Theory of pulsar magnetospheres

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Summary

The Crab pulsar : a standard example ?

Pulsed emission

- in radio
- IR/optical/UV
- X-ray
- γ-ray
- all pulses in phase !

$\Rightarrow$ Does it mean that there is only one emission site ?

$\Rightarrow$ Production of coherent and incoherent radiation at the same place !

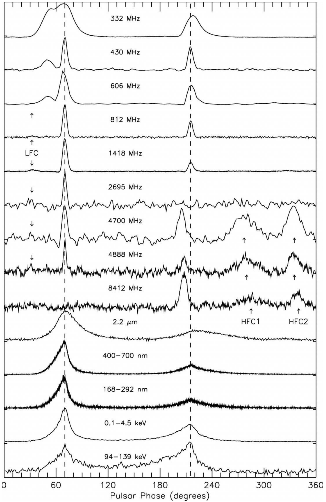

FIGURE : *Crab pulsar light-curves* (Hankins & Eilek, 2007).

Key questions (still unanswered)

- where does radio emission come from ?
- where does high energy emission come from ?
- what is the coherent radio emission mechanism ?
- where are particles accelerated ?
- to what energies are they accelerated ?
- what is the plasma composition, its density ?
- where is this plasma located within the magnetosphere ?

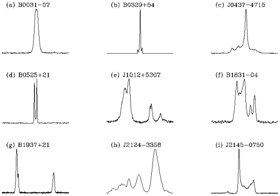

FIGURE : *Some radio pulse profiles (Lorimer, 2008).*

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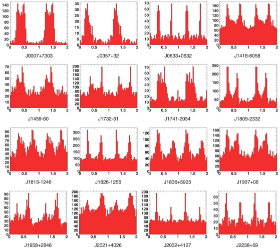

FIGURE : *Gamma-ray light-curves* (Abdo et al., 2009).

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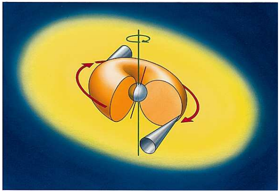

FIGURE : *Emission model from Larousse.*

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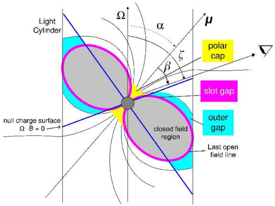

FIGURE : *Acceleration zones in pulsars.*

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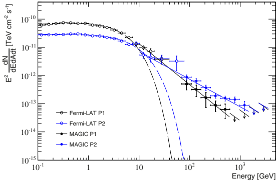

FIGURE : *Very high energy emission in the Crab (Ansoldi et al., 2016).*

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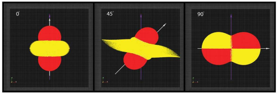

FIGURE : *Electron-positron electrosphere* (McDonald & Shearer, 2009).

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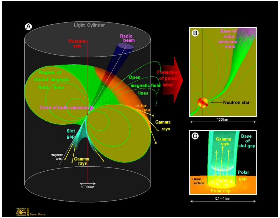


FIGURE : *Magneosphere view.*

Electromagnetic and gravitational field characteristics

- electric field induced at the stellar crust

$$E = \Omega\, B\, R = 10^{13}\ \mathrm{V/m}$$

$\Rightarrow$ instantaneous acceleration at ultra-relativistic speeds, Lorentz factor $\gamma \gg 1$ ($\tau_{\rm acc} < 10^{-20}$ s)

- negligible gravitational force for protons ! ! !

$$\frac{F_{\rm grav}}{F_{\rm em}} \approx \frac{G\, M\, m_p / R^2}{e\, \Omega\, B\, R} \approx 10^{-12} \ll 1$$

even smaller for electrons/positrons (m_e/m_p)

$\Rightarrow$ dynamic of the magnetosphere dominated by the electromagnetic field

Exact solutions

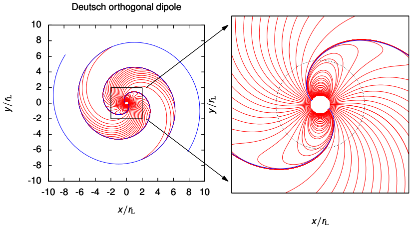
- exact analytical solution for a rotating dipole in vacuum (Deutsch, 1955)
- spindown power due to magnetodipole losses. For a perpendicular rotator

$$L_\perp^{\rm vac} = \frac{8\,\pi\,B^2\,\Omega^4\,R^6}{3\,\mu_0\,c^3}$$

- torque exerted on the surface by charges and currents (Michel & Goldwire, 1970; Davis & Goldstein, 1970)

$\Rightarrow$ secular evolution of the inclination angle

- two singular open field lines leading to a two armed archimedean spiral



- exact analytical solutions for multipoles also exist (Bonazzola et al., 2015; Pétri, 2015)

$\Rightarrow$ useful to enhance the pair production rate at the polar caps (Harding & Muslimov, 2011)

Consequences : spindown luminosity and torque

- an aligned rotator in vacuum does not radiate
- for an oblique rotator

$$L^{\mathrm{vac}} = L_{\perp}^{\mathrm{vac}} \sin^2 \chi$$

- braking index definition from the slow down rate

$$\dot{\Omega} = -K \, \Omega^n$$

  It is computed and measured observationally according to the first and second derivatives

$$n = \frac{\Omega \, \ddot{\Omega}}{\dot{\Omega}^2}$$

- integral of motion $\Omega \cos \chi = \mathrm{cst}$ and therefore a braking index

$$n = 3 + 2 \cot^2 \chi \geqslant 3.$$

These are predictions of the theory. What about observations ?

Consequences : braking index

Pulsar	Distance (kpc)	Période P (s)	$\dot{P}$ (10^{-15})	Indice de freinage n	Références
B0531+21	2.0	0.033	421	2.509 ± 0.001	Lyne et al. (1993)
J0537-6910	51	0.0161	0.0518	-1.5 ± 0.1	Middleditch et al. (2006)
B0540-693	51.5	0.050	479	2.140 ± 0.009	Livingstone et al. (2005)
B0833-45	0.29	0.089	124	1.40 ± 0.20	Lyne et al. (1996)
B1509-58	5.81	0.150	1490	2.837 ± 0.001	Kaspi et al. (1994)
J1846-0258	5.10	0.325	7083	2.65 ± 0.01	Livingstone et al. (2006)
				2.16 ± 0.13	Livingstone et al. (2011)
J1119-6127	8.40	0.408	4021	2.91 ± 0.05	Weltevrede et al. (2011)
J1734-3333		1.170	2280	0.9 ± 0.2	Espinoza et al. (2011)
J1833-1034				1.857	Roy et al. (2012)
J1640-4631	12	0.206	975.8	3.15	Archibald et al. (2016)

TABLE : Observational properties of some typical pulsars.

Useful values are

- magnetized wind $n = 1$
- vacuum dipole rotator $n = 3$
- vacuum quadrupole rotator (or gravitational waves) $n = 5$

$\Rightarrow$ a wind/dipole rotator combination could explain $1 < n < 3$

but in 2016 a pulsar found with $n = 3.15$.

The role of the plasma

A plasma is required observationally to produce the broadband radiation detected on Earth. Particles are needed to furnish charges and currents in the magnetosphere whenever and wherever imposed by Maxwell equations.

Analytical study intractable, recent progress via numerical simulations of which most extensively studied

- force-free electrodynamics (FFE or magnetodynamics) : zero mass limit. No energy dissipation.
- resistive magnetodynamics : transfer of energy from field to particles. Prescription not unique. Plasma motion not solved.
- magnetohydrodynamics (MHD) : particle inertia taken into account and the full stress-energy tensor, matter and field, is solved. Ideal and resistive MHD regimes.
- multi-fluids : evolve each species independently, coupling through electromagnetic interactions.
- fully kinetic treatment : individual particle acceleration that are out of thermal equilibrium. Needs to solve the full Vlasov-Maxwell equations.
- radiation reaction limit : acceleration compensated by radiation reaction. Particle motion solved analytically in terms of the external electromagnetic field.

The “standard model” of a pulsar

Basic underlying assumption : force-free magnetosphere

$$\rho_{\mathrm{e}} \vec{E} + \vec{j} \wedge \vec{B} = \vec{0}$$

magnetic energy density $\frac{B^2}{2\,\mu_0} \gg$ any other energy densities

- particle inertia neglected : zero mass limit.
- no dissipation : ideal MHD

$$\vec{E} + \vec{v} \wedge \vec{B} = \vec{0}$$

- no pressure : cold plasma.

Two interpretations

- charge-separated plasma $\Rightarrow$ low particle density.
- MHD model $\Rightarrow$ quasi-neutral plasma, high particle density.

Simplest approach to pulsar electrodynamics
ideal MHD without particle inertia and without radiation

- Maxwell equations

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{\sqrt{\gamma}} \partial_t(\sqrt{\gamma}\,\mathbf{B})$$

$$\nabla \cdot \mathbf{D} = \rho_{\rm e}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{1}{\sqrt{\gamma}} \partial_t(\sqrt{\gamma}\,\mathbf{D})$$

- FFE current prescription (constraints $\mathbf{E} \cdot \mathbf{B} = 0$ and $E < c\,B$)

$$\mathbf{J} = \rho_{\rm e} \frac{\mathbf{E} \wedge \mathbf{B}}{B^2} + \frac{\mathbf{B} \cdot \nabla \times \mathbf{B}/\mu_0 - \varepsilon_0\,\mathbf{E} \cdot \nabla \times \mathbf{E}}{B^2}\,\mathbf{B}$$

$$\rho_{\rm e} = \varepsilon_0\,\nabla \cdot \mathbf{E}$$

No fluid quantities enter into the system to be solved. (Spitkovsky, 2006; Komissarov, 2006; McKinney, 2006; Pétri, 2012; Paschalidis & Shapiro, 2013; Cao et al., 2016)

Force-free magnetospheres

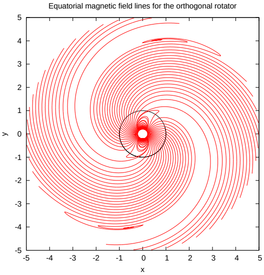


FIGURE : *Magnetic field of the perpendicular rotator $\chi = 90^o$.*

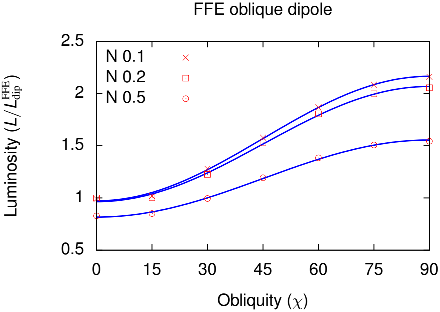


FIGURE : *Spin-down luminosity vs χ from simulations in red and fit in blue.*

Plasma filled magnetosphere spindown

$$L_{\rm sp}^{FFE} \approx \frac{3}{2} L_{\perp}^{\rm vac} (1 + \sin^2 \chi)$$

to be compared with vacuum

$$L_{\rm sp}^{vac} \approx L_{\perp}^{\rm vac} \sin^2 \chi$$

(Spitkovsky, 2006; Pétri, 2012)

Dissipative/resistive "FFE" magnetospheres

Tells you where magnetic energy is dissipated into radiation. $\Rightarrow$ Should trace the location of the gaps

- Maxwell equations as before
- and a reasonable current prescription
- no constraints for $\mathbf{E} \cdot \mathbf{B} = 0$ or $E < c\,B$.
  - strong field electrodynamics (Gruzinov, 2008)

$$\mathbf{J} = \frac{\rho_{\mathrm{e}}\,\mathbf{E} \wedge \mathbf{B} + \sqrt{\rho_{\mathrm{e}}^2 + \gamma^2\,\sigma^2\,E_0^2/c^2}\,(E_0\mathbf{E}/c + c\,B_0\mathbf{B})}{B^2 + E_0^2/c^2}$$

  - a kind of Ohm's law for a relativistic quasi neutral plasma (Li et al., 2012)

$$\mathbf{J} = \frac{\rho_{\mathrm{e}}\,\mathbf{E} \wedge \mathbf{B} + \gamma\,\sigma\,E_0/c\,(E_0\,\mathbf{E}/c + c\,B_0\,\mathbf{B})}{B^2 + E_0^2/c^2}$$

⚠ σ is not interpreted as the resistivity in the first expression

Dissipative/resistive “FFE” magnetospheres

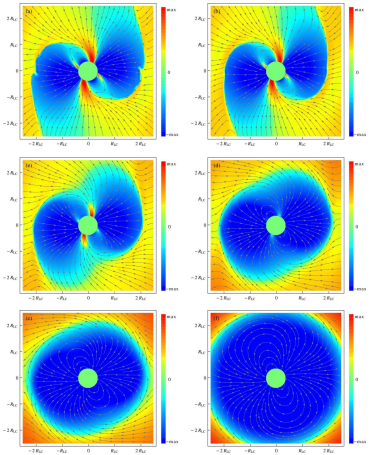


FIGURE : *From FFE to vacuum magnetosphere depending on “resistivity” σ.*

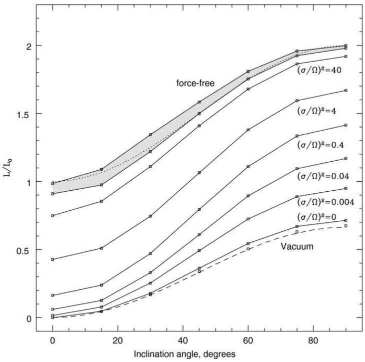


FIGURE : *Spin-down luminosity vs obliquity χ and “resistivity” σ.*

(Li et al., 2012)

Includes particle inertia but not particle acceleration

$$\nabla_\mu(\rho_{\rm m}\, u^\mu) = 0$$
$$\nabla_\mu(T^{\alpha\,\mu(em)} + T^{\alpha\,\mu(fl)}) = 0$$

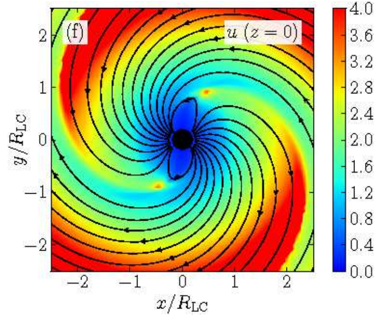


FIGURE : *Perpendicular rotator* $\chi = 90^o$

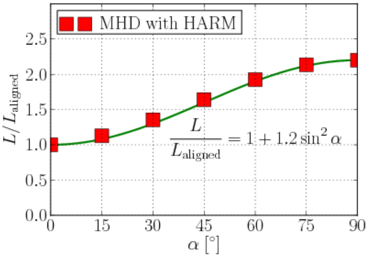


FIGURE : *Spin-down luminosity vs obliquity* χ.

(Tchekhovskoy et al., 2013)

Includes particle inertia AND particle acceleration self-consistently

Equation of motion for a particle (Lorentz force)

$$m \frac{du^\alpha}{d\tau} = q \, F^{\alpha\,\mu} \, u_\mu$$

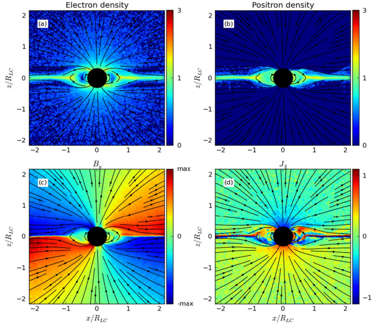


FIGURE : *Lepton properties for an aligned rotator* $\chi = 0^o$

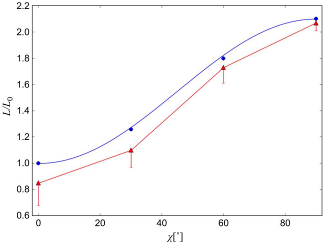


FIGURE : *Spin-down luminosity vs obliquity* χ*. Blue curve is MHD, blue points PIC at high pair density, red triangles PIC with "realistic" pair creation.*

(Philippov et al., 2015b)

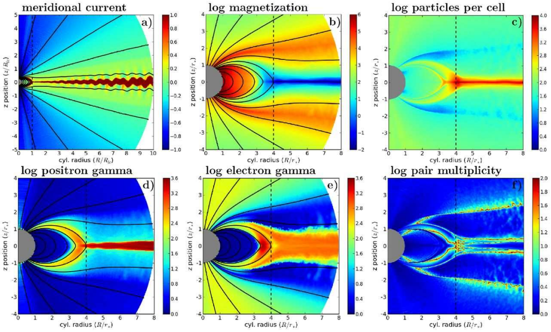


FIGURE : *Plasma properties*

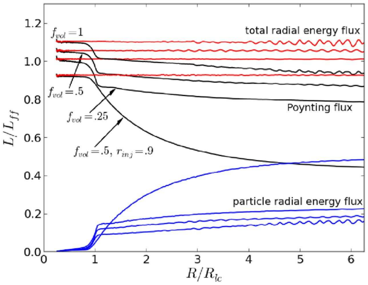


FIGURE : *Particle and Poynting energy flux*

(Belyaev, 2015)

Equation of motion for a particle (Lorentz force + radiation reaction)

$$m \frac{du^\alpha}{d\tau} = q F^{\alpha\,\mu}\, u_\mu + g^\alpha$$

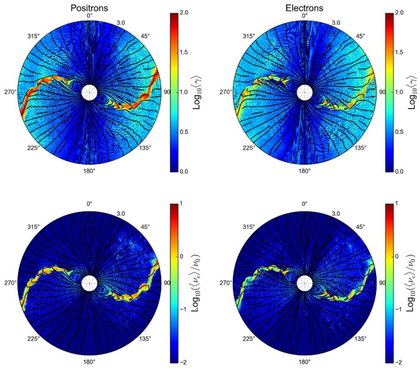


FIGURE : *Lorentz factor and characteristics synchrophoton frequency.*

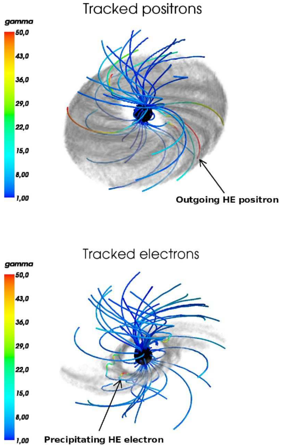


FIGURE : *Electron and positron trajectories.*

Do we need general relativity ?

Space-time curvature and frame dragging

modify the structure of the electromagnetic field close to the neutron star surface

- amplification of the intensity of the electric field in the neighborhood of the stellar surface because of the gravitational field
  (Muslimov & Tsygan, 1992).
- dynamics of the polar caps changed
  (Beskin, 1990).

Consequences on the magnetosphere

Quantitative modifications of

- the geometry of the polar caps, opening angle.
- the shape of the radio pulses.
- the modulation of the light-curves in X-rays for accreting pulsars.
- spin-down luminosity.
  (Pétri, 2013, 2014).

Poynting flux for the perpendicular rotator in flat space-time

$$L_{\rm dip} = \frac{8\,\pi}{3\,\mu_0\,c^3}\,\Omega^4\,B^2\,R^6$$

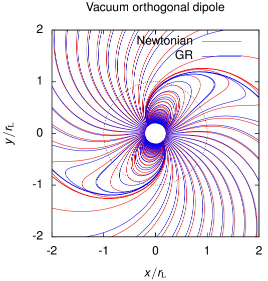


FIGURE : Magnetic field lines in the equatorial plane for $r_{\rm L}/R = 10$.

(Pétri, 2016)

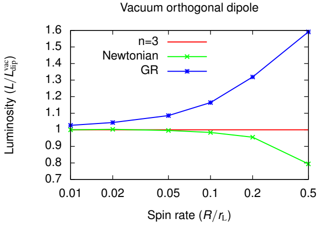


FIGURE : Poynting flux $L/L_{\rm dip}$ compared to the Deutsch solution. (Corrections for Ω-redshift and B-amplification omitted, see (Rezzolla & J. Ahmedov, 2004)).

Spin parameter

$$\frac{a}{R_{\rm s}} = \frac{2}{5}\,\frac{R}{R_{\rm s}}\,\frac{R}{r_{\rm L}}$$

with $R = 2\,R_{\rm s}$.

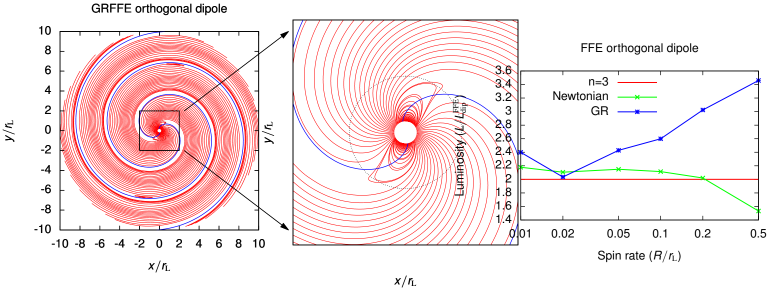


FIGURE : *Magnetospheric structure of the perpendicular rotator for a general relativistic dipole magnetic field with $R/r_L = 0.2$ and $R/R_s = 2$. The distances are normalized to the light cylinder radius. A spiral arm form where field lines change polarity. This special geometry is at the heart of the striped wind model*

FIGURE : *Poynting flux of the force-free orthogonal rotator normalized to L_{dip}^{FFE} and for* $R/r_L = \{0.01, 0.02, 0.05, 0.1, 0.2, 0.5\}$.

(Pétri, 2016)

Do we need general relativity ?

In FFE simulations, GR is required to account for efficient pair production in the polar caps through frame-dragging effect lowering the corotation charge density leading to space-like electric current.

Corotation charge density

$$\rho_e = -2\,\varepsilon_0\,\frac{(\Omega - \omega_{LT})}{\alpha}\,B$$

Current density from flat spacetime simulations $j_r = \rho_e\,c$ thus

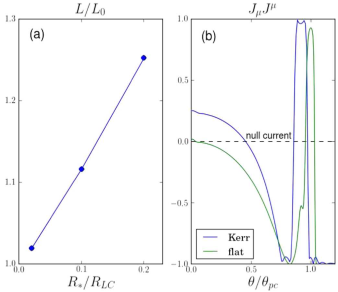
$$\left.\frac{j_r}{\rho_e\,c}\right|_{GR} = \left.\frac{j_r}{\rho_e\,c}\right|_{flat}\frac{1}{1-\omega_{LT}/\Omega} > 1$$

Current density is spacelike at the polar caps

$$J^\mu\,J_\mu < 0 \Leftrightarrow j > \rho_e\,c$$

Physically, it means that the current cannot be supported by a charge separated plasma simply by convection terms $\rho_e\,v \lesssim \rho_e\,c$.

(Belyaev & Parfrey, 2016; Gralla et al., 2016)



(Philippov et al., 2015a)

On small scales obviously yes for pair creation.
The critical magnetic field is

$$B_{\rm qed} = \frac{m_{\rm e}^2 \, c^2}{e \, \hbar} \approx 4.4 \times 10^9 \; \mathrm{T}$$

Maxwell equations become non-linear for $B \gtrsim B_{\rm qed}$.
$\Rightarrow$ Corrections to lowest order by expansion of Euler-Heisenberg Lagrangian
post-Mawxellian parameters like
post-Newtonian gravity
$\Rightarrow$ Quantum vacuum equivalent to a medium : $\mathbf{D}(\mathbf{E},\mathbf{B}), \mathbf{H}(\mathbf{E},\mathbf{B})$

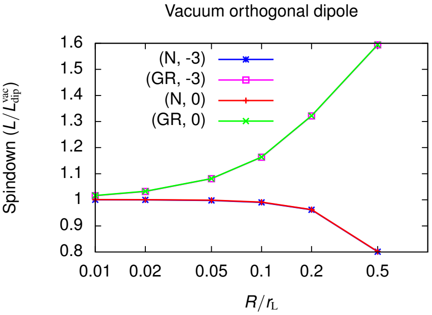


FIGURE : *Spindown luminosity for different rotation rates, magnetic field strengths given by* $\log(B/B_{\rm qed})$ *and gravitational field (Newtonian or GR).*

(Pétri, submitted)

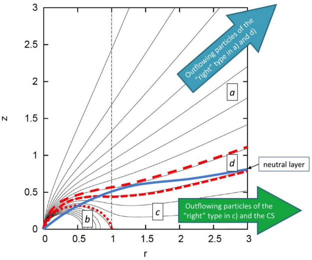
QED does not influence neutron star global vacuum electrodynamics.
Same conclusions for FFE magnetospheres.

Towards general agreements about pulsar magnetospheres

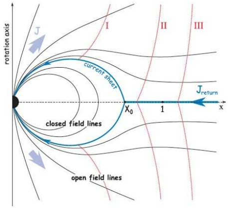
- filled with electron/positron pairs almost everywhere
- FFE approximation satisfactory on a global scale
- formation of an equatorial current sheet
- efficient particle acceleration and emission in this sheet
- Y-point of great importance for the dynamics/spindown losses

$$\dot{E}_Y \approx \left(\frac{r_{\rm L}}{R_Y}\right)^2 \dot{E}_L \geqslant \dot{E}_L$$

  maybe solution for braking index ?
- breakdown of ideal MHD/FFE in some small regions
- magnetic reconnection invoked in these regions and in the sheet



(Contopoulos, 2016)



(Timokhin, 2006)

Conclusions

O Pulsar magnetosphere and wind

- global structure well constrained
- global FFE picture satisfactory
- good agreement between FFE/MHD and PIC simulations
- magnetosphere naturally linked to its striped wind

⚠ Caveats

- some dissipation regions required for emission
- self-consistent acceleration of particles only through PIC/Vlasov simulations
- particle injection rate unknown
- a lot of microphysics still missing
- realistic fully kinetic simulations impossible because

$$\frac{\omega_B}{\Omega} \gtrsim 10^{12} - 10^{18}$$

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