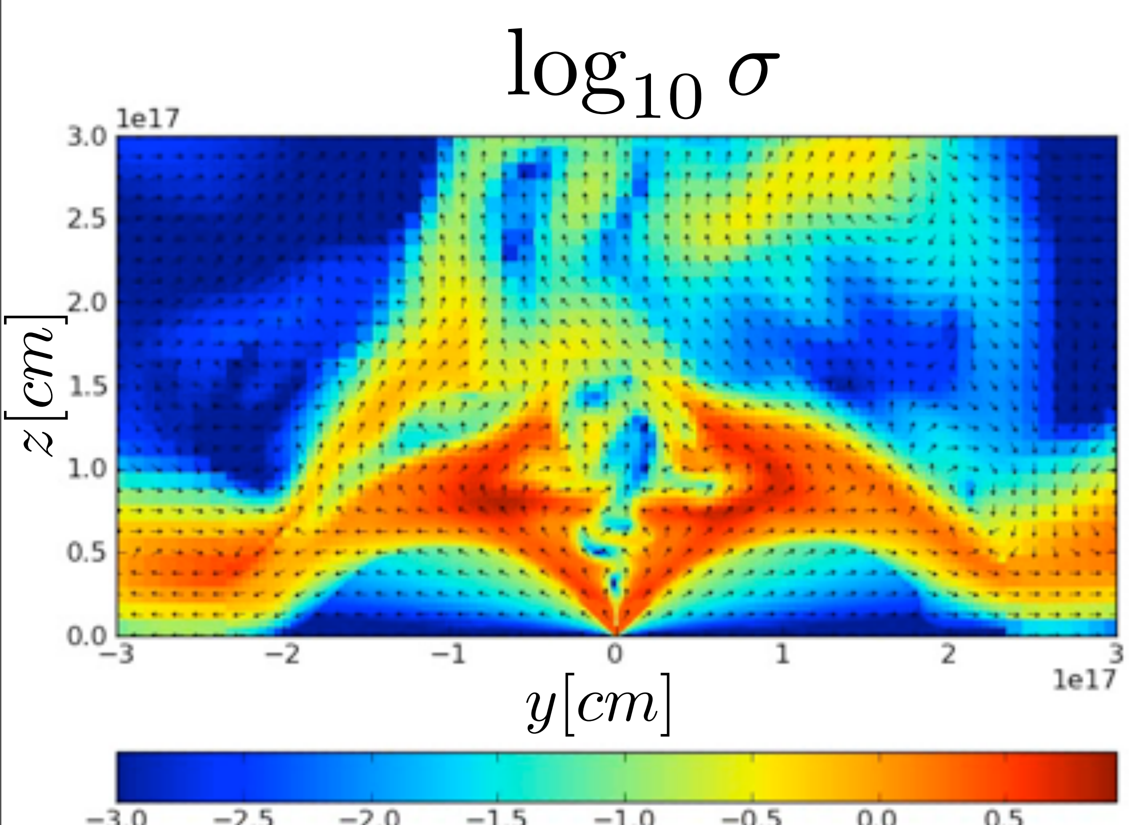


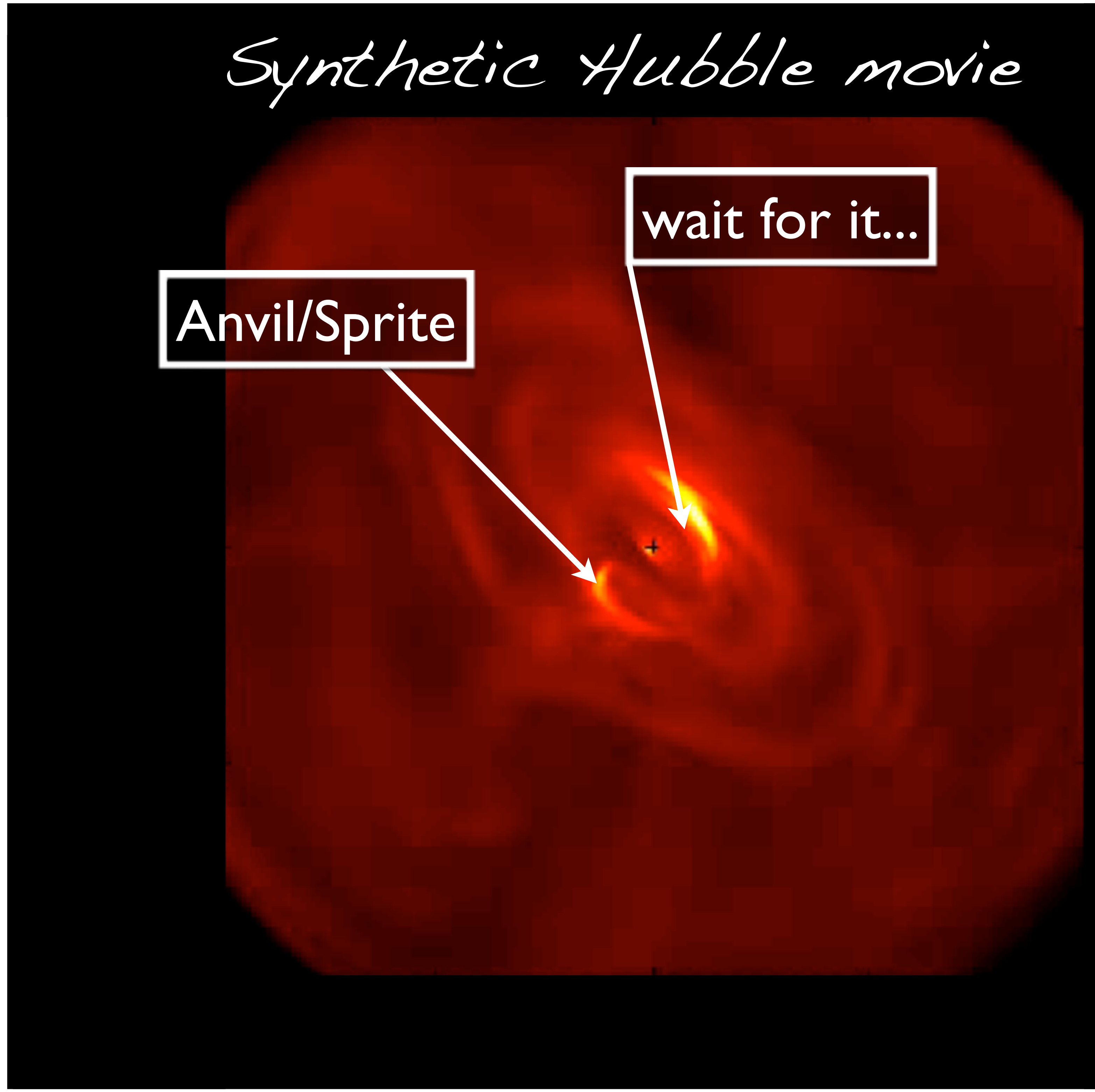
Good resemblance with Hubble observations of Crab

But: No jet in (hard) Xray?

Anvil

- Highly variable feature at the base of the jet
- Tempting: candidate for γ-ray Flares (Tavani et al 2011, Abdo et al 2011)?
- Not seen in 2D simulations



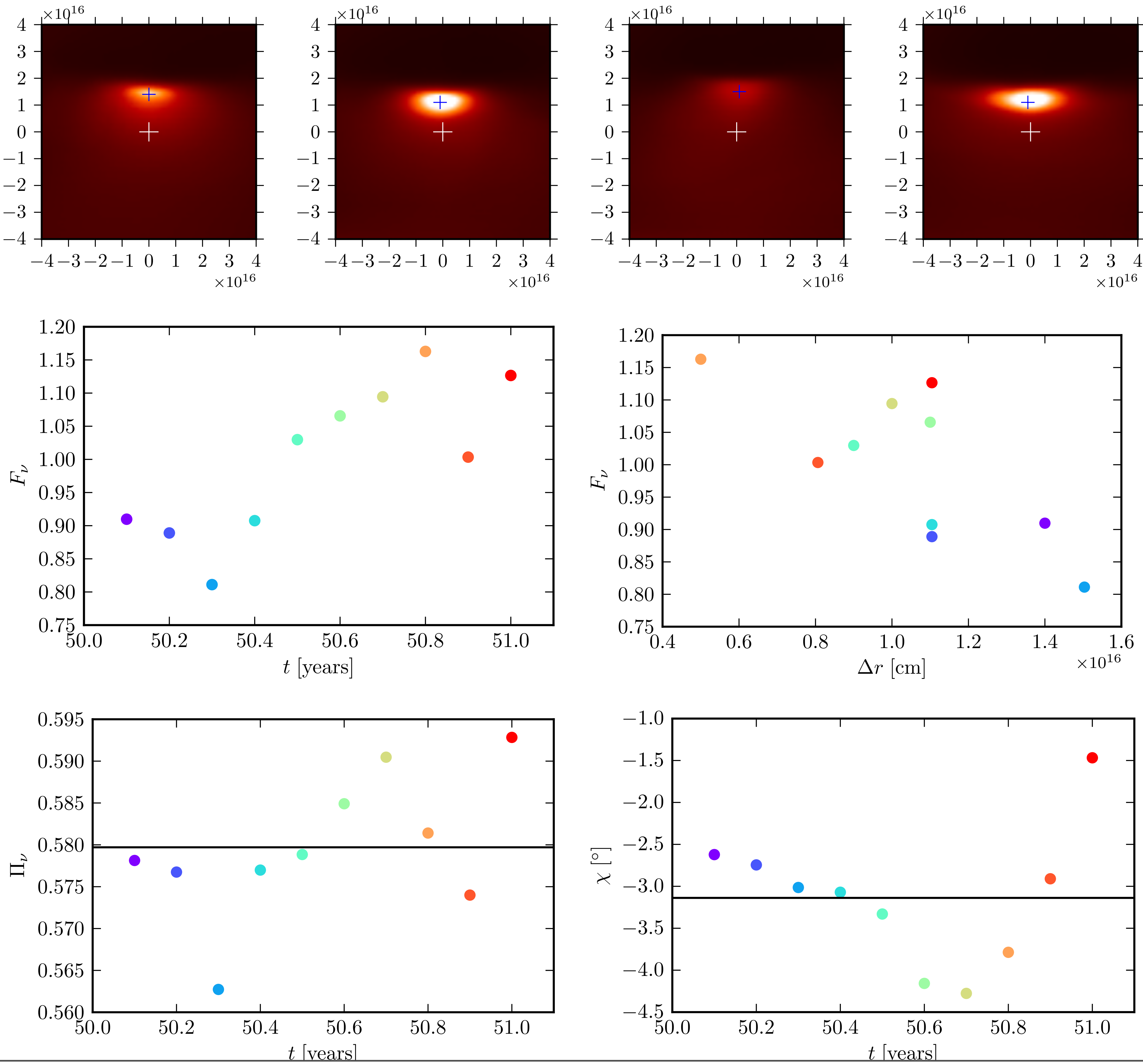


square root filtered intensity ~40 days between frames (1year total)

Compare with
Moran et al. (2013)
Rudy et al. (2015)

Variability of Knot

$\alpha = 45°,\ \sigma_0 = 1$



- optical intensity (linear scale) of the knot measured ~ 1 month apart
- Flux as a function of time and as function of displacement
- Unresolved polarisation degree and direction of the Knot
- Consistent with Komissarov & Lyubarsky (2004)
- Significant flux variability ~20%
- Closer in <-> brighter
- Stable polarisation signal at a degree of 60%

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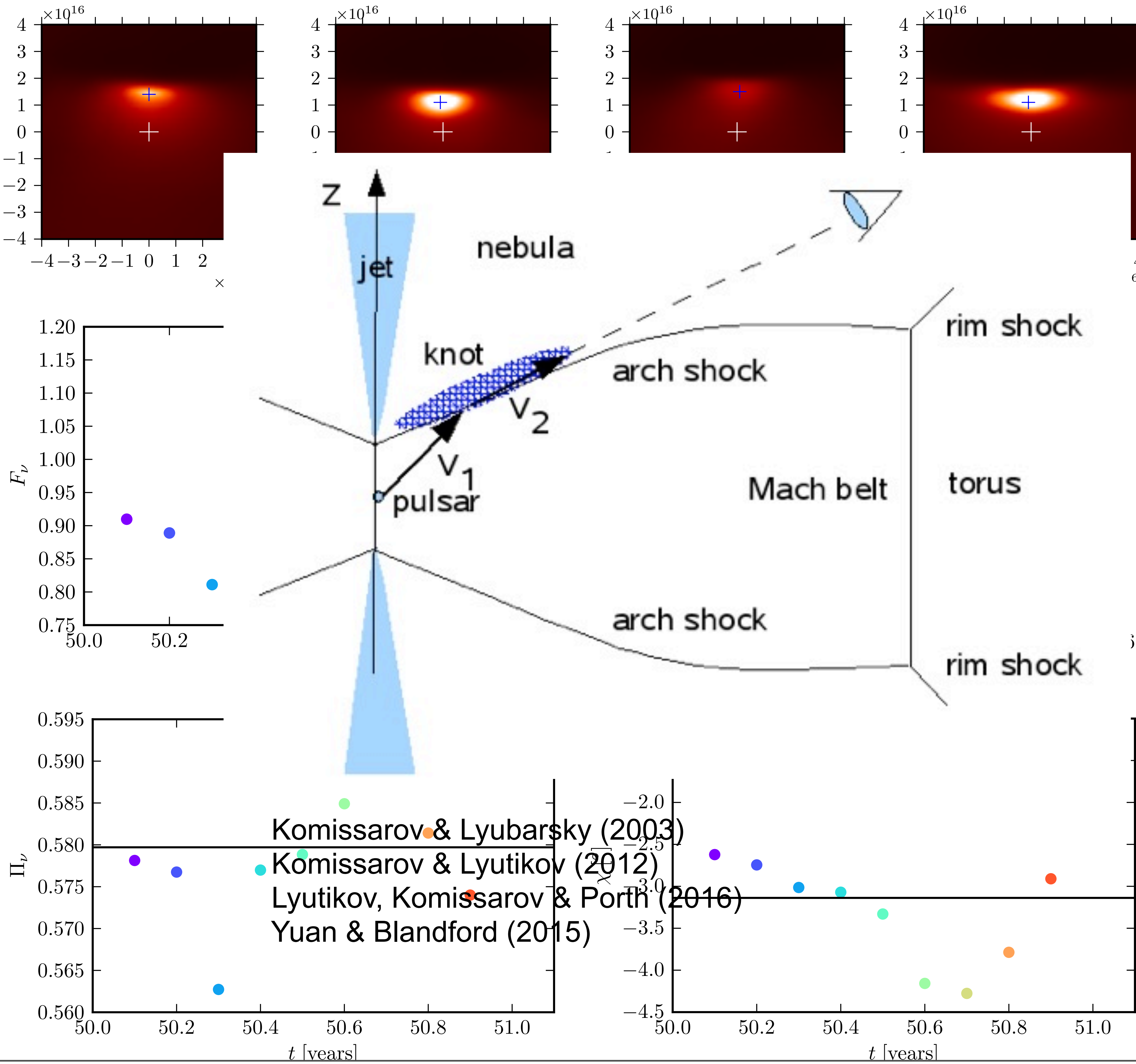

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3D models tell us

- Three mechanisms involved in the solution of the sigma problem:
    - Dissipation in striped Wind (here: assumption)
    - Nebula turbulence (field randomization)
    - We might not get the exact rate correct, but Turbulent magnetic dissipation in the nebula is dynamically important
- 3D RMHD models for Crab with $\sigma_0=3$ (>1)
- MHD model of Crab can explain many observed features: shock variability, jet, torus, wisps, knot1, robust in 3D!
- The jets form downstream of the termination shock where the magnetic hoop stress causes collimation of the flow lines that pass through the shock at intermediate latitudes.
- Jets don't drill through the nebula bubble, z-pinch magnetohydrostatic configurations obtained in 2D unphysical! Total nebula pressure mostly uniform.
- Illuminating the jet (v up to 0.7c) might require particle acceleration in addition to the striped wind region at the termination shock.

Particle transport in PWN

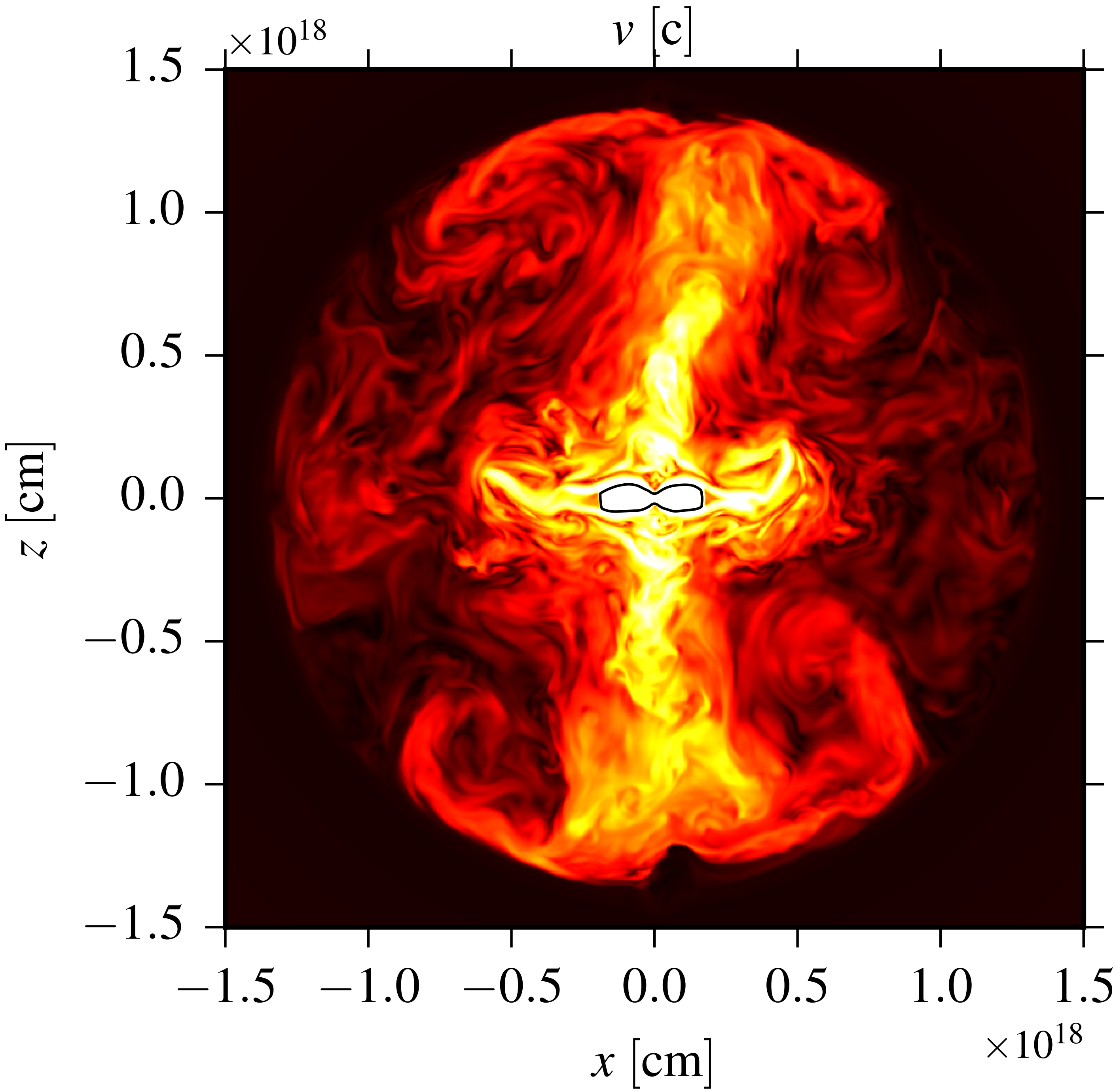
With **Michael Vorster**, Eugene Engelbrecht and Maxim Lyutikov
MNRAS, accepted

arXiv: 1604.03352

Particle transport: Motivation

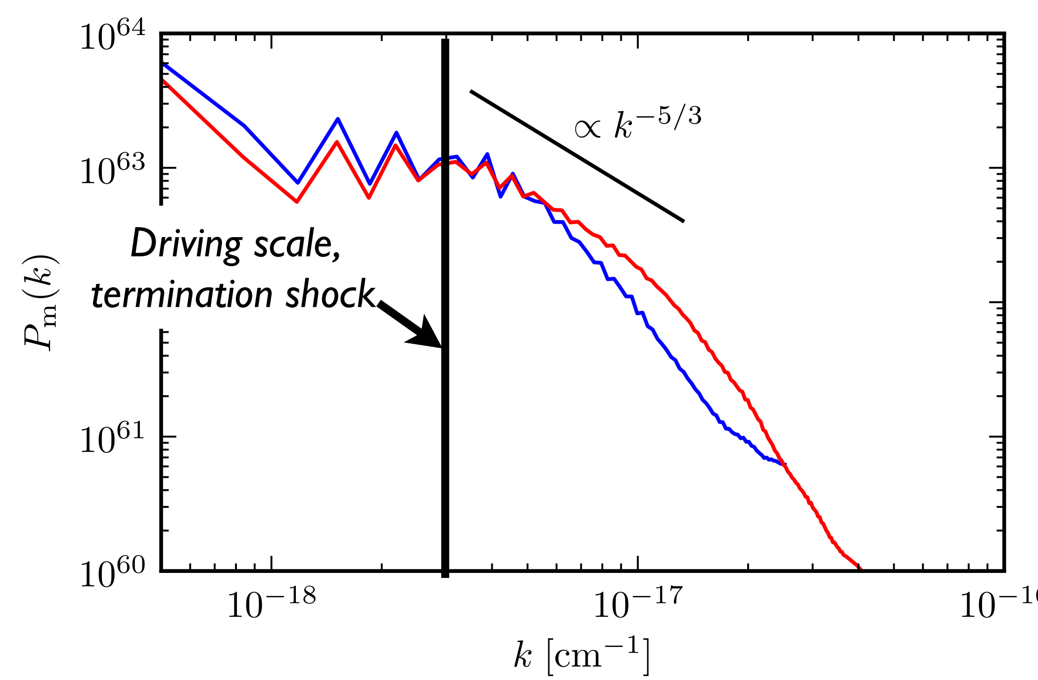
- PWN are often center filled: relativistic leptons injected centrally, transport and cool
- Spectral index maps and radiative fluxes can help to constrain parameters, depend on:
  - Particle injection and acceleration
  - Dynamics of PWN
  - Particle transport and cooling
- May help to understand particle acceleration: Only at shock or in-situ in the nebula?
- Escape of PWN accelerated particles: Might explain properties of cosmic rays, e.g. positron excess?

The PWN is a turbulent environment

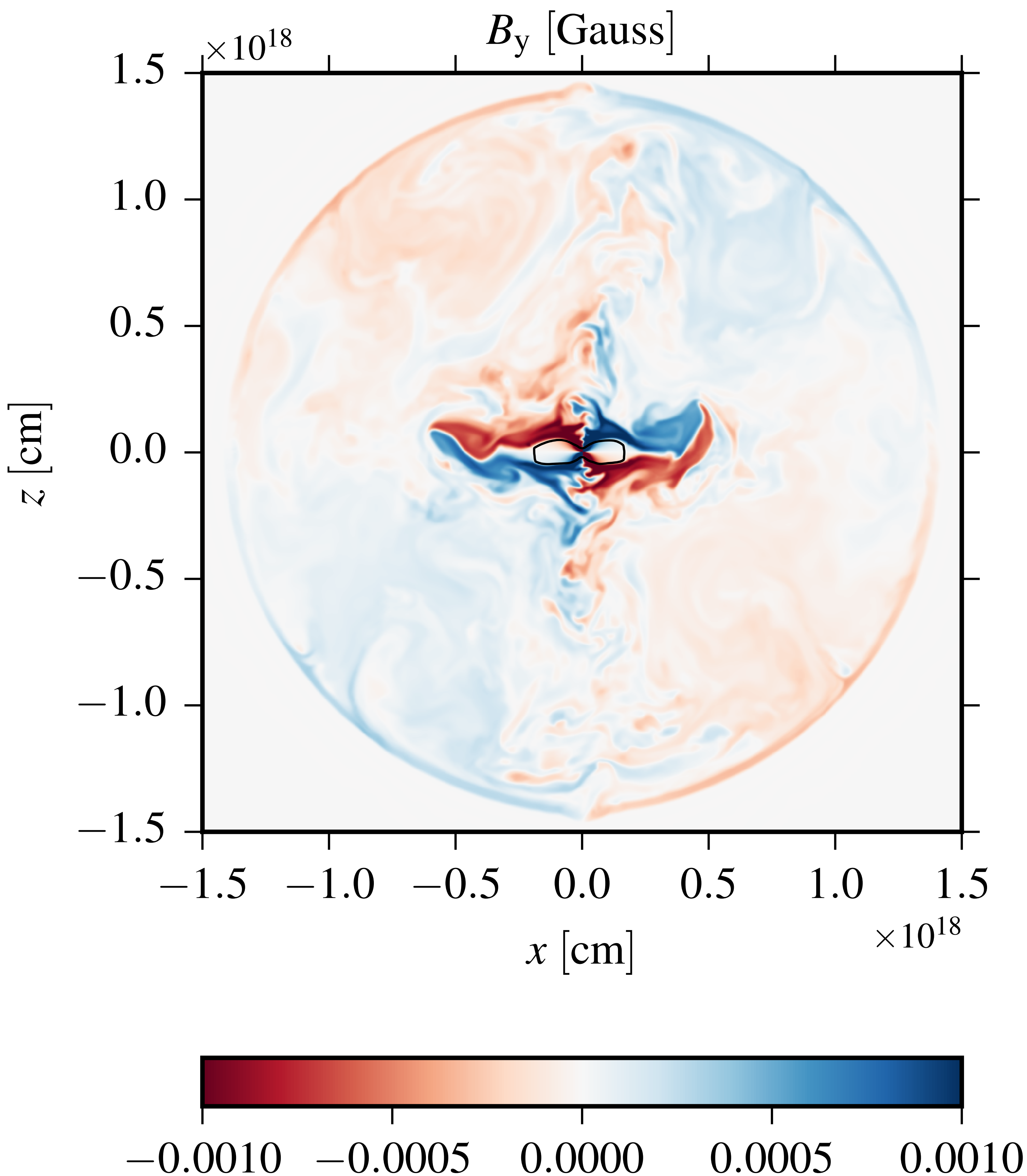


- Turbulence driven by kink-instability of the jet and equatorial shear-flow
- Injection scale: R_{ts}
- Less vigorous and decaying further out.
- Hemispherical guide-field still present

Slice through a 3D simulation



The PWN is a turbulent environment



Turbulence driven by kink-instability of the jet and equatorial shear-flow

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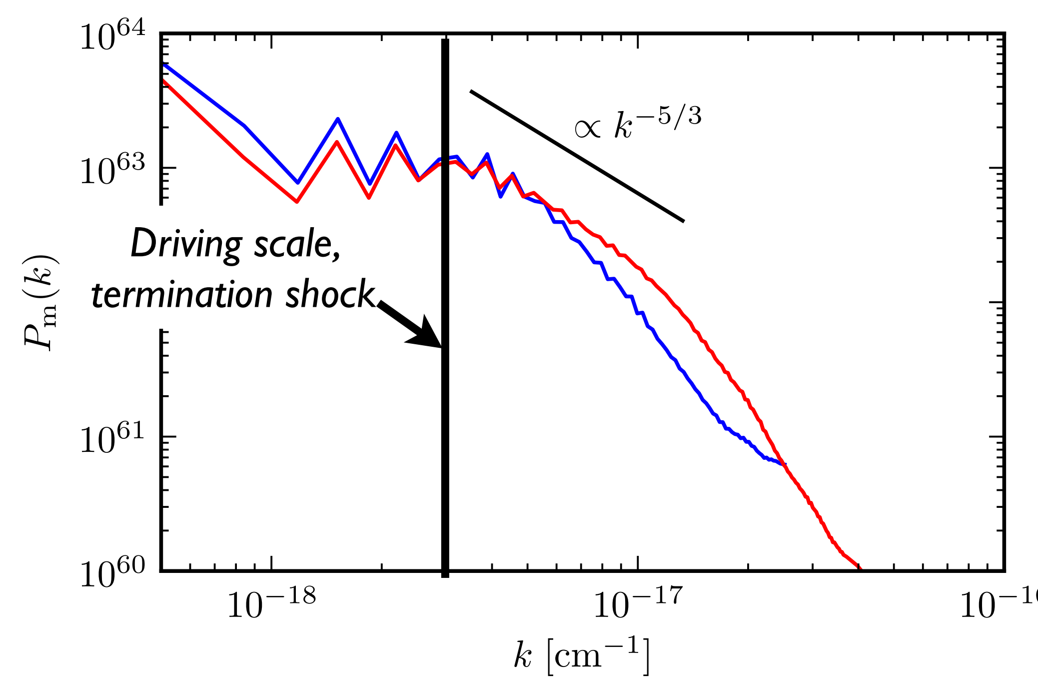
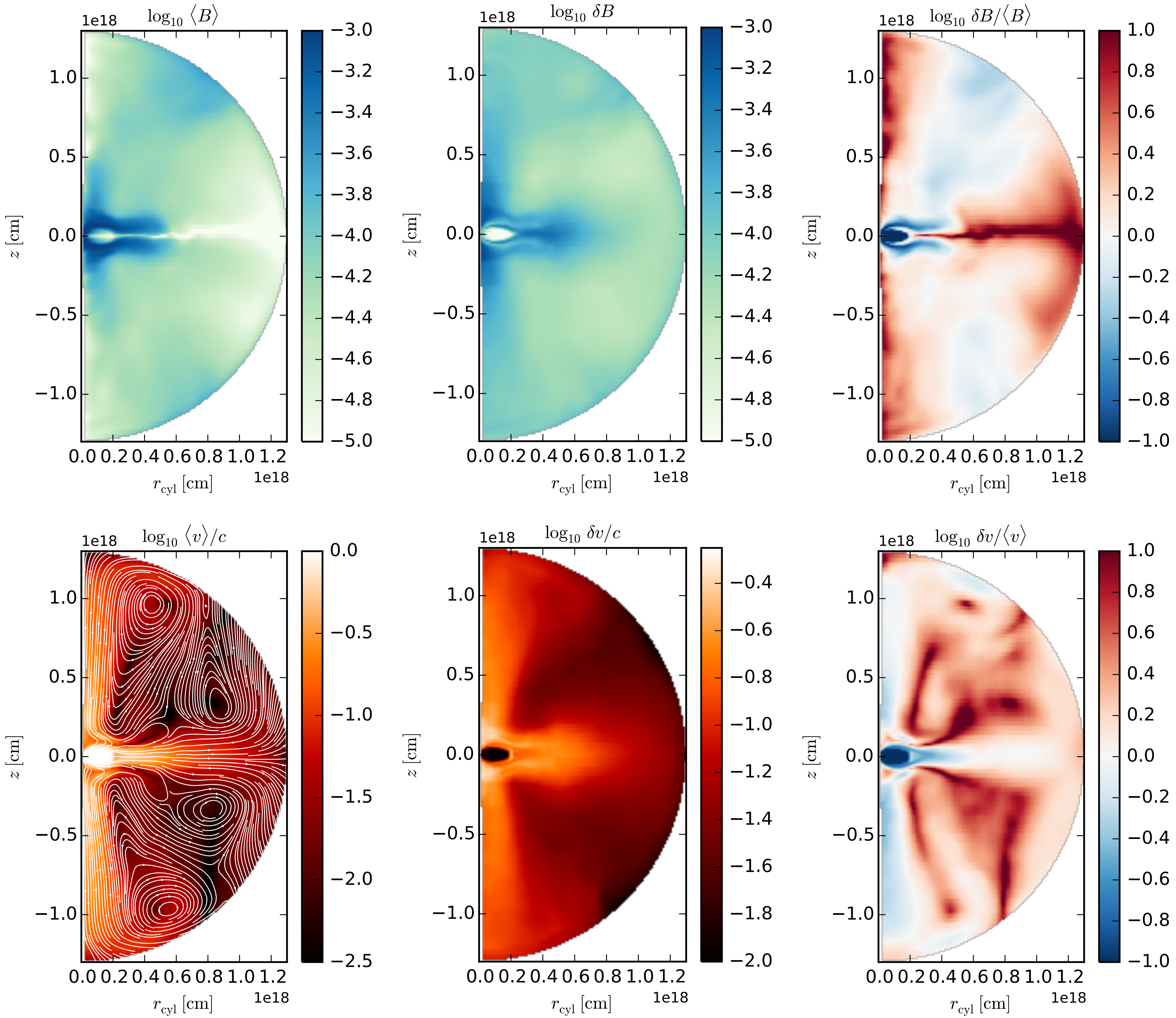


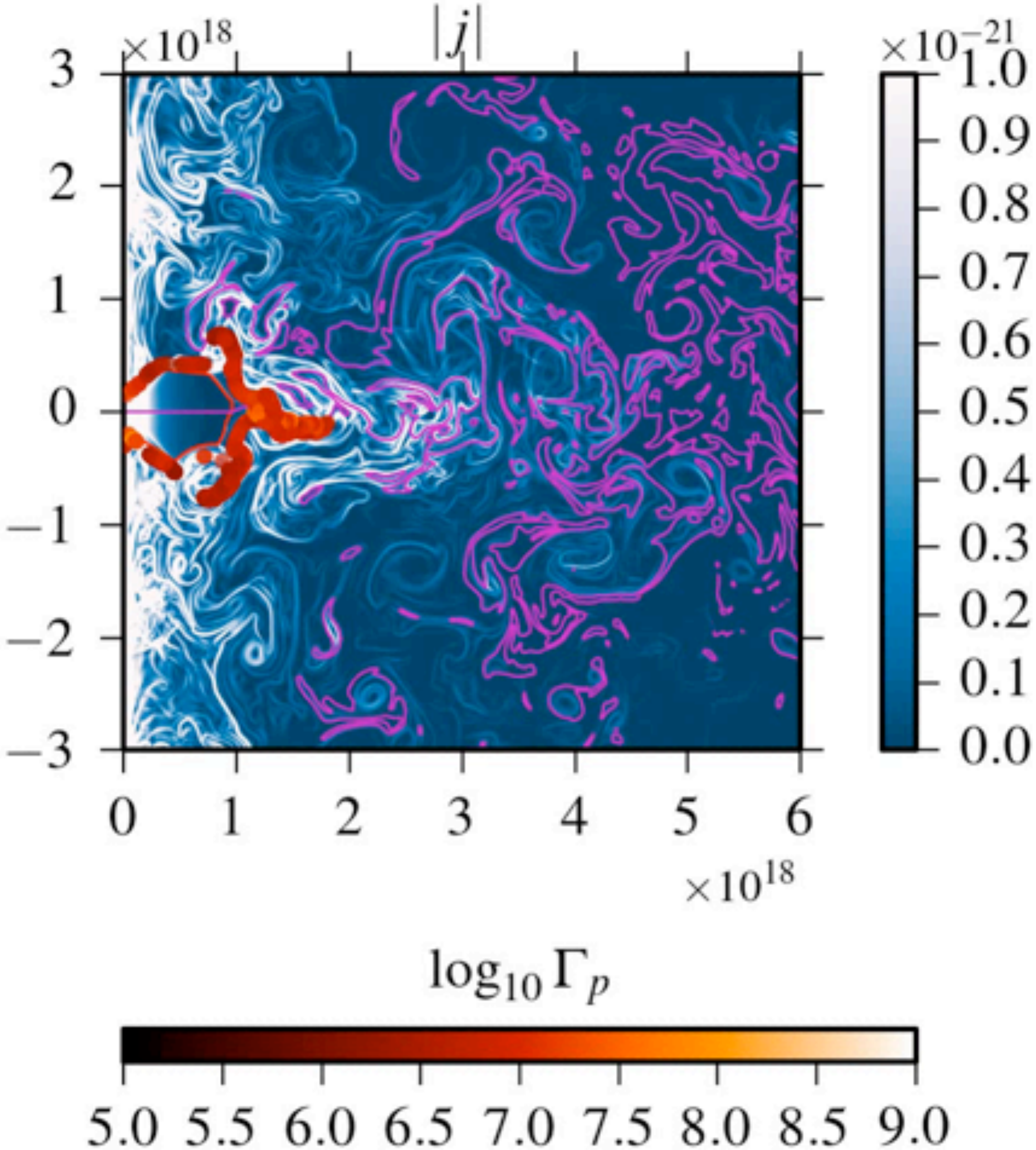


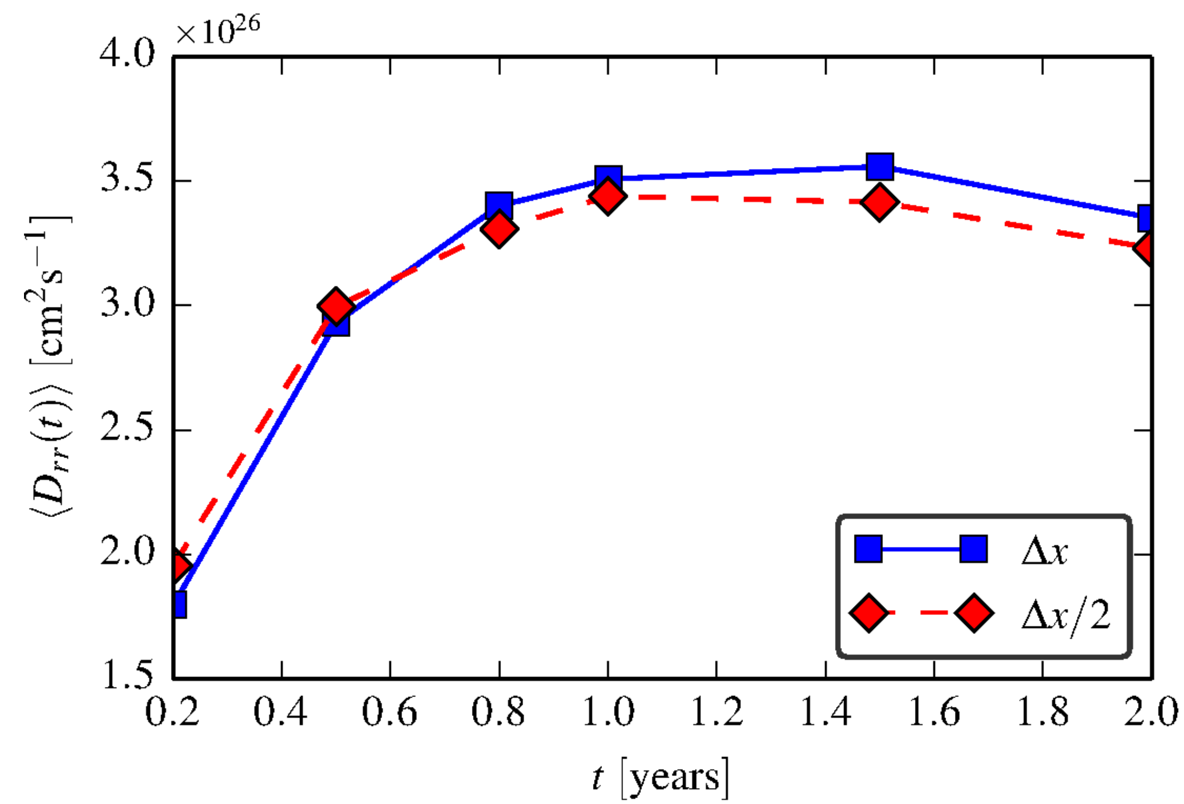
$\log_{10}\langle B\rangle$
$\log_{10}\,\delta B$
$\log_{10}\,\delta B/\langle B\rangle$
$\log_{10}\langle v\rangle/c$
$\log_{10}\,\delta v/c$
$\log_{10}\,\delta v/\langle v\rangle$
z [cm]
$r_{\rm cyl}$ [cm]
1e18

Suppose X-ray emitting particles are injected at the termination shock. *How are they transported through the nebula?*

$$\frac{d\mathbf{u}}{dt} = \frac{q}{mc}\left(\mathbf{E} + \frac{\mathbf{u}\times\mathbf{B}}{c\Gamma_{\mathrm{p}}}\right)$$

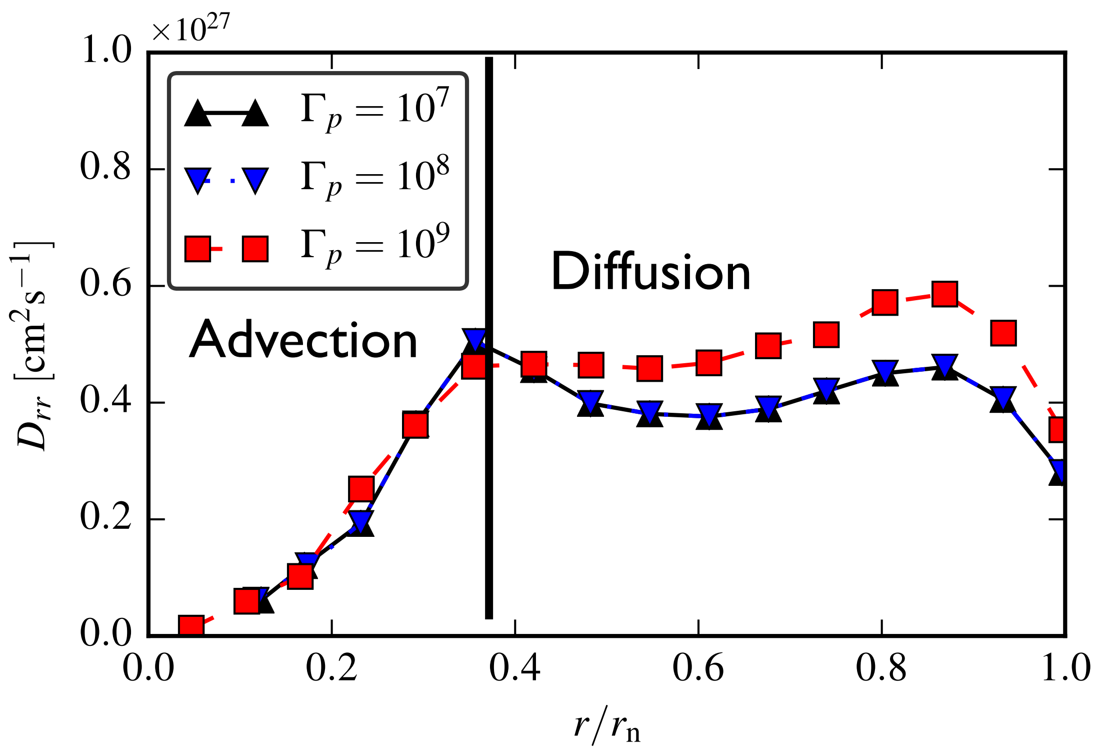
Particle Lorentz factors $\Gamma_{\mathrm{p}} \geq 10^{7}$





$$D_{rr}(t) = \frac{1}{2}\frac{\langle(\Delta r(t))^2\rangle_{r,\phi,\theta}}{t}$$

Particle transport in Pulsar Wind Nebulae



Average profile of the radial diffusion coefficient for increasing particle energies.

Bohm Diffusion:

$$D^{\rm B} = \frac{1}{3} r_g^2 \omega_g \tag{13}$$

$$= 1.7 \times 10^{24} \left(\frac{\Gamma_{\rm p}}{10^7}\right) \left(\frac{B}{100\mu{\rm G}}\right)^{-1} {\rm cm}^2\ {\rm s}^{-1} \tag{14}$$

Turbulent Eddy diffusion:

$$D^{\rm E}_{Ls} = \frac{1}{3} v_f L_{\rm s} \tag{15}$$

$$= 1 \times 10^{27} \left(\frac{v_f}{0.5c}\right) \left(\frac{L_{\rm s}}{0.2{\rm Ly}}\right) {\rm cm}^2\ {\rm s}^{-1}\ . \tag{16}$$

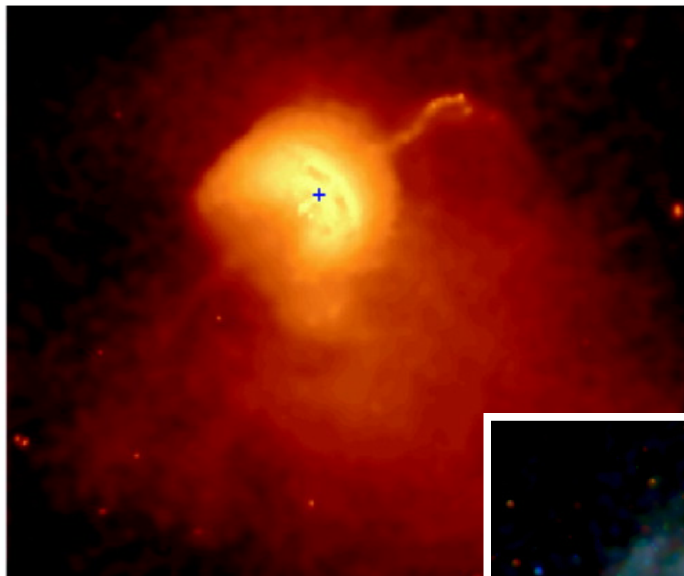
L_s : Scale of largest Eddy, termination shock $\sim 2 \times 10^{17}\,{\rm cm}$. v_f: Velocity at this scale $\sim 1/2c$.

$$r_g = \frac{p_\perp c}{eB} = 1.7 \times 10^{16} \left(\frac{\Gamma_{\rm p}}{10^9}\right) \left(\frac{B}{100\mu G}\right)^{-1} {\rm cm} \tag{17}$$

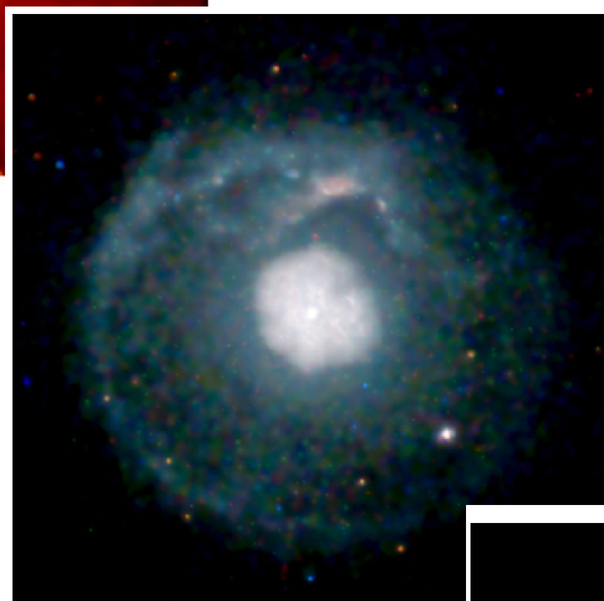
Diffusion becomes energy dependent when $r_g \geq L_s$, thus for $\Gamma_p = 10^{10}$, these particles have too short synchrotron lifetimes however $\Rightarrow$ Diffusion always energy independent!

lets do some modeling

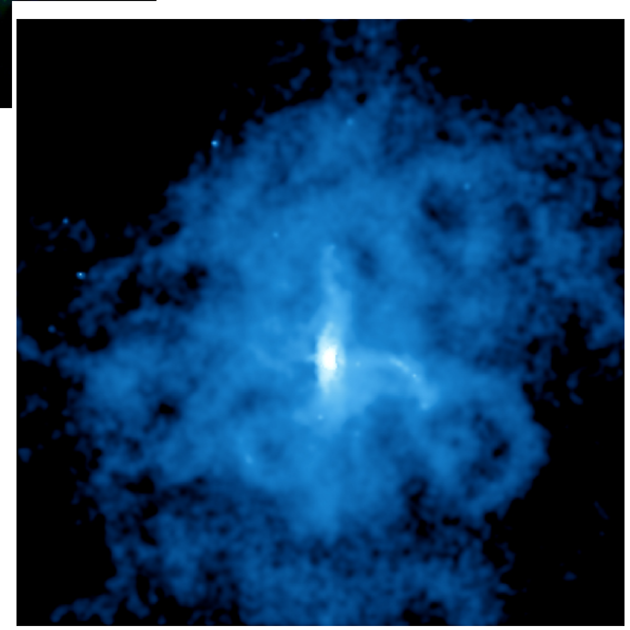
What do X-ray observations tell us about...



Vela



G21.5-09



3C58

?

Particle transport in Pulsar Wind Nebulae

Back to the transport equation:

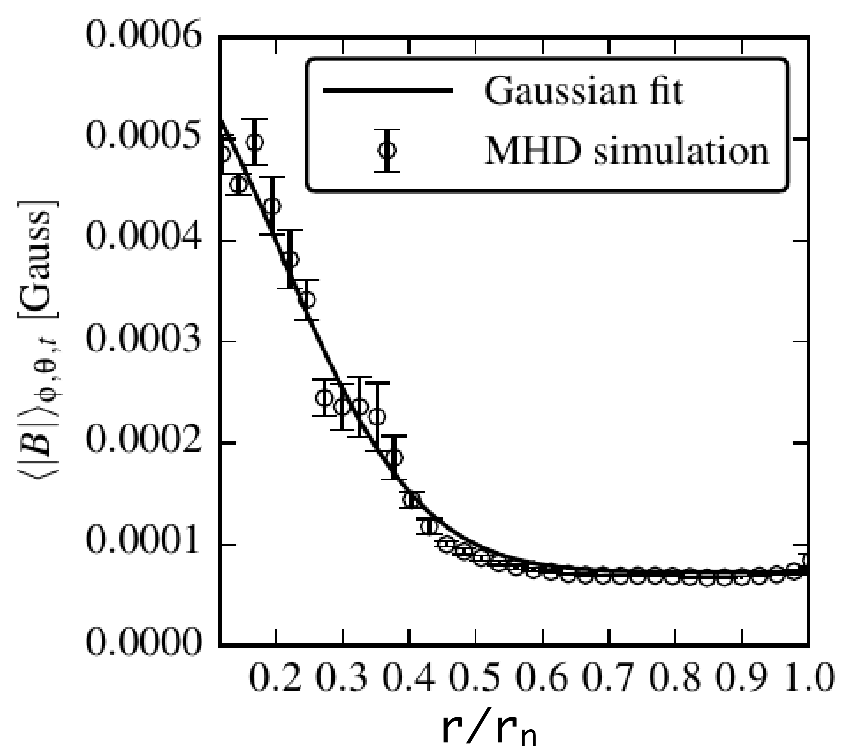
$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = \frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu}\right) + \frac{1}{p^2}\frac{\partial}{\partial p}\left(D_{pp}\frac{\partial f}{\partial p}\right) \tag{18}$$

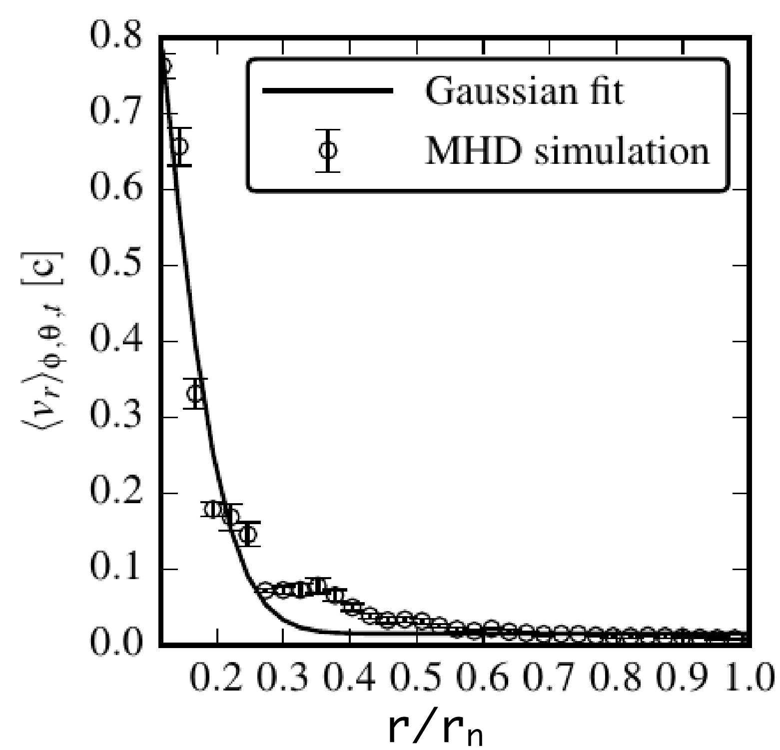
Look for steady state solutions for the radial transport and including **adiabatic** and **radiative** losses:

$$D_{rr}(r)\frac{\partial^2 f}{\partial r^2} + \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_{rr}(r)\right) - V(r)\right]\frac{\partial f}{\partial r} + \left[\frac{1}{3r^2}\frac{\partial}{\partial r}\left(r^2 V(r)\right) + z_p p\right]\frac{\partial f}{\partial \ln p} + 4 z_p p f = 0, \tag{19}$$

with the Synchrotron loss term

$$z_p(r) = \frac{4\sigma_{\mathrm{T}}}{3\,(m_e c)^2}\frac{B^2(r)}{8\pi} \tag{20}$$





Averaged profiles from a fiducial MHD simulation

Fokker-Planck spectral modeling, e.g. Tang & Chevalier (2012)

Particle transport in Pulsar Wind Nebulae

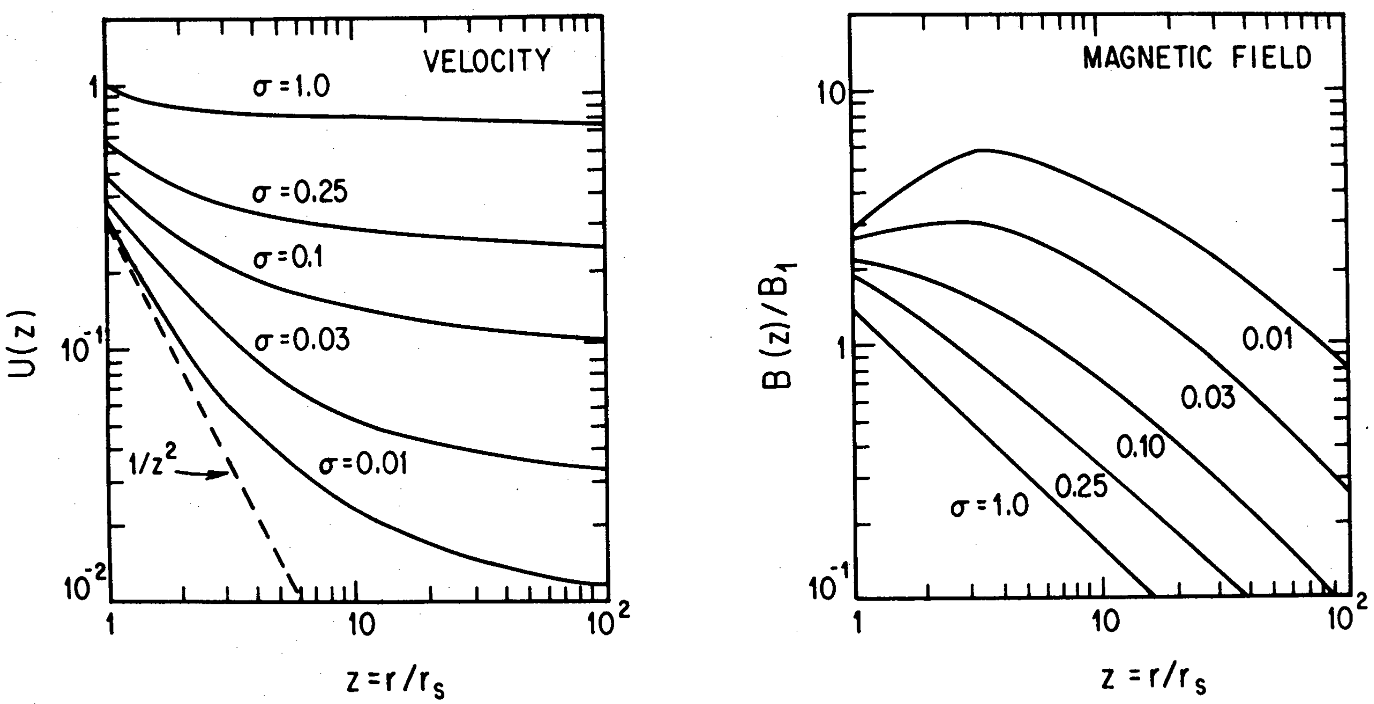
Back to the transport equation:

$$\Big) + \frac{1}{p^2}\frac{\partial}{\partial p}\left(D_{pp}\frac{\partial f}{\partial p}\right) \quad (18)$$

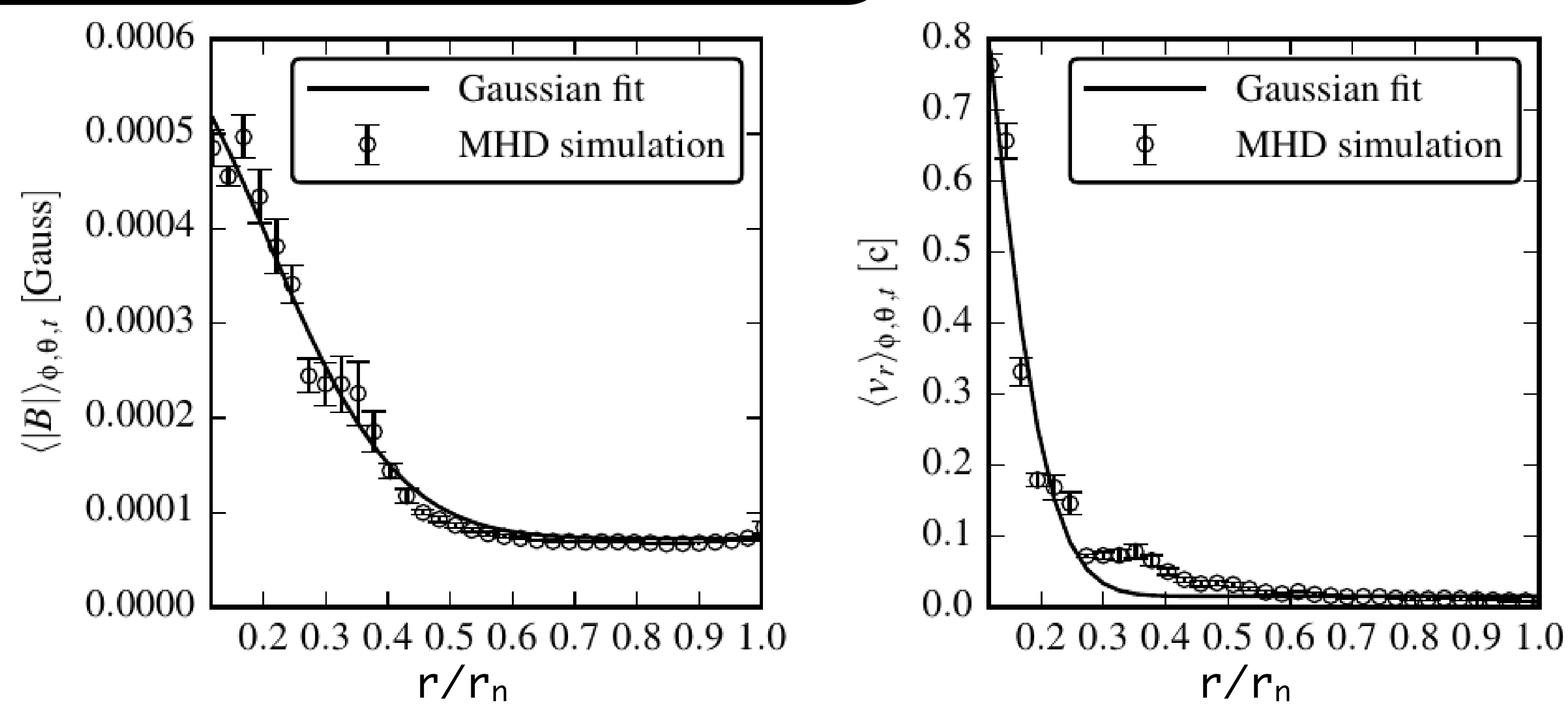
ansport and including **adiabatic** and

$$\frac{1}{3r^2}\frac{\partial}{\partial r}\left(r^2 V(r)\right) + z_p p\Big]\frac{\partial f}{\partial \ln p} + 4 z_p p f = 0, \quad (19)$$

$$\Big)^2 \frac{B^2(r)}{8\pi} \quad (20)$$



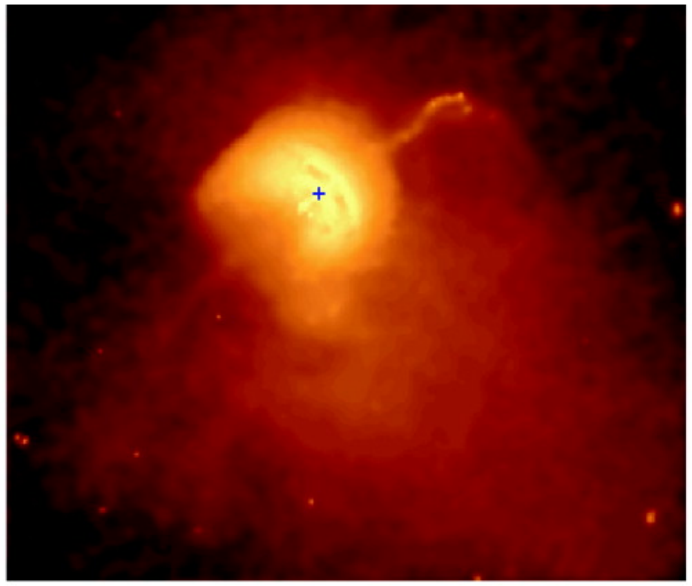
Kennel & Coroniti (1984) laminar model



Averaged profiles from a fiducial MHD simulation

Fokker-Planck spectral modeling, e.g. Tang & Chevalier (2012)

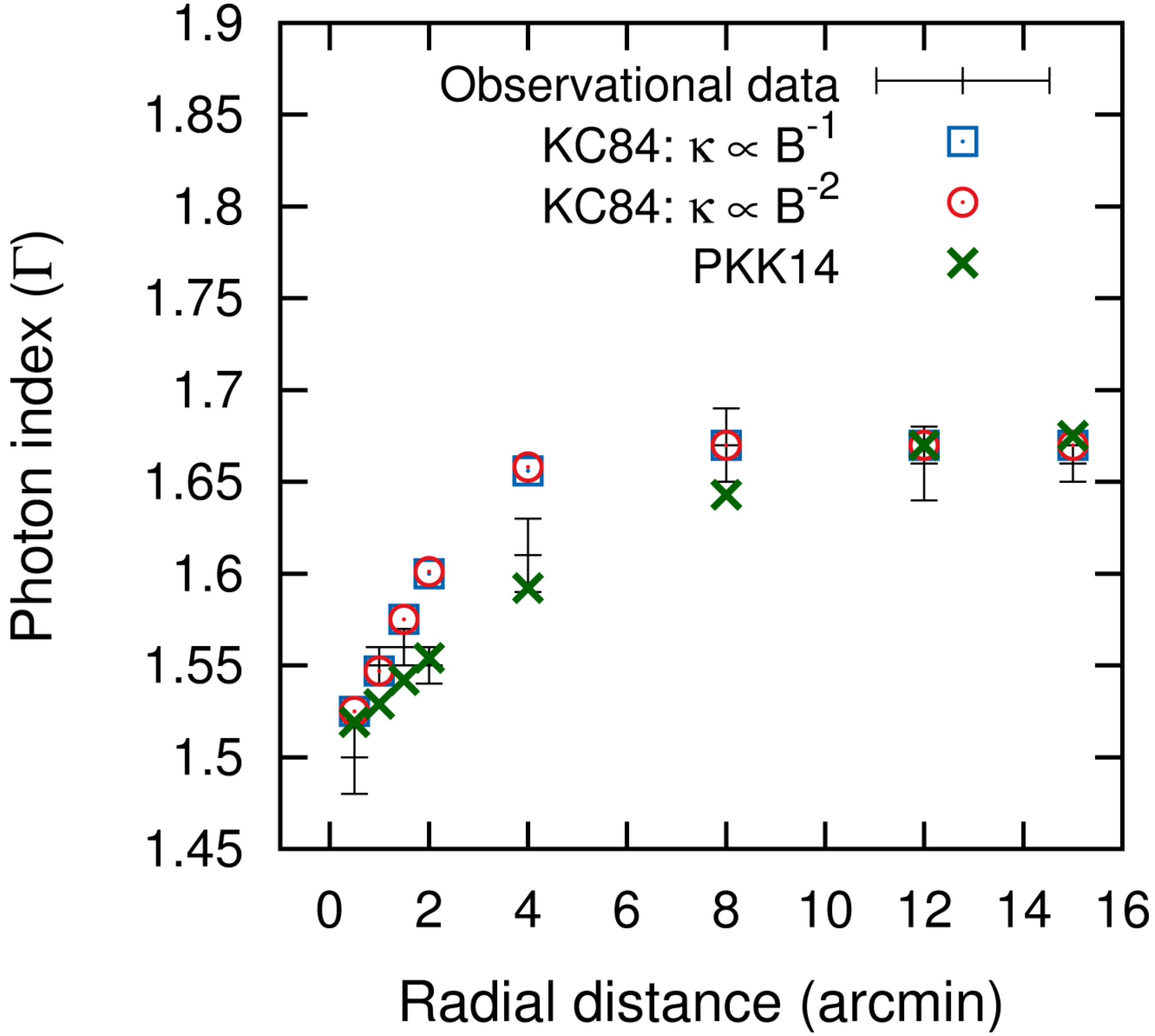
Particle transport in Pulsar Wind Nebulae: **Vela**

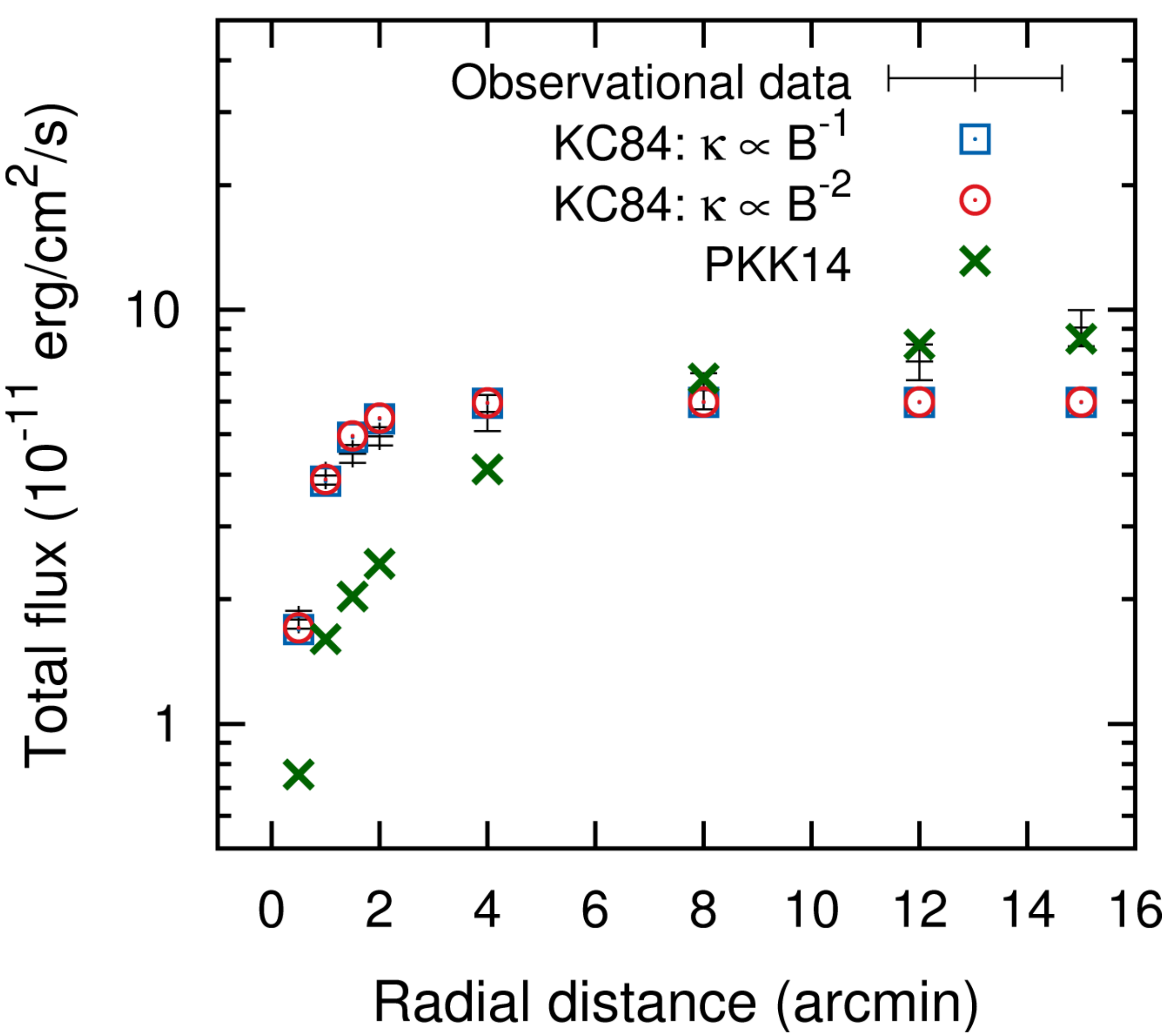


$r_n = 40''$

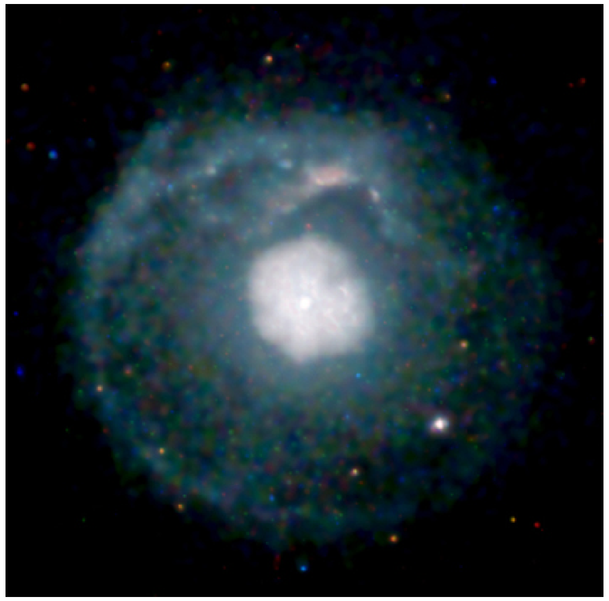
$r_0 = 21''$

$v_n = 1000 \mathrm{km/s}$

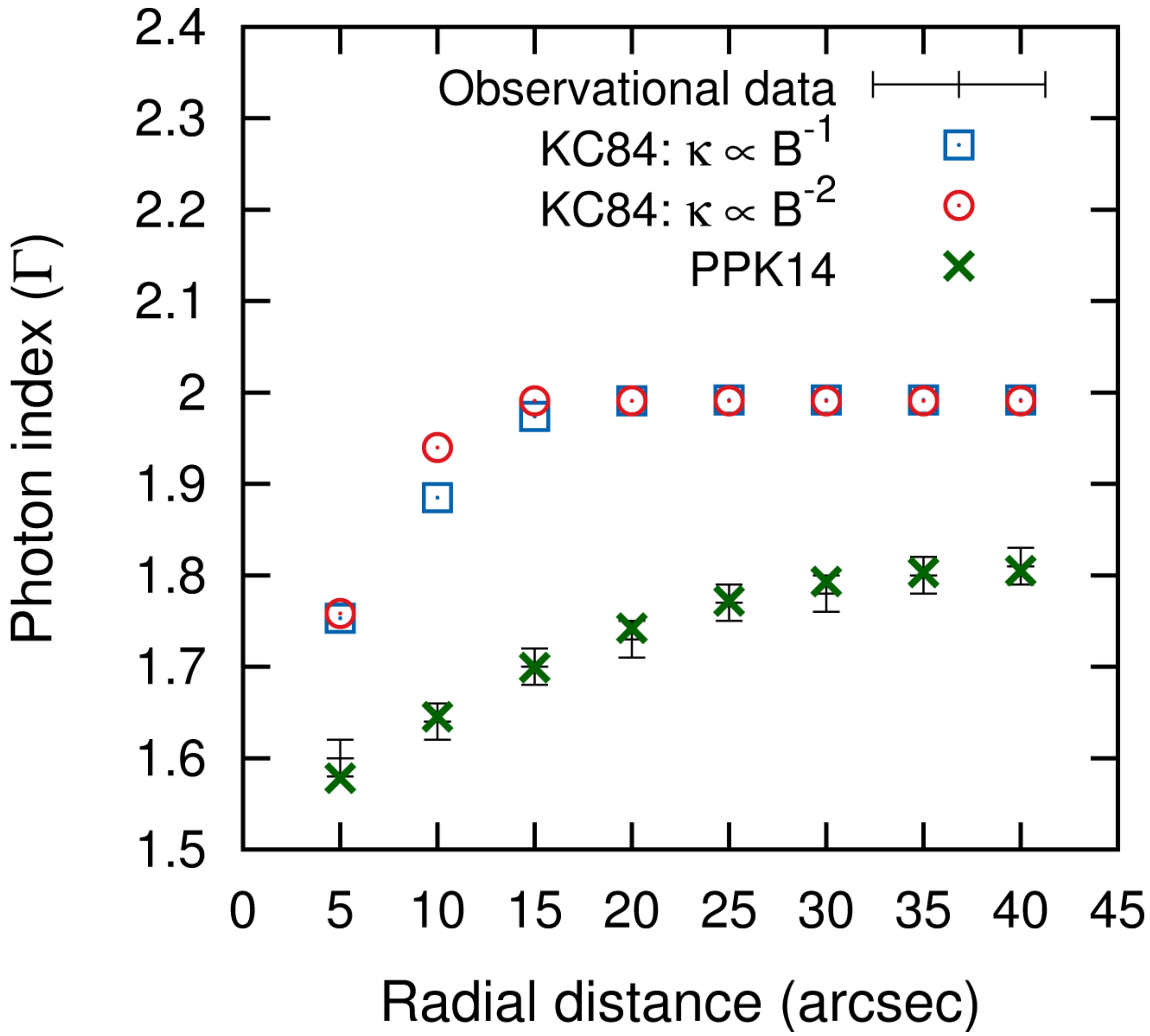


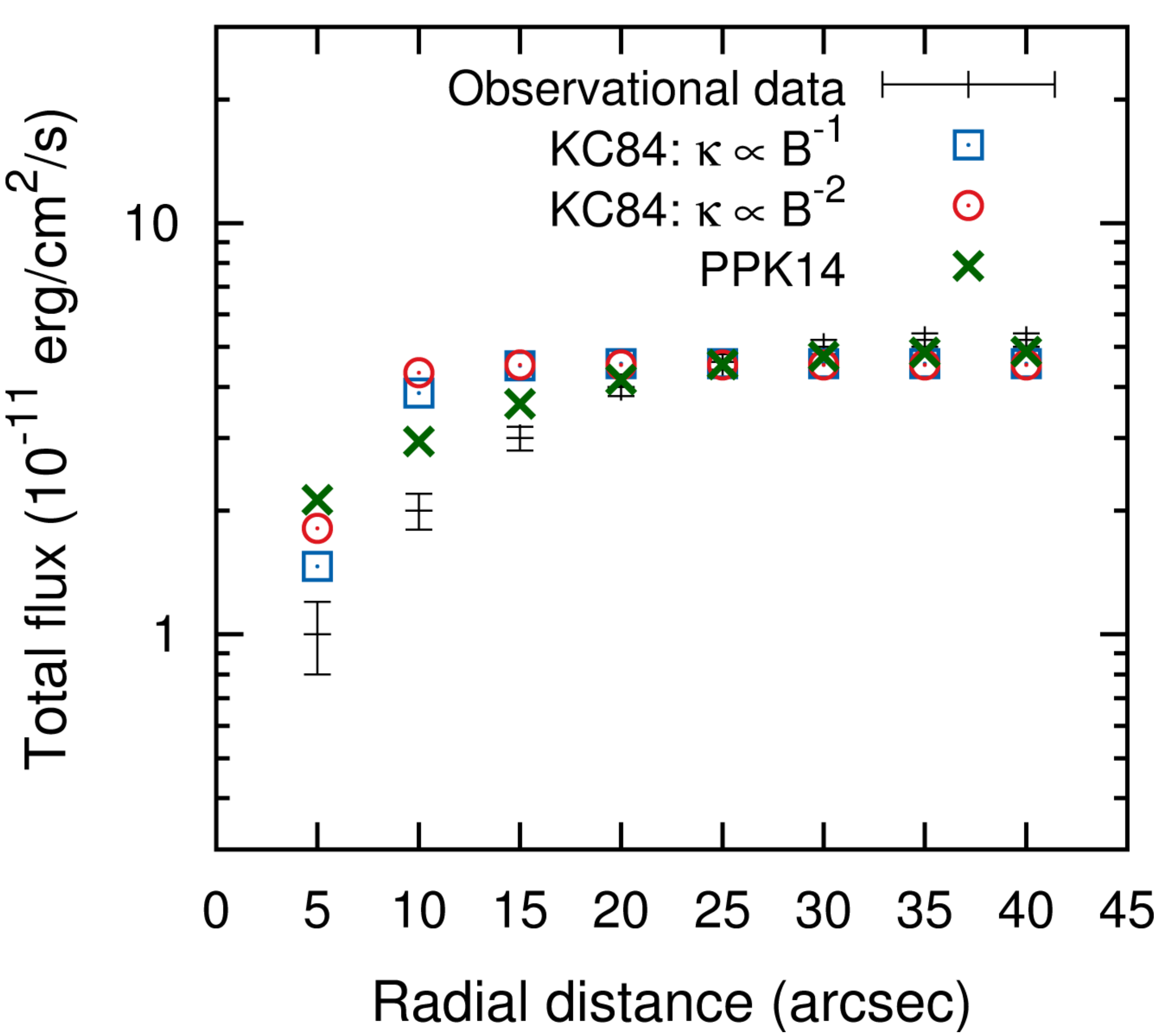


Particle transport in Pulsar Wind Nebulae: **G21.5-0.9**

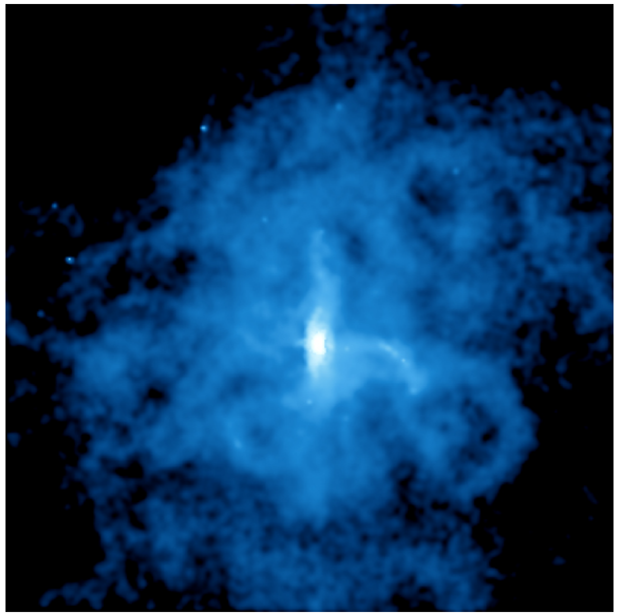


$$r_n = 40''$$
$$L_s = 1''.5$$
$$v_n = 910\mathrm{km/s}$$

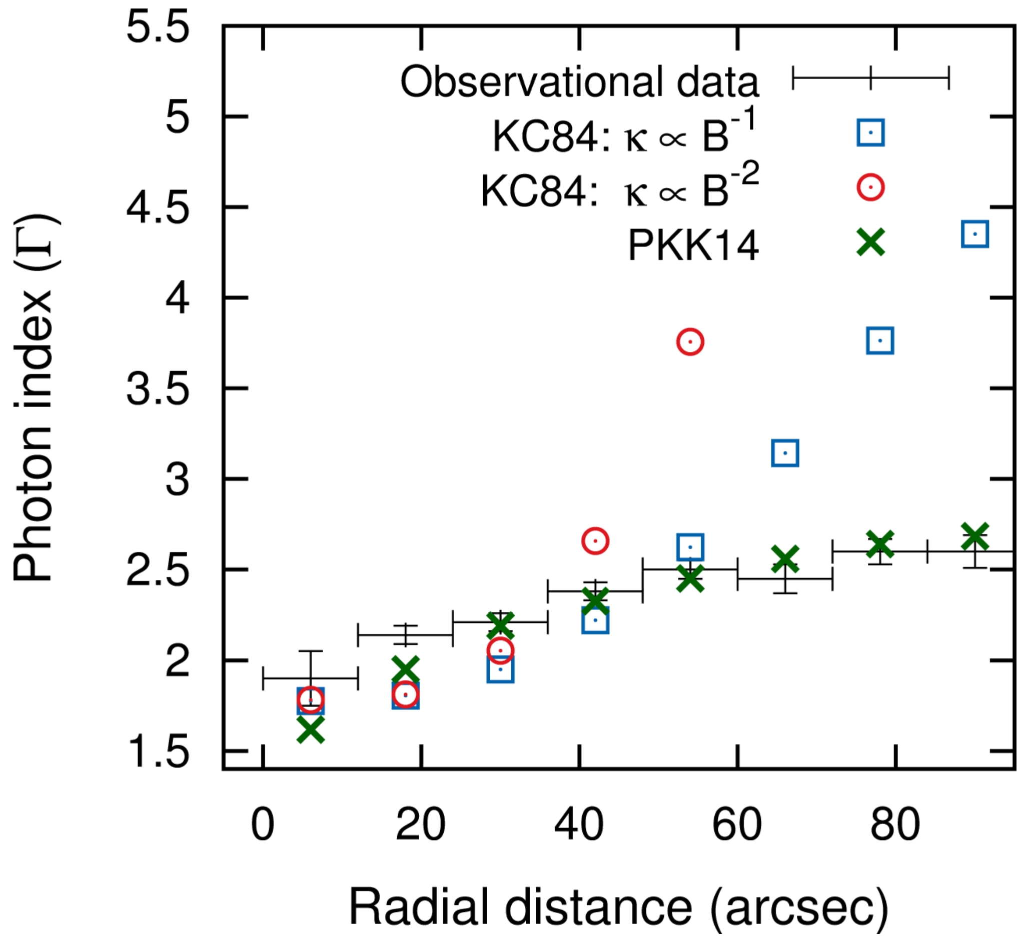




Particle transport in Pulsar Wind Nebulae: **3C58**



Parameter	G21.5-0.9 KC84(a)	G21.5-0.9 KC84(b)	G21.5-0.9 PKK14	Vela KC84(a)	Vela KC84(b)	Vela PKK14	3C 58 KC84(a)	3C 58 KC84(b)	3C 58 PKK14
B_s (μG)	33	33	283	264	264	38	8	8	300
V_s (units of c)	0.36	0.36	0.51	0.52	0.52	0.51	0.35	0.35	0.51
κ_s (10^{26} cm^2 s^{-1})	8.5	6.0	5.7	0.5	0.5	1.4	27.8	17.1	13.3
σ (10^{-3})	1.3	1.3	−−	142	143	−−	0.6	0.6	−−
η (10^{-2})	8.8	22.5	4.5	0.3	0.3	2.1	−−	−−	−−
$\bar{B}$ (μG)	158	158	43	30	30	5.8	63	63	46
$\bar{V}$ (10^{-3}, units of c)	4.2	4.2	3.1	159	159	3.3	4.5	4.5	2.6
$\bar{\kappa}$ (10^{26} cm^2 s^{-1})	1.2	0.3	5.7	1.1	0.9	1.4	1.8	0.3	13.3
$\bar{\xi}$	2.0	9.7	0.3	129	186	2.1	2.0	15	0.2



- Re-scaled simulation give better fits for photon index maps
- Typically, lower average magnetic field strength and velocities obtained
- Peclet number $\bar{\xi} = \frac{vr}{D_{rr}} = O(1)$

  thus diffusion important transport mechanism!

$$v_n = 730\mathrm{km/s} \quad r_0 = 5''.25 \quad r_n = 90''$$

Conclusions Particle transport

- Turbulence in PWN gives rise to high diffusive particle transport, most likely dominates over kinetic diffusion. Peclet numbers of ~1
- Propose a scaling of diffusion with the predominant scale: Termination shock
- Particle escape time in Crab: ~300 years
- Test particle simulations find energy independent diffusive regime, not Bohm.
- Performed fits of X-ray flux and spectral index for three young PWN
- Yields acceptable fits also for σ_{wind}~1 in particular for the X-ray spectral index
- Don‘t use Kennel-Coroniti model! Its wrong and does not tell you anything about σ_{wind}