



# A NOVEL MAPPING OF THE NON-LOCAL FOCK TERMS ONTO THE RELATIVISTIC HARTREE THEORY

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Pulsars and their environments

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# COVARIANT DFT WITH LOCALIZED FOCK

Mapping

## Idea

### Hartree

*Properties*

Momentum *independent*

No exchange (Fock) part in the interaction

Phenomenological density dependence

**Local**



### Hartree-Fock

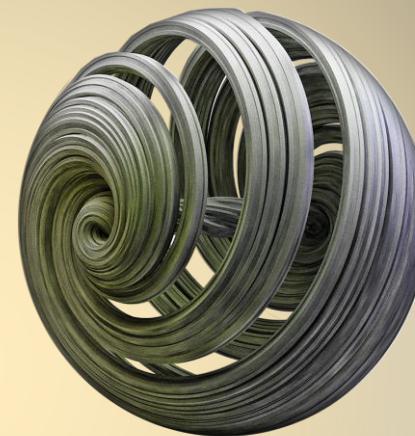
**Non-local**

*Properties*

Momentum *dependent*

Exchange (Fock) correlations

Intrinsic density dependence



# COVARIANT DFT WITH LOCALIZED FOCK

Mapping

## Idea

Hartree

Properties

Momentum *independent*

No exchange (Fock) part in the interaction

Phenomenological density dependence

Local



Hartree-Fock

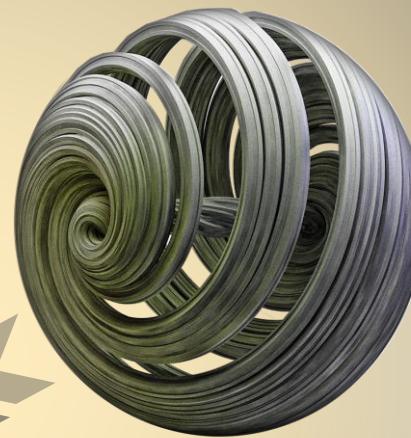
Non-local

Properties

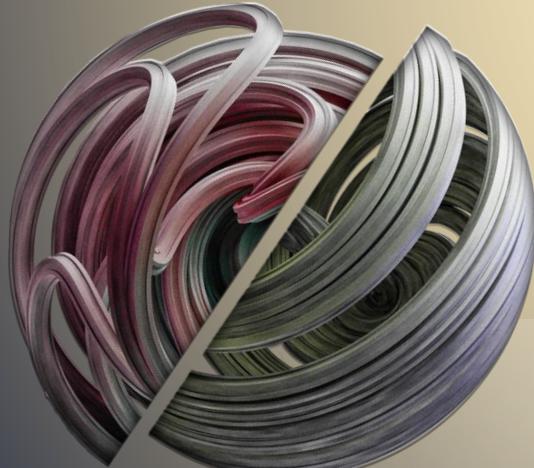
Momentum *dependent*

Exchange (Fock) correlations

Intrinsic density dependence



Hartree +



Covariant DFT with localized Fock terms

Properties

Momentum independent – *Locality*

Precisely accounted exchange correlations – *Fock effects*

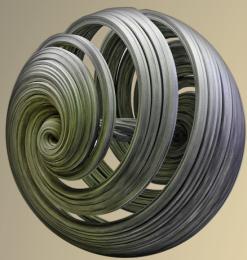
Hartree-Fock *density dependence*

# COVARIANT DFT WITH LOCALIZED FOCK

## Relativistic Hartree-Fock Approach

### Starting point

RHF with coupling constants



- + no new parameters
- + local densities
- + high accuracy
- symmetric matter

### RHF Lagrangian density

$$\mathcal{L}_{\text{HF}} = \mathcal{L}_{\text{HF}}^{\text{fermi}} + \mathcal{L}_{\text{HF}}^{\text{bose}} + \mathcal{L}_{\text{HF}}^{\text{int}}$$

$$\mathcal{L}_{\text{HF}}^{\text{fermi}} = + \bar{\psi} [i\gamma^\mu \partial_\mu - m] \psi$$

$$\begin{aligned} \mathcal{L}_{\text{HF}}^{\text{bose}} = & + \frac{1}{2} \partial_\mu \hat{\boldsymbol{\pi}} \cdot \partial^\mu \hat{\boldsymbol{\pi}} - \frac{1}{2} m_\pi^2 \hat{\boldsymbol{\pi}}^2 \\ & + \frac{1}{2} \partial_\mu \hat{\sigma} \partial^\mu \hat{\sigma} - \frac{1}{2} m_\sigma^2 \hat{\sigma}^2 - \frac{1}{4} \hat{\omega}_{\mu\nu} \hat{\omega}^{\mu\nu} + \frac{1}{2} m_\omega^2 \hat{\omega}_\mu \hat{\omega}^\mu \\ & + \frac{1}{2} \partial_\mu \hat{\boldsymbol{\delta}} \cdot \partial^\mu \hat{\boldsymbol{\delta}} - \frac{1}{2} m_\delta^2 \hat{\boldsymbol{\delta}}^2 - \frac{1}{4} \hat{\boldsymbol{\rho}}_{\mu\nu} \cdot \hat{\boldsymbol{\rho}}^{\mu\nu} + \frac{1}{2} m_\rho^2 \hat{\boldsymbol{\rho}}_\mu \cdot \hat{\boldsymbol{\rho}}^\mu \\ & - \frac{1}{4} \hat{A}_{\mu\nu} \hat{A}^{\mu\nu}, \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{HF}}^{\text{int}} = & + \bar{\psi} [g_\sigma \hat{\sigma} + g_\delta \hat{\boldsymbol{\delta}} \cdot \boldsymbol{\tau} - g_\omega \gamma^\mu \hat{\omega}_\mu - g_\rho \gamma^\mu \hat{\boldsymbol{\rho}}_\mu \cdot \boldsymbol{\tau} \\ & - \frac{f_\pi}{m_\pi} \gamma_5 \gamma^\mu \partial_\mu \hat{\boldsymbol{\pi}} \cdot \boldsymbol{\tau} - \frac{f_\rho}{2m} \sigma^{\mu\nu} \partial_\mu \hat{\boldsymbol{\rho}}^\nu \cdot \boldsymbol{\tau} \\ & - e \gamma^\mu Q \hat{A}_\mu] \psi, \end{aligned}$$

$\pi_{\text{pv}}$   
 $\sigma$   
 $\omega$   
 $\delta$   
 $\rho_v$   
 $\rho_t$   
 $\rho_{vt}$

A. Bouyssy, et al., PRC 36, 380 (1987)

	HF <sub>1</sub>	HF <sub>2</sub>	HF <sub>3</sub>	HF <sub>4</sub>
$g_\sigma$	8.3437	8.1994	5.3409	7.2302
$g_\omega$	12.4021	12.4930	11.2099	11.8529
$g_\pi$	—	13.6433	13.6433	13.6433
$f_\pi$	—	1.00265	1.00265	1.00265
$g_\rho$	—	—	2.62897	2.62897
$f_\rho$	—	—	17.35123	9.72719
$\kappa$	—	—	6.6	3.7

# COVARIANT DFT WITH LOCALIZED FOCK

## Relativistic Hartree-Fock Approach

### Dirac equation

$$[(\gamma^\mu k_\mu - \gamma^0 \Sigma_o - \boldsymbol{\gamma} \cdot \hat{\mathbf{k}} \Sigma_v) - (m + \Sigma_s)] \psi = 0$$

### Energy density

### 2 ENERGY DENSITY

$$\mathcal{E} = \langle \psi_0 | \mathcal{H} | \psi_0 \rangle = \epsilon_{kin} + \epsilon_D + \epsilon_E$$

HARTREE

$$\epsilon^D = \sum_{\sigma, \omega, \rho, \delta} \epsilon_i^D$$

$$\epsilon_\sigma^D = -\frac{1}{2} \frac{g_\sigma^2}{m_\sigma^2} \rho_S^2$$

$$\epsilon_\omega^D = +\frac{1}{2} \frac{g_\omega^2}{m_\omega^2} \rho_B^2$$

$$\epsilon_\delta^D = -\frac{1}{2} \frac{g_\delta^2}{m_\delta^2} \rho_{S_3}^2$$

$$\epsilon_\rho^D = +\frac{1}{2} \frac{g_\rho^2}{m_\rho^2} \rho_{B_3}^2$$

### 1 SELF-ENERGY

$$\Sigma(k, k_F) = \Sigma_s(k, k_F) + \gamma^0 \Sigma_o(k, k_F) + \boldsymbol{\gamma} \cdot \hat{\mathbf{k}} \Sigma_v(k, k_F)$$



Functional derivatives  
with respect to density

FOCK

$$\epsilon^E = \sum_{\sigma, \omega, \delta} \frac{1}{2} \frac{1}{(2\pi)^4} \sum_{\tau, \tau'} f_{\tau, \tau'} \int_0^{k_{F\tau}} k dk \int_0^{k_{F\tau'}} q dq \left[ A_j(k, q) + \hat{M}(k) \hat{M}(q) B_j(k, q) + \hat{P}(k) \hat{P}(q) C_j(k, q) \right]$$

Motivation

- Momentum dependent meson propagators
  - Fierz transformation
  - Heavy meson mass limit

# COVARIANT DFT WITH LOCALIZED FOCK

## Localized Fock Approximation

### Energy density

$$\mathcal{E}^E = \mathcal{E}_o^E + \mathcal{E}_s^E + \mathcal{E}_v^E + \mathcal{E}_{\rho_{vt}}^E$$

$$\mathcal{E}_o^E = \sum_{\sigma, \omega, \delta, \rho_v} \frac{\gamma}{4(2\pi)^4} \int_0^{k_F} k dk \int_0^{k_F} q dq \Theta_j(k, q),$$

$$\mathcal{E}_s^E = \sum_{\sigma, \omega, \delta, \rho_v} \frac{\gamma}{4(2\pi)^4} \int_0^{k_F} k dk \hat{M}(k) \int_0^{k_F} q dq \hat{M}(q) \Theta_j(k, q),$$

$$\mathcal{E}_v^E = \sum_{\sigma, \omega, \delta, \rho_v} \frac{\gamma}{4(2\pi)^4} \int_0^{k_F} k dk \hat{P}(k) \int_0^{k_F} q dq \hat{P}(q) \Phi_i(k, q)$$

$$+ \frac{\gamma}{4m_\pi^2} \frac{1}{(2\pi)^4} \int_0^{k_F} k dk \hat{P}(k) \int_0^{k_F} q dq \hat{P}(q) \Lambda_\pi(k, q)$$

$$+ \frac{\gamma}{4m_\rho^2} \frac{1}{(2\pi)^4} \int_0^{k_F} k dk \hat{P}(k) \int_0^{k_F} q dq \hat{P}(q) \Psi_\rho(k, q),$$

$$\mathcal{E}_{\rho_{vt}}^E = \frac{\gamma}{4m} \frac{1}{(2\pi)^4} \int_0^{k_F} k dk \hat{P}(k) \int_0^{k_F} q dq \hat{M}(q) \Omega_\rho(k, q).$$

$$\hat{M} = \frac{m^*}{E^*}, \quad \hat{P} = \frac{p^*}{E^*}$$

### Angular exchange functions

$$F = (\Theta_j, \Phi_i, \Lambda_\pi, \Psi_\rho, \Omega_\rho)$$

TIME-LIKE VECTOR

SCALAR

SPACE-LIKE VECTOR

$$I_F = \int_0^{k_F} k dk \int_0^{k_F} q dq F(k, q)$$

Analytic  
solution

LFA  
for Energy density

$$J_F \rho_B^2 \approx I_F \rho_S^2$$

$$K_F \rho_B^2 \approx I_F \rho_V^2$$

$$L_F \rho_B^2 \approx I_F \rho_V \rho_S$$

Densities

$$\rho_B = \frac{\gamma}{2\pi^2} \int_0^{k_F} k^2 dk = \frac{\gamma}{2} \frac{k_F^3}{3\pi^2}$$

$$\rho_S = \frac{\gamma}{2\pi^2} \int_0^{k_F} k^2 dk \hat{M}(k)$$

$$\rho_V = \frac{\gamma}{2\pi^2} \int_0^{k_F} k^2 dk \hat{P}(k)$$

# COVARIANT DFT WITH LOCALIZED FOCK

## Localized Fock Approximation

### Self-energy



momentum  
dependent

$$\Sigma_o^E = \sum_{\sigma, \omega, \delta, \rho_v} \frac{\bar{A}_j}{(4\pi)^2} \frac{1}{k} \int_0^{k_F} q dq \Theta_j(k, q),$$

$$\Sigma_s^E = \sum_{\sigma, \omega, \delta, \rho_v} \frac{\bar{B}_j}{(4\pi)^2} \frac{1}{k} \int_0^{k_F} q dq \hat{M}(q) \Theta_j(k, q)$$

$$+ \frac{1}{2} \frac{1}{m} \frac{\bar{D}_{\rho_{vt}}}{(4\pi)^2} \frac{1}{k} \int_0^{k_F} q dq \hat{P}(q) \Omega_\rho(k, q),$$

$$\Sigma_v^E = \sum_{\sigma, \omega, \delta, \rho_v} \frac{\bar{C}_i}{(4\pi)^2} \frac{1}{k} \int_0^{k_F} q dq \hat{P}(q) \Phi_i(k, q)$$

$$+ \frac{1}{m_\pi^2} \frac{\bar{C}_{\pi_{pv}}}{(4\pi)^2} \frac{1}{k} \int_0^{k_F} q dq \hat{P}(q) \Lambda_\pi(k, q)$$

$$+ \frac{1}{m_p^2} \frac{\bar{C}_{\rho_t}}{(4\pi)^2} \frac{1}{k} \int_0^{k_F} q dq \hat{P}(q) \Psi_\rho(k, q)$$

$$+ \frac{1}{2} \frac{1}{m} \frac{\bar{D}_{\rho_{vt}}}{(4\pi)^2} \frac{1}{k} \int_0^{k_F} q dq \hat{M}(q) \Omega_\rho(k, q).$$

$$I_F^\Sigma = \langle I_F^\Sigma(k) \rangle_k$$

$$J_F^\Sigma = \langle J_F^\Sigma(k) \rangle_k$$

$$K_F^\Sigma = \langle K_F^\Sigma(k) \rangle_k$$

k-average  
over Fermi sphere

TIME-LIKE VECTOR

SCALAR

SPACE-LIKE VECTOR

LFA  
for Self-energy 2

$$I_F^\Sigma \rho_B^2 = \frac{\gamma}{2\pi^2} I_F \rho_B$$

$$J_F^\Sigma \rho_B^2 \approx \frac{\gamma}{2\pi^2} I_F \rho_S$$

$$K_F^\Sigma \rho_B^2 \approx \frac{\gamma}{2\pi^2} I_F \rho_V$$

Analytic  
solution

$$I_F = \int_0^{k_F} k dk \int_0^{k_F} q dq F(k, q)$$

Angular exchange functions

$$F = (\Theta_j, \Phi_i, \Lambda_\pi, \Psi_\rho, \Omega_\rho)$$

$$\hat{M} = \frac{m^*}{E^*}, \quad \hat{P} = \frac{p^*}{E^*}$$

# COVARIANT DFT WITH LOCALIZED FOCK

## Localized Fock Approximation

### New coupling functions

#### Energy density

$$\varepsilon_o = \varepsilon_o^D + \varepsilon_o^E = +\frac{1}{2} \left[ \frac{g_\omega^2}{m_\omega^2} + \sum_{\sigma, \omega, \delta, \rho_v} \frac{A_j}{2\gamma} \frac{g_j^2}{m_j^2} X_{\Theta_j}(k_F, m_j) \right] \rho_B^2 = +\frac{1}{2} \frac{G_\omega^2}{m_\omega^2} \rho_B^2,$$

$$\varepsilon_s = \varepsilon_s^D + \varepsilon_s^E = -\frac{1}{2} \left[ \frac{g_\sigma^2}{m_\sigma^2} - \sum_{\sigma, \omega, \delta, \rho_v} \frac{B_j}{2\gamma} \frac{g_j^2}{m_j^2} X_{\Theta_j}(k_F, m_j) \right] \rho_s^2 = -\frac{1}{2} \frac{G_\sigma^2}{m_\sigma^2} \rho_s^2,$$

$$\varepsilon_v = \varepsilon_v^E = +\frac{1}{2} \left[ \sum_{\sigma, \omega, \delta, \rho_v} \frac{C_j}{2\gamma} \frac{g_j^2}{m_j^2} Y_j(k_F, m_j) \right] \rho_v^2 = +\frac{1}{2} \frac{G_v^2}{m_\pi^2} \rho_v^2,$$

$$\varepsilon_{\rho_{vt}}^E = +\frac{1}{2} \left[ \frac{D_{\rho_{vt}}}{2\gamma} \frac{g_\rho^2}{m_\rho^2} Z_{\Omega_\rho}(k_F, m_\rho, m) \right] \rho_V \rho_S = +\frac{1}{2} \frac{G_{\rho_{vt}}^2}{m_\rho^2} \rho_V \rho_S.$$

#### Self-energy

$$\Sigma_o = \Sigma_o^D + \Sigma_o^E = +\left[ \frac{g_\omega^2}{m_\omega^2} + \sum_{\sigma, \omega, \delta, \rho_v} \frac{A_j}{2\gamma} \frac{g_j^2}{m_j^2} X_{\Theta_j}(k_F, m_j) \right] \rho_B = +\frac{G_\omega^2}{m_\omega^2} \rho_B,$$

$$\Sigma_s = \Sigma_s^D + \Sigma_s^E = -\left[ \frac{g_\sigma^2}{m_\sigma^2} - \sum_{\sigma, \omega, \delta, \rho_v} \frac{B_j}{2\gamma} \frac{g_j^2}{m_j^2} X_{\Theta_j}(k_F, m_j) \right] \rho_S = -\frac{G_\sigma^2}{m_\sigma^2} \rho_S,$$

$$\Sigma_v = \Sigma_v^E = +\left[ \sum_{\sigma, \omega, \delta, \rho_v} \frac{C_j}{2\gamma} \frac{g_j^2}{m_j^2} Y_j(k_F, m_j) \right] \rho_V = +\frac{G_v^2}{m_\pi^2} \rho_V,$$

$$\Sigma_{\rho_{vt}} = +\frac{1}{2} \left[ \frac{D_{\rho_{vt}}}{2\gamma} \frac{g_\rho^2}{m_\rho^2} Z_{\Omega_\rho}(k_F, m_\rho, m) \right] (\rho_V + \rho_S) = +\frac{1}{2} \frac{G_{\rho_{vt}}^2}{m_\rho^2} (\rho_V + \rho_S).$$

#### TIME-LIKE VECTOR

#### SCALAR

#### SPACE-LIKE VECTOR

#### FOCK

#### HARTREE

$$\begin{aligned} \frac{G_\sigma^2}{m_\sigma^2} &= +\frac{g_\sigma^2}{m_\sigma^2} - \frac{g_\sigma^2}{m_\sigma^2} \frac{1}{2\gamma} X_{\Theta_\sigma} + \frac{g_\omega^2}{m_\omega^2} \frac{2}{\gamma} X_{\Theta_\omega} \\ &\quad + \frac{g_\rho^2}{m_\rho^2} \frac{2(\gamma-1)}{\gamma} X_{\Theta_\rho} - \frac{g_\delta^2}{m_\delta^2} \frac{(\gamma-1)}{2\gamma} X_{\Theta_\delta} \\ &\quad + \frac{g_\pi^2}{m_\pi^2} \frac{(\gamma-1)}{2\gamma} \frac{m_\pi^2}{4m^2} X_{\Theta_\pi} + \frac{g_\rho^2}{m_\rho^2} \frac{(\gamma-1)}{2\gamma} \frac{3\kappa^2 m_\rho^2}{4m^2} X_{\Theta_\rho} \end{aligned}$$

#### example

$$X_{\Theta_j}(k_F, m_j) = \left[ \frac{9}{4} \frac{m_j^2}{k_F^6} I_{\Theta_j} \right],$$

$$Y_{\Phi_i}(k_F, m_i) = \left[ \frac{9}{4} \frac{m_i^2}{k_F^6} I_{\Phi_i} \right],$$

$$Y_{\Lambda_\pi}(k_F, m_\pi) = \left[ \frac{9}{4} \frac{1}{k_F^6} I_{\Lambda_\pi} \right],$$

$$Y_{\Psi_\rho}(k_F, m_\rho) = \left[ \frac{9}{4} \frac{1}{k_F^6} I_{\Psi_\rho} \right],$$

$$Z_{\Omega_\rho}(k_F, m_\rho, m) = \left[ \frac{9}{4} \frac{1}{m} \frac{m_\rho^2}{k_F^6} I_{\Omega_\rho} \right].$$

**New coupling function**

**Exchange functions**

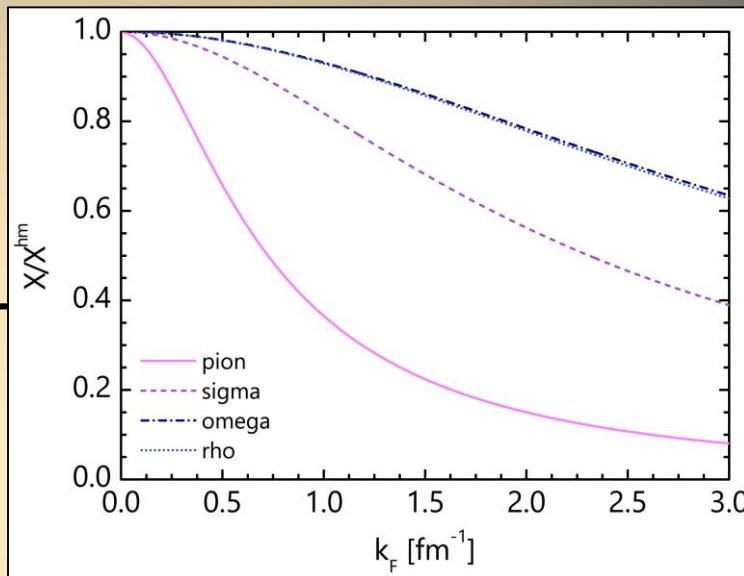
# COVARIANT DFT WITH LOCALIZED FOCK

## Localized Fock Approximation

### New coupling functions

#### Energy density

$$\begin{aligned}\varepsilon_o &= \varepsilon_o^D + \varepsilon_o^E = +\frac{1}{2} \left[ \frac{g_\omega^2}{m_\omega^2} + \sum_{\sigma, \omega, \delta, \rho_v} \frac{A_j}{2\gamma} \frac{g_j^2}{m_j^2} X_{\Theta_j}(k_F, m_j) \right] \rho_B^2 = +\frac{1}{2} \frac{G_\omega^2}{m_\omega^2} \rho_B^2, \\ \varepsilon_s &= \varepsilon_s^D + \varepsilon_s^E = -\frac{1}{2} \left[ \frac{g_\sigma^2}{m_\sigma^2} - \sum_{\sigma, \omega, \delta, \rho_v} \frac{B_j}{2\gamma} \frac{g_j^2}{m_j^2} X_{\Theta_j}(k_F, m_j) \right] \rho_S^2 = -\frac{1}{2} \frac{G_\sigma^2}{m_\sigma^2} \rho_S^2, \\ \varepsilon_v &= \varepsilon_v^E = +\frac{1}{2} \left[ \sum_{\sigma, \omega, \delta, \rho_v} \frac{C_j}{2\gamma} \frac{g_j^2}{m_j^2} Y_j(k_F, m_j) \right] \rho_V^2 = +\frac{1}{2} \frac{G_\pi^2}{m_\pi^2} \rho_V^2, \\ \varepsilon_{\rho_{vt}}^E &= +\frac{1}{2} \left[ \frac{D_{\rho_{vt}}}{2\gamma} \frac{g_\rho^2}{m_\rho^2} Z_{\Omega_\rho}(k_F, m_\rho, m) \right] \rho_V \rho_S = +\frac{1}{2} \frac{G_{\rho_{vt}}^2}{m_\rho^2} \rho_V \rho_S.\end{aligned}$$



**New  
coupling  
function**

#### Self-energy

$$\begin{aligned}\Sigma_o &= \Sigma_o^D + \Sigma_o^E = +\left[ \frac{g_\omega^2}{m_\omega^2} + \sum_{\sigma, \omega, \delta, \rho_v} \frac{A_j}{2\gamma} \frac{g_j^2}{m_j^2} X_{\Theta_j}(k_F, m_j) \right] \rho_B = +\frac{G_\omega^2}{m_\omega^2} \rho_B, \\ \Sigma_s &= \Sigma_s^D + \Sigma_s^E = -\left[ \frac{g_\sigma^2}{m_\sigma^2} - \sum_{\sigma, \omega, \delta, \rho_v} \frac{B_j}{2\gamma} \frac{g_j^2}{m_j^2} X_{\Theta_j}(k_F, m_j) \right] \rho_S = -\frac{G_\sigma^2}{m_\sigma^2} \rho_S, \\ \Sigma_v &= \Sigma_v^E = +\left[ \sum_{\sigma, \omega, \delta, \rho_v} \frac{C_j}{2\gamma} \frac{g_j^2}{m_j^2} Y_j(k_F, m_j) \right] \rho_V = +\frac{G_\pi^2}{m_\pi^2} \rho_V, \\ \Sigma_{\rho_{vt}} &= +\frac{1}{2} \left[ \frac{D_{\rho_{vt}}}{2\gamma} \frac{g_\rho^2}{m_\rho^2} Z_{\Omega_\rho}(k_F, m_\rho, m) \right] (\rho_V + \rho_S) = +\frac{1}{2} \frac{G_{\rho_{vt}}^2}{m_\rho^2} (\rho_V + \rho_S).\end{aligned}$$

#### example

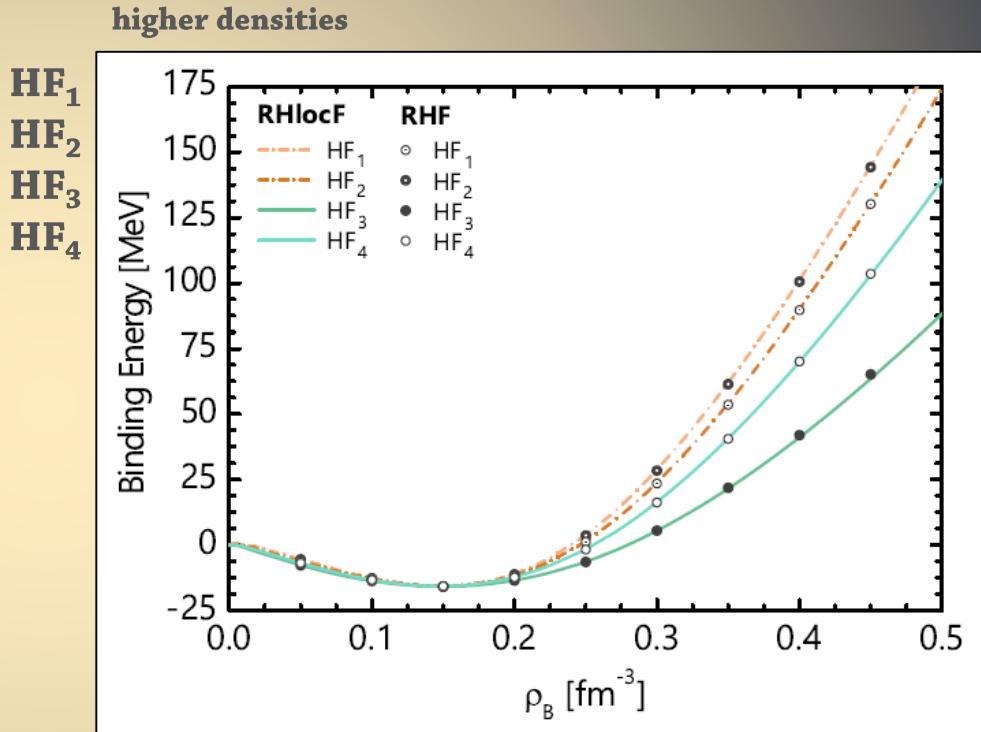
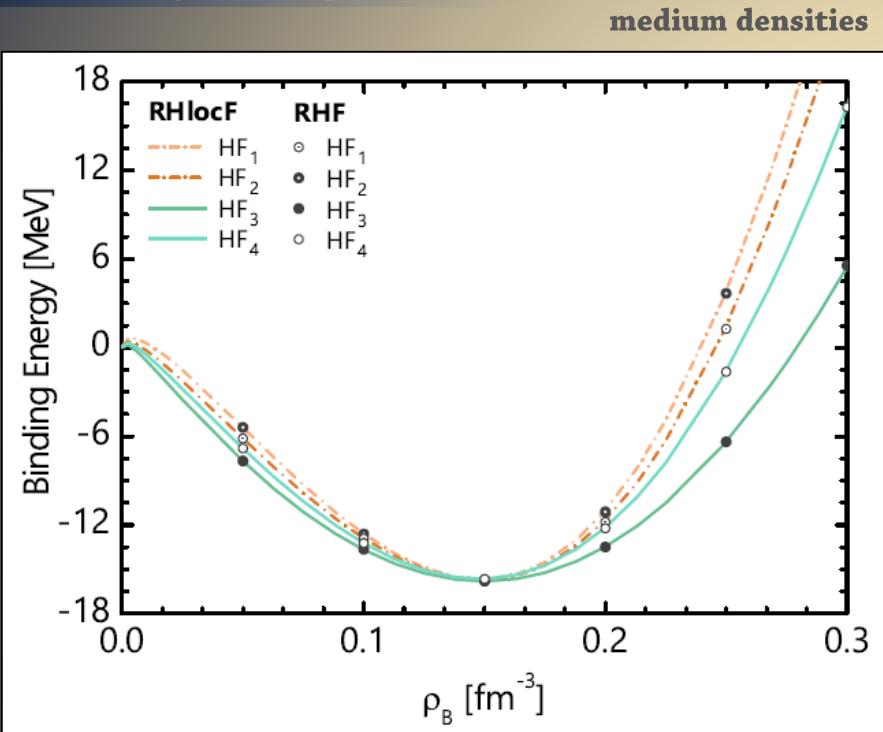
$$\begin{aligned}X_{\Theta_j}(k_F, m_j) &= \left[ \frac{9}{4} \frac{m_j^2}{k_F^6} I_{\Theta_j} \right], \\ Y_{\Phi_i}(k_F, m_i) &= \left[ \frac{9}{4} \frac{m_i^2}{k_F^6} I_{\Phi_i} \right], \\ Y_{\Lambda_\pi}(k_F, m_\pi) &= \left[ \frac{9}{4} \frac{1}{k_F^6} I_{\Lambda_\pi} \right], \\ Y_{\Psi_\rho}(k_F, m_\rho) &= \left[ \frac{9}{4} \frac{1}{k_F^6} I_{\Psi_\rho} \right], \\ Z_{\Omega_\rho}(k_F, m_\rho, m) &= \left[ \frac{9}{4} \frac{1}{m} \frac{m_\rho^2}{k_F^6} I_{\Omega_\rho} \right].\end{aligned}$$

**Exchange  
functions**

# COVARIANT DFT WITH LOCALIZED FOCK

## Results – Symmetric matter

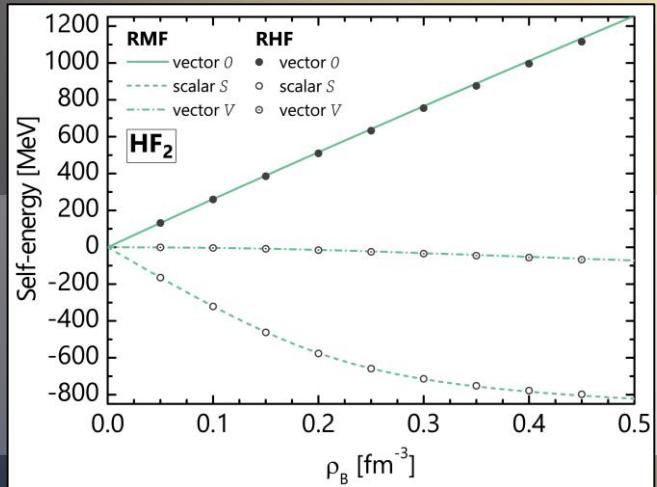
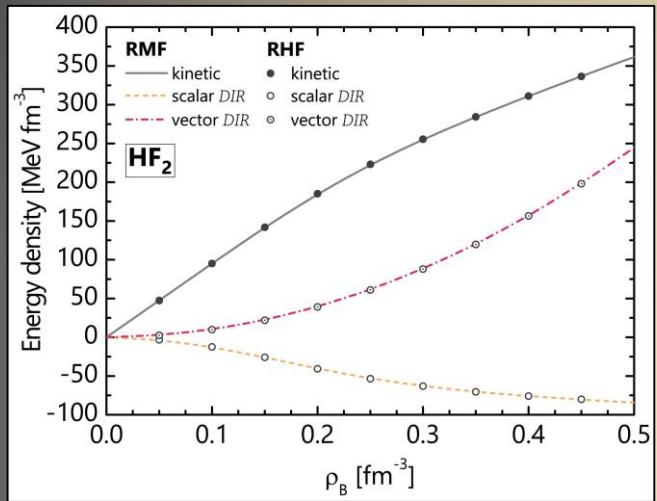
### Binding energy



### Results

- Excellent reproduction up to  $3\rho_{\text{sat}}$
- All parametric sets show no relevant deviations
  - RHF saturation densities well reproduced

# COVARIANT DFT WITH LOCALIZED FOCK

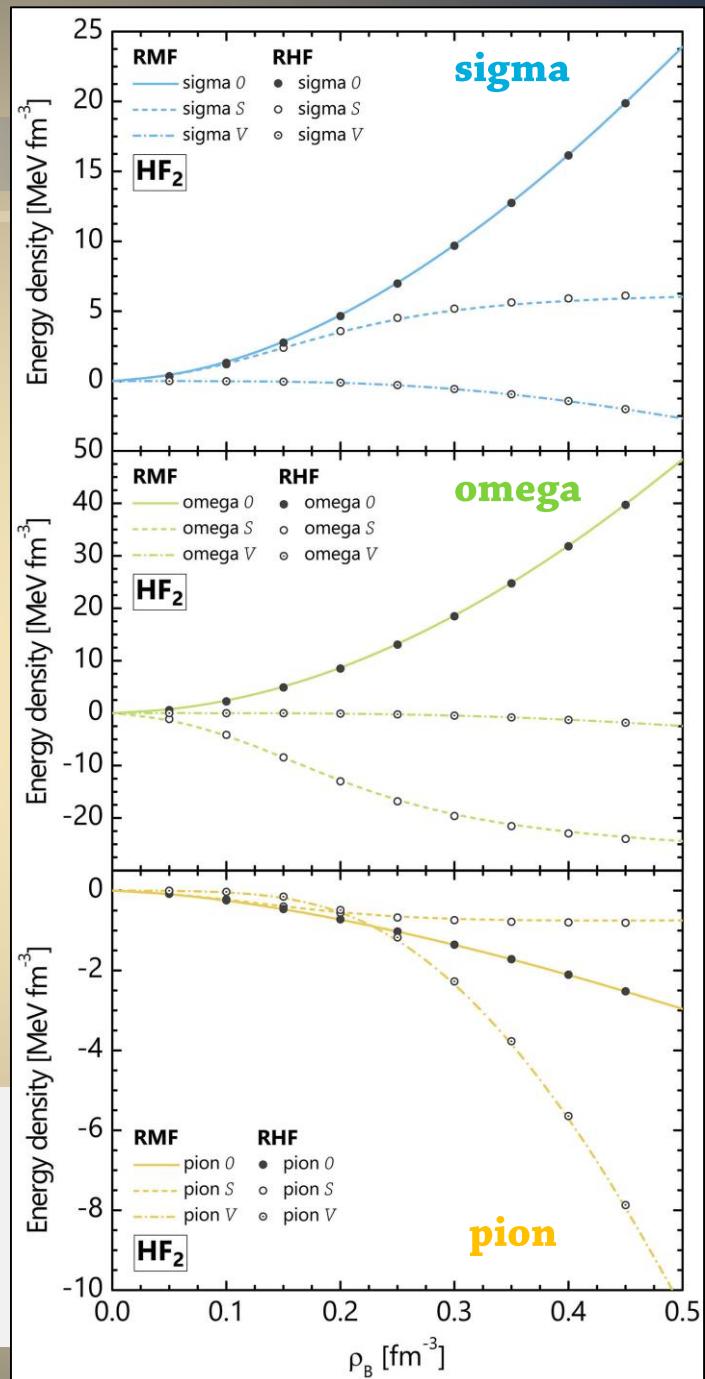


**kinetic**

## **vector DIRECT**

scalar DIRECT

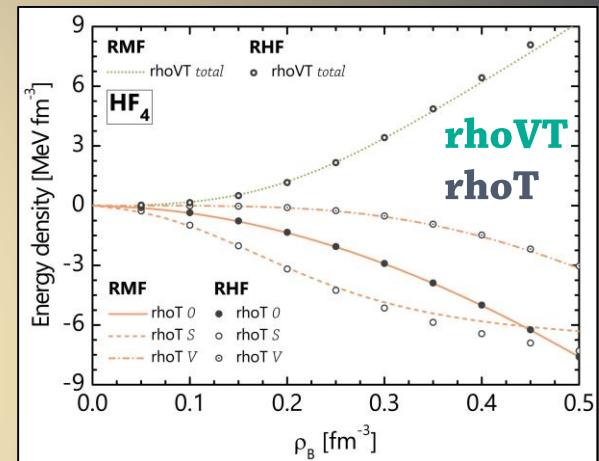
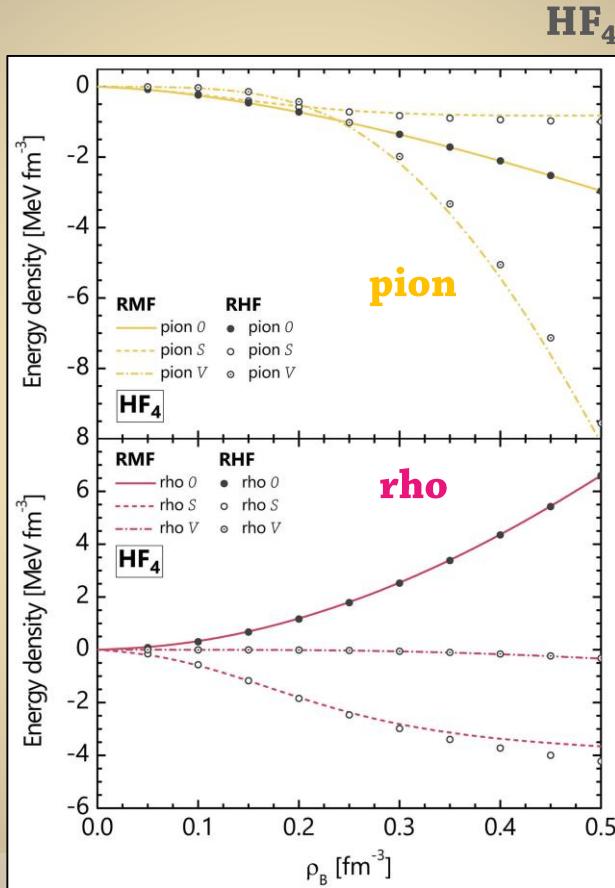
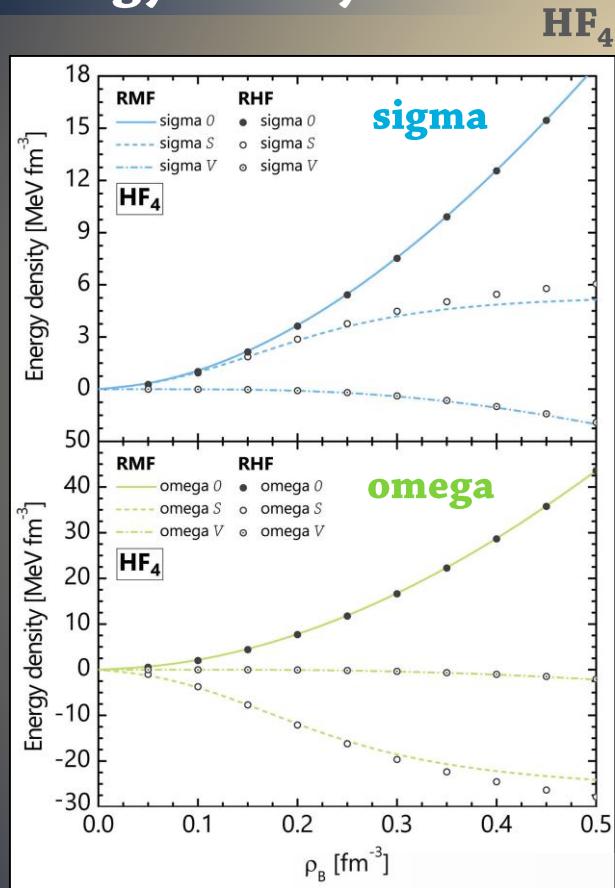
HF<sub>2</sub>



# COVARIANT DFT WITH LOCALIZED FOCK

## Results – Symmetric matter

### Energy density



Results

- Excellent reproduction up to  $2\rho_{\text{sat}}$
- Mild deviations in the SCALAR channel for higher densities
- Scalar density responsible for the differences

# COVARIANT DFT WITH LOCALIZED FOCK

## Future

1

### *Density-dependent models*

*PKO1-3, PKA1 parametrizations for RHF model*

*Rearrangement self-energy*

2

### *Asymmetric nuclear matter*

*LFA in the asymmetric case*

*Neutron matter studies*

*Extension to neutron stars*

3

### *Finite nuclei calculations*

*Finite nuclei in local density approximation*

THANK YOU!