Collective modes in neutron star inner crust

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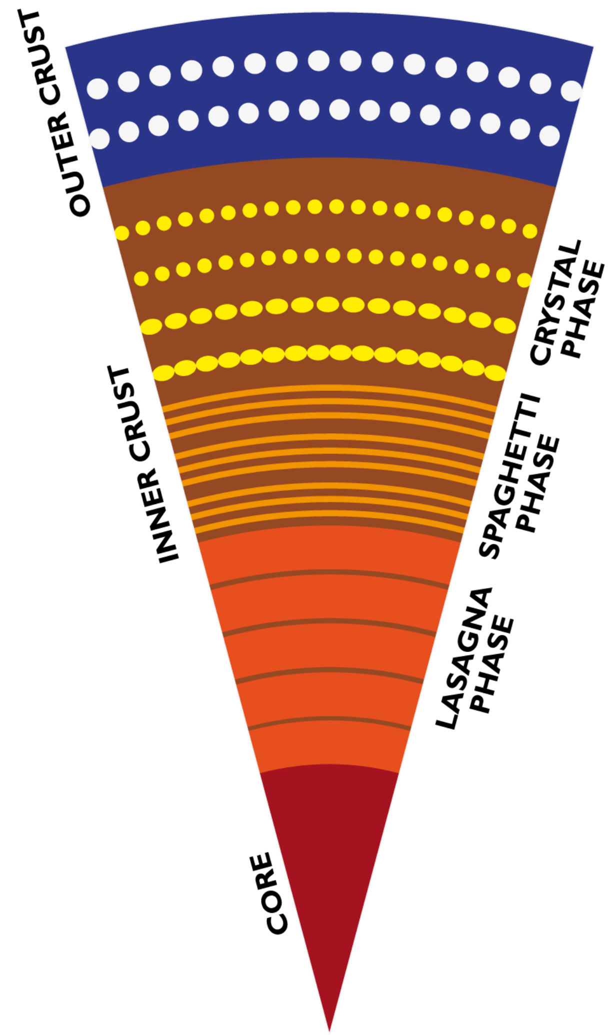
Introduction – Neutron star structure

Neutrons stars are expected to be organized in **layers:**

- <u>**Outer crust:**</u> nuclei in a gas of relativistic electrons. **Crystalline arrangement** because of the Coulomb interaction.
- <u>**Inner crust:**</u> unbound neutrons drip out of the nuclei and form a **dilute neutron gas** between the clusters. **Different geometries** (crystal, rods, plates) expected.
- <u>**Core:**</u> homogeneous *npe* matter. Perhaps: hyperons, quark matter?

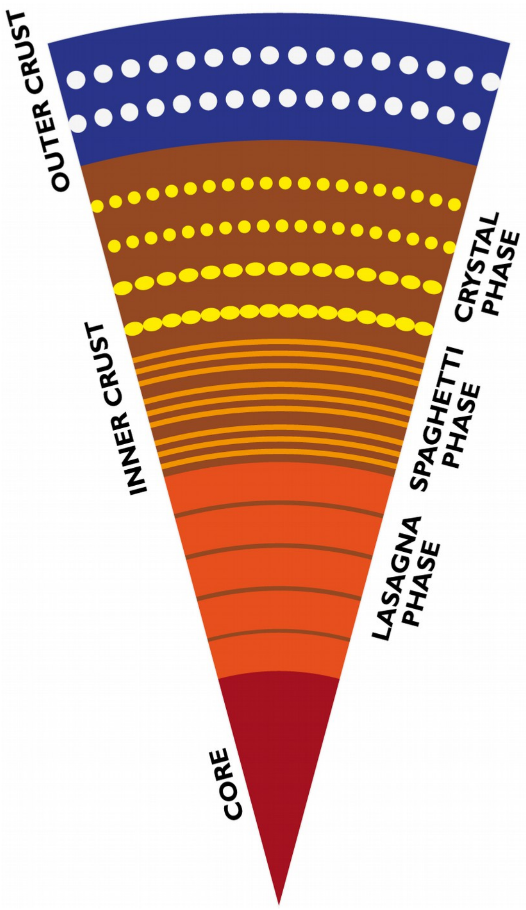


Not to scale

Introduction – Inner crust structure

Henceforth we include **clusters**.

- **Clusters** can have **different geometries** *[Ravenhall et al., PRL 50, 2066 (1983)] :*
  - **Crystal (3D) : spherical clusters,**
  - **Cylinders (2D, « spaghetti ») ,**
  - **Planes (1D, « lasagna »).**
- At **core-crust** transition **inverse phases are expected:**
  - **Tubes,**
  - **Holes** – “Swiss cheese”.



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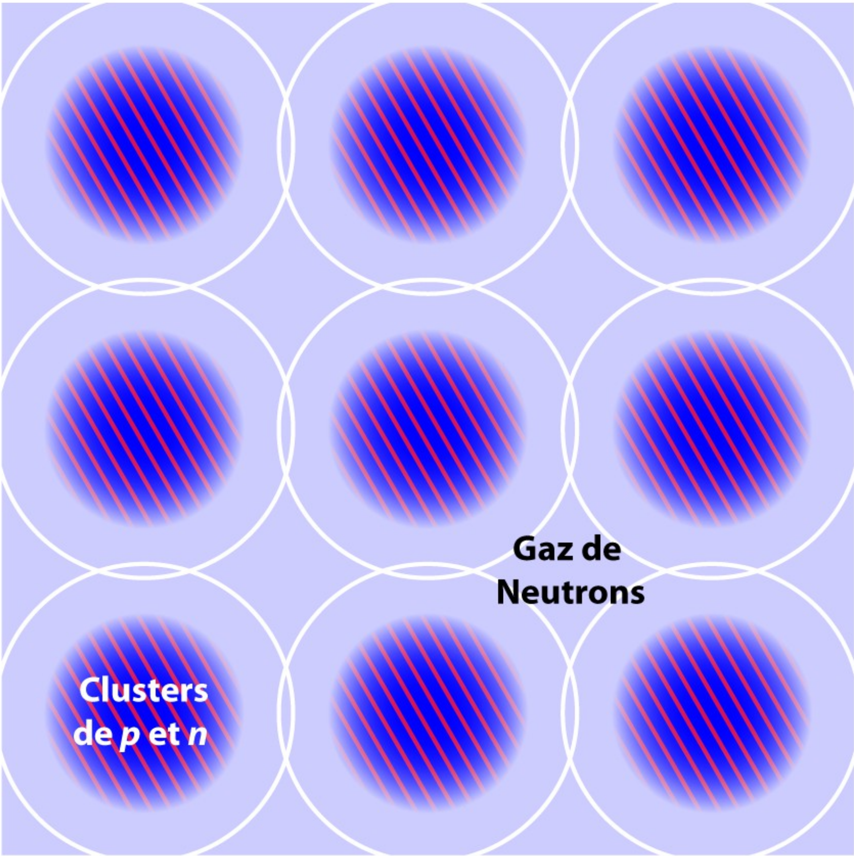
Inner crust description

N.M. and M. Urban, Phys. Rev. C **92**, 015803 (2015)

Extended Thomas-Fermi

- **Energy minimization** with a **parametrized surface** (diffusivity).
- **Coulomb energy** calculated within **Wigner-Seitz** approximation.

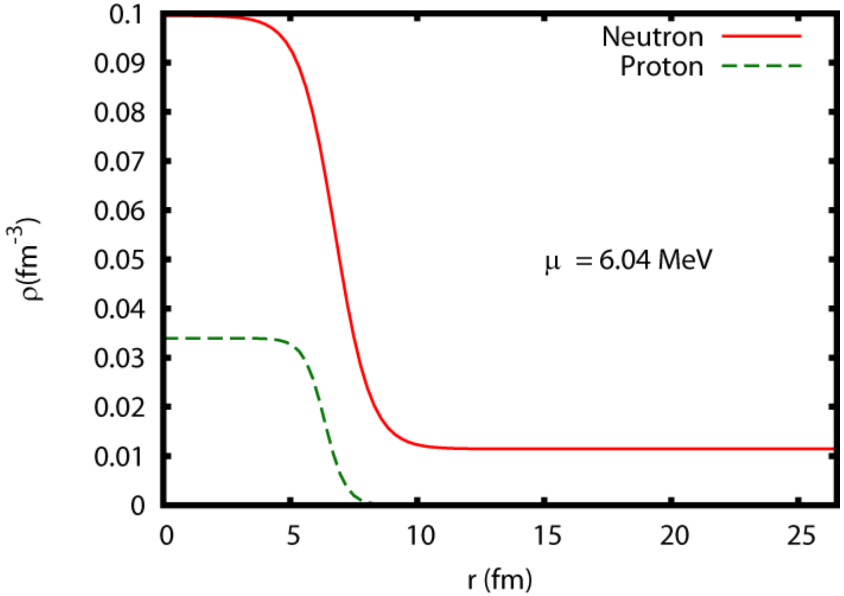


Extended Thomas-Fermi

- **Surface density profile** to minimize **is parametrized:**

$$\rho_q(r)=\rho_q^{gas}+\frac{\rho_q^{liq}-\rho_q^{gas}}{1+\exp((r-r_q)/a_q)} \qquad (q=n,p)$$

→ 9 parameters to minimize.



- **Thermodynamic potential** to minimize:

$$\omega=\frac{1}{V_{WS}}[E_{Skyrme}+E_e+E_C+E_{ex}-\mu_n(N+Z)]$$

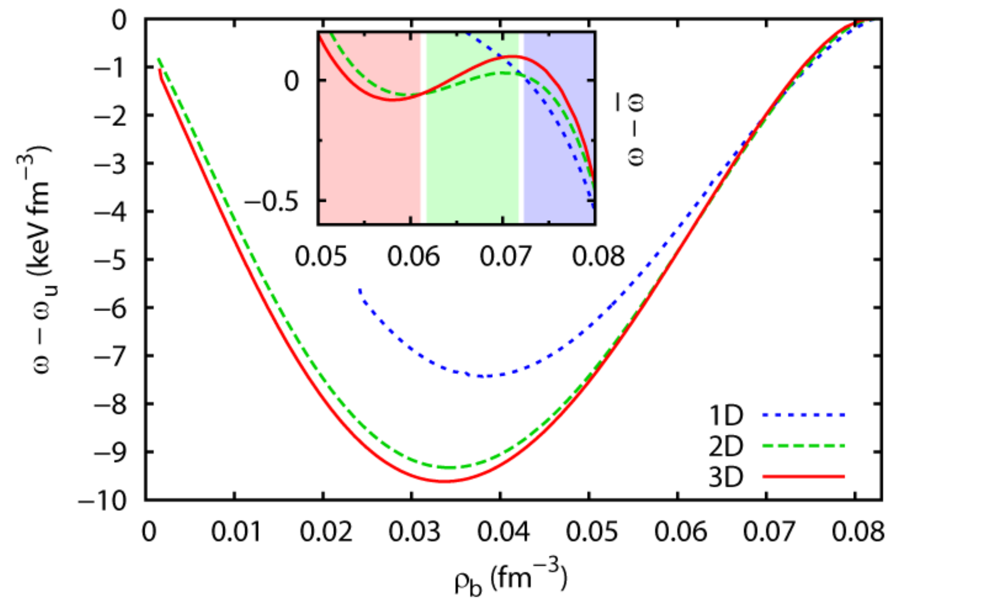
- **Strong interaction** obtained from **Skyrme** (SLy4), with **ETF** approximation in **τ**.
- **Coulomb** energy: $E_c=\frac{1}{2}\int_0^{R_{WS}} d^d r\,\rho_c(r)V(r)$
- **Exchange term for relativistic electrons:**

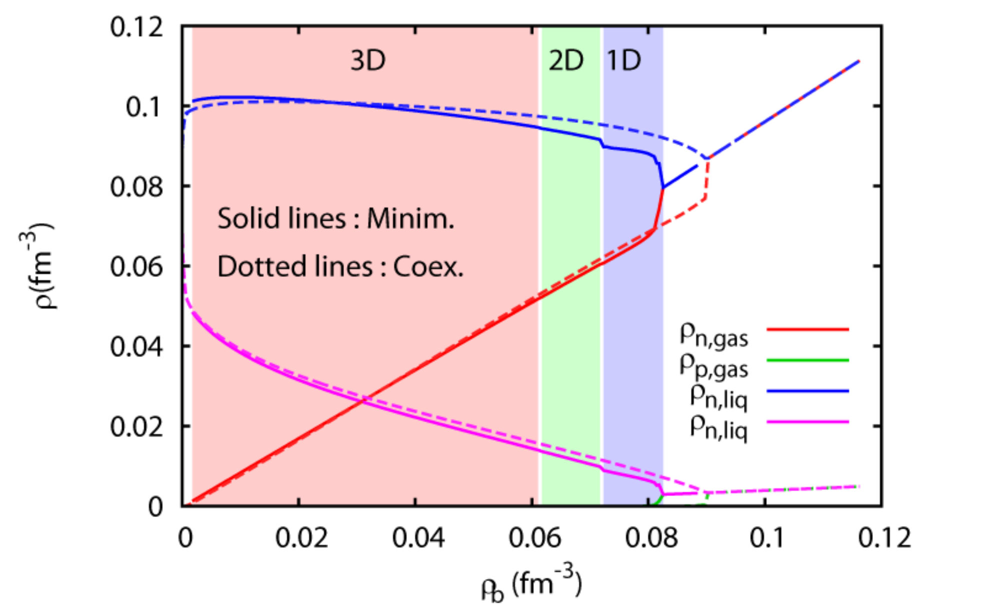
$$\epsilon_{ex,e}=\frac{e^2 k_{F,e}^4}{8\pi^3}\left\{3\left[\sqrt{1+\frac{1}{x_e^2}}-\frac{\sinh^{-1}(x_e)}{x_e^2}\right]-2\right\}$$

become positive for x_e ($=k_{F,e}/m_e$) ≳ 2.53
[E. Salpeter, ApJ. (1961), A.K. Rajagopal, J.Phys.C (1978)]

Energy minimization

- We **minimize ω** by adjusting parameters.
- **Phase transitions**:
  - 3D → 2D ~ 0,06 fm^{-3}
  - 2D → 1D ~ 0,07 fm^{-3}
  - 1D → **Uni.** ~ 0,08 fm^{-3}
- Phase coexistence and **minimization similar results** for the **densities**.
- **Crust-core** transition **earlier** in **ETF** (surface and Coulomb effects).
- **No inverse phase obtained** (Y_p too small because of β-equilibrium).

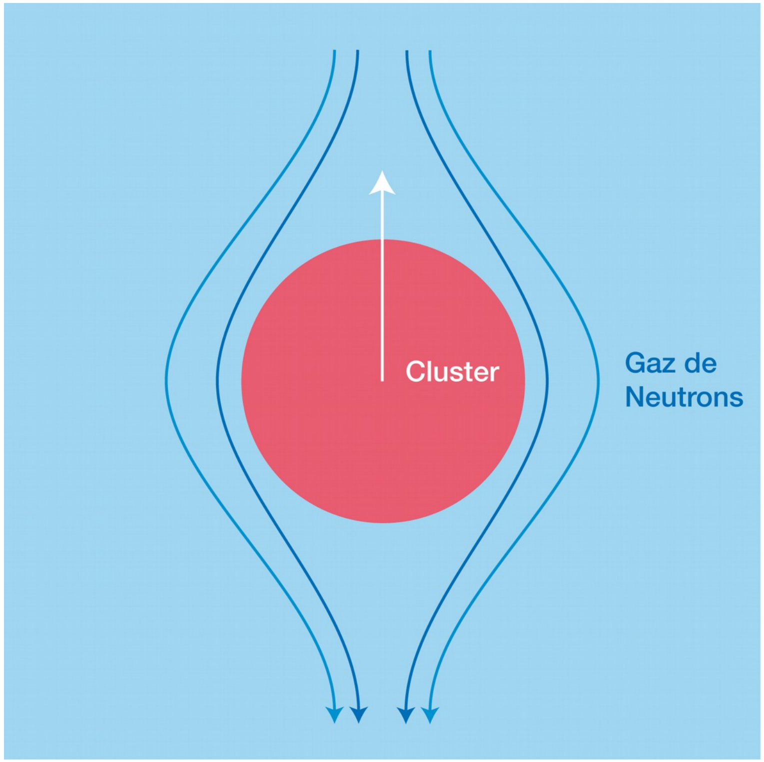




Neutron hydrodynamics and entrainment

Entrainment

- **Proton** and **neutron fluids do not move independently** each other, because some neutrons are **bound to the cluster.**
- We talk about **"bound" and "conduction"** neutrons, or in terms of cluster **"effective mass"**.
- Dramatic effect on lattice **phonons, heat capacity**, etc…



Band theory *(solid state physics) [N. Chamel]*

- **Clusters entrain** neutrons with them $A_{eff} > A$.
- **Superfluidity not included.**

Superfluid hydrodynamics *[A. Sedrakian, P. Magierski and A.Bulgac]*

- **Neutrons** go **through** clusters $A_{eff} < A$.
- Considered as an **isolated nucleus** inside an **infinite gas.**

Superfluid hydrodynamics

- **Superfluid hydrodynamics** is valid for low-energy excitations (cf. QRPA) :

$$\boldsymbol{v}_n(\boldsymbol{r}) = \frac{1}{2m} \nabla \phi(\boldsymbol{r}) \qquad \phi = \text{phase of } \Delta_n$$

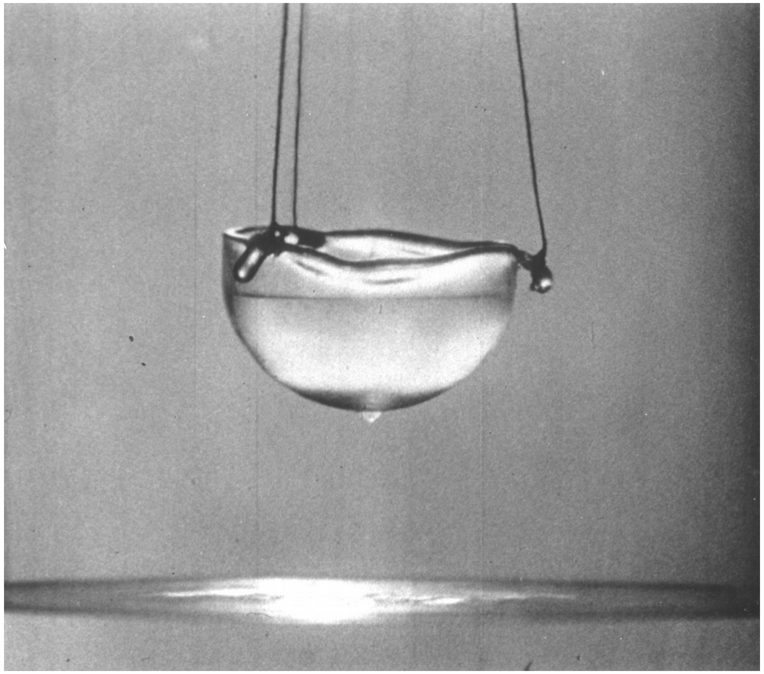
- Ground state: **hydrostatic equilibrium**
  - Clusters with **sharp surface** (**phase coexistence**),
  - Needs **boundary conditions at liquid-gas** interface**,**

$$\phi^{\text{in}} = \phi^{\text{out}}$$

$$\rho_n^{\text{in}} v_{n\perp}^{\text{in}} - \rho_n^{\text{out}} v_{n\perp}^{\text{out}} = (\rho_n^{\text{in}} - \rho_n^{\text{out}}) v_{p\perp}$$

  - **Periodic lattice,**

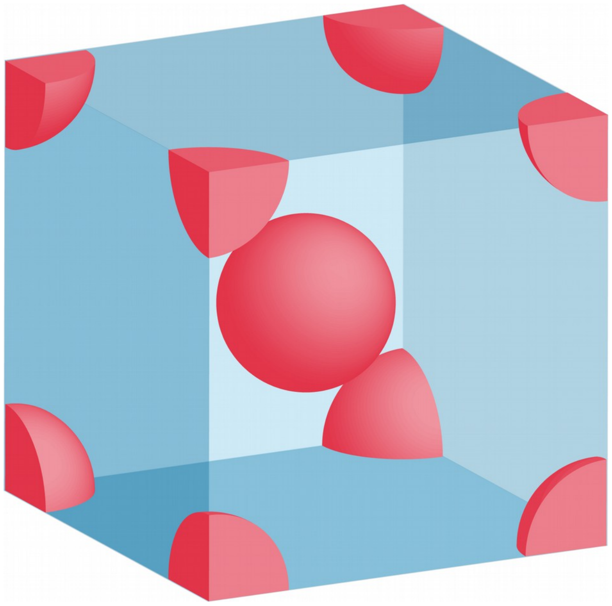
$$\phi(\boldsymbol{r}) = \phi(\boldsymbol{r} + \boldsymbol{L})$$



3D and 2D geometries

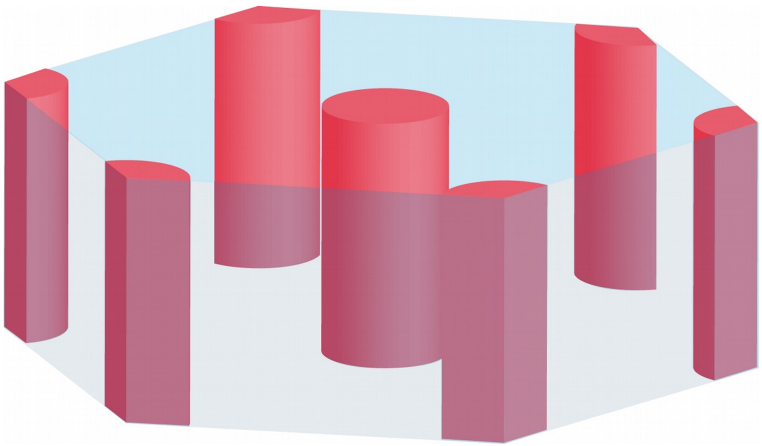
Crystalline phase (3D)

- Inner crust
- **Body Centered Cubic** (BCC) cell
- **Two clusters** per unit cell



« Spaghetti » phase (2D)

- Inner crust
- **Hexagonal** cell
- **One cluster** per unit cell



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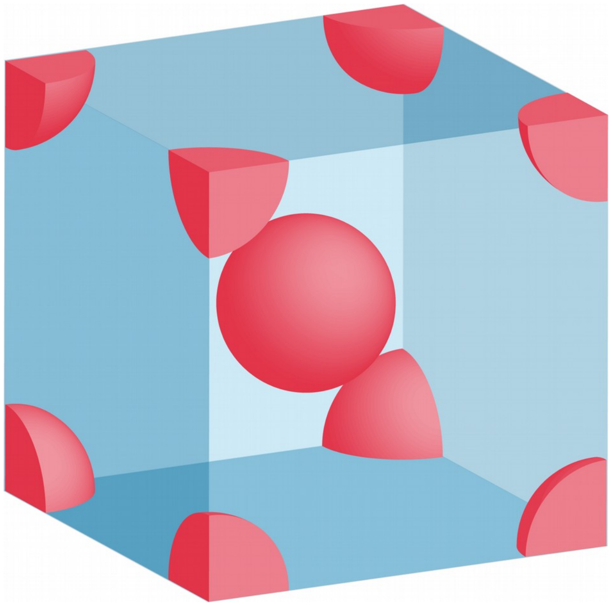

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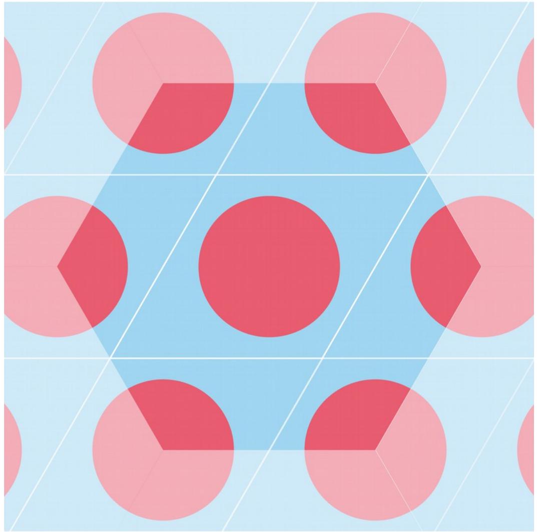
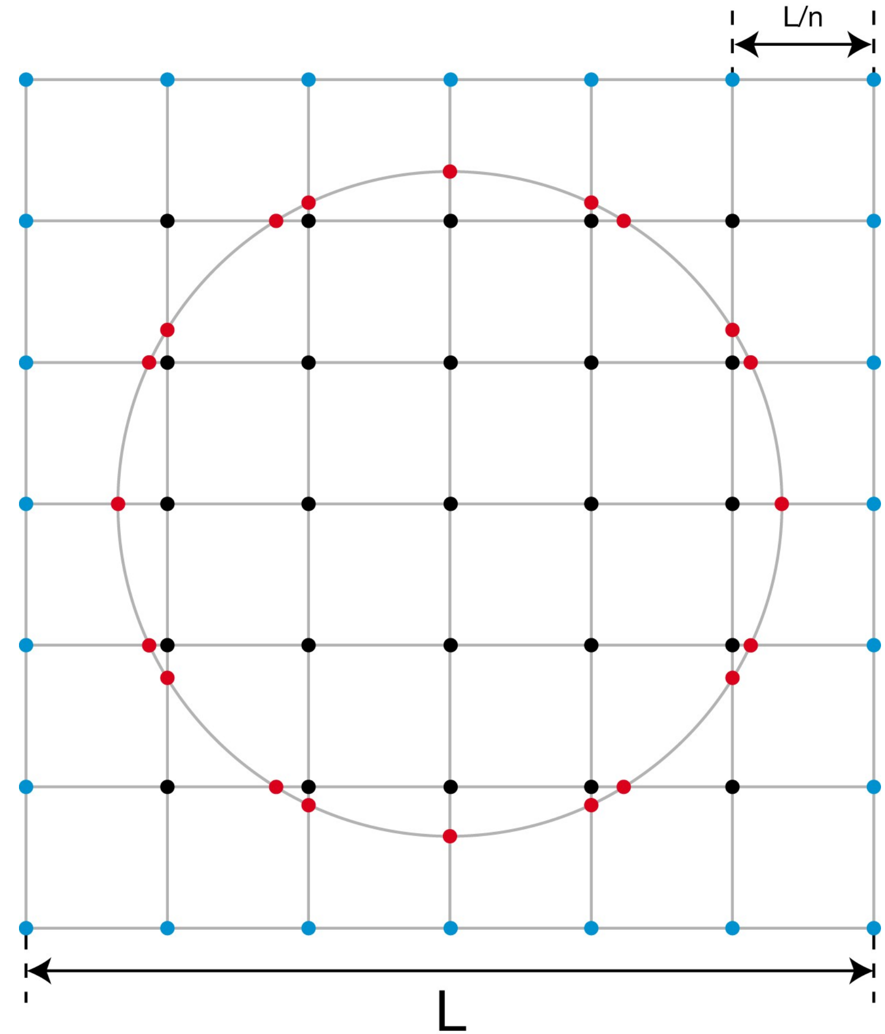


Numerical methods

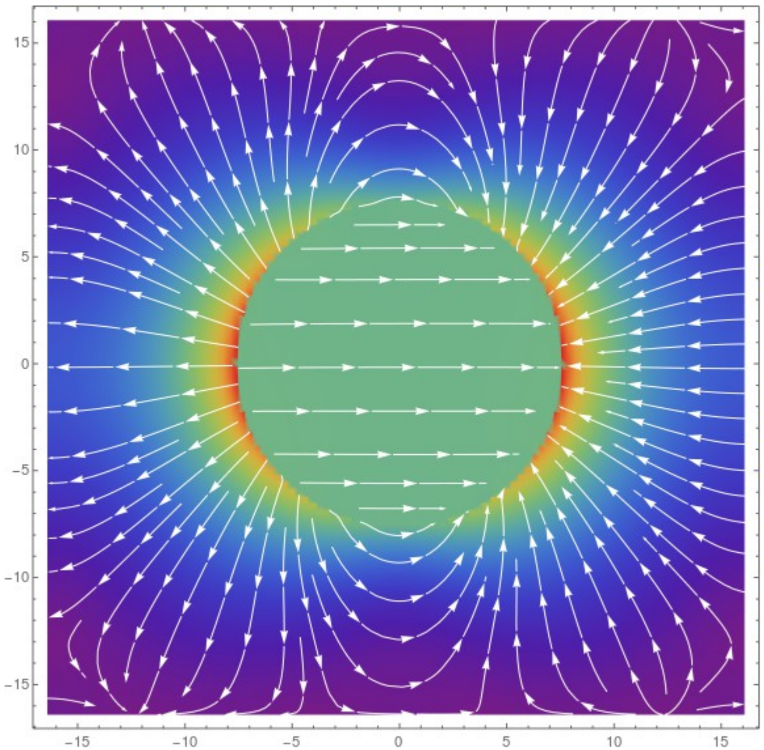
Equations to solve

$$\Delta\phi=0,$$
$$\rho_{in}\left(\frac{\partial\phi_{in}}{\partial r}\bigg|_{r=R}-\vec{v}\cdot\vec{n}\right)=\rho_{out}\left(\frac{\partial\phi_{out}}{\partial r}\bigg|_{r=R}-\vec{v}\cdot\vec{n}\right),$$
$$\phi(x,y,z)=\phi(x+L,y,z)\quad\text{(analogous for y,z)}$$

- **We discretize equations,**
  - Regular **mesh**,
  - **Surface** points (lying on grid axis),
  - **Periodicity.**
- Solving a huge **linear system of equations** (~300 GB) and **"sparse matrix"** storage.
- Solved with PARDISO (MKL Intel) until ~160^3 points (done at C.C. IN2P3, Lyon).



Crystalline phase – Streamlines



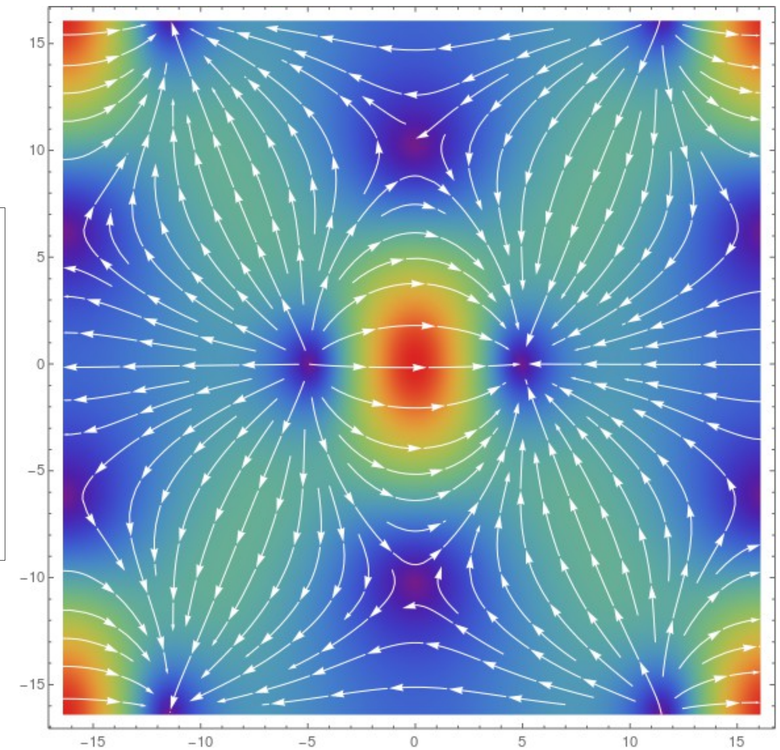
Properties

$R_{CLUSTER}$ = 7.54 fm

L_{CELL} = 38.1fm

ρ_{iN} = 0.0973 fm^{-3}

ρ_{OUT} = 0.0412 fm^{-3}



z = 0 fm
Cluster center

Colors → Norm of *v*
White → Streamlines

z = 8,6 fm
Between clusters

Crystalline phase – Potential Φ

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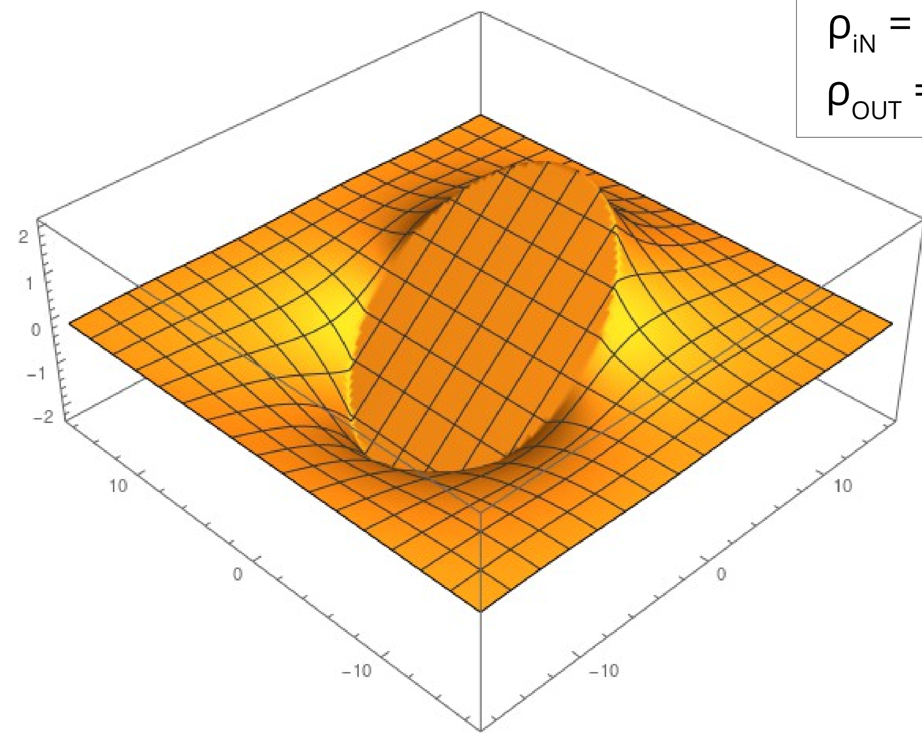
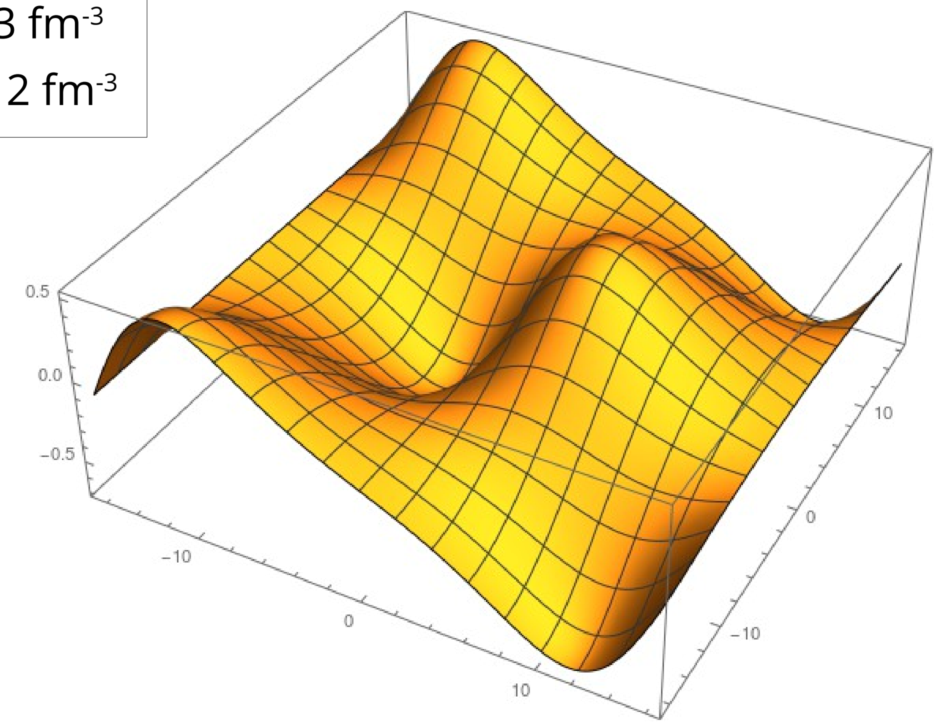


z = 0 fm

Cluster center



z = 8,6 fm

Between clusters

Cluster properties obtained from ETF calculation

“Spaghetti” phase – Streamlines

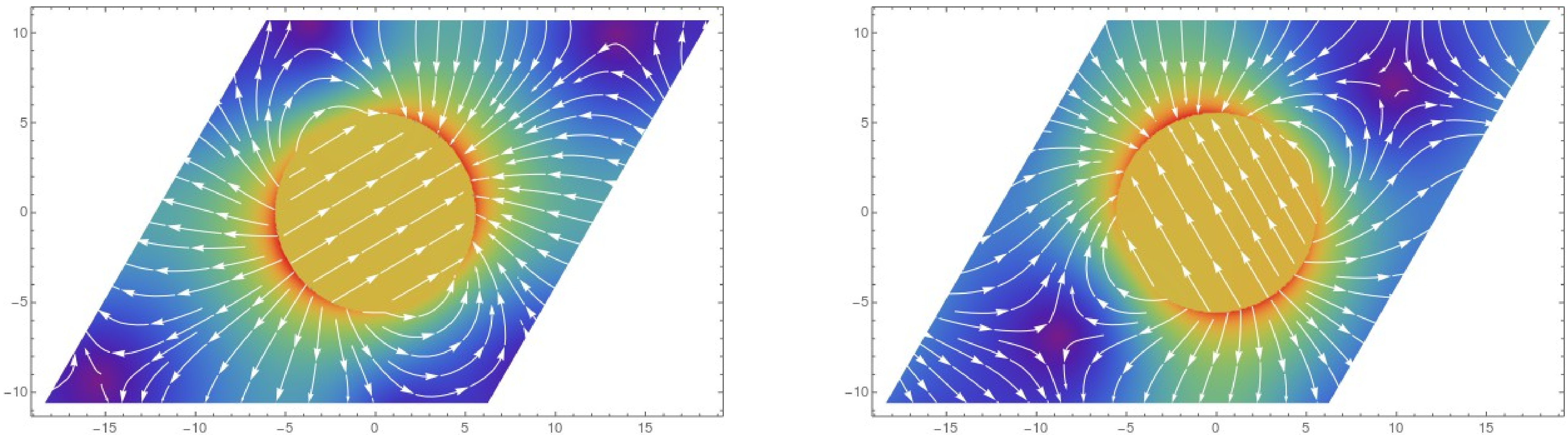
Properties

$R_{CLUSTER}$ = 5.53 fm

L_{CELL} = 24.7 fm

ρ_{iN} = 0.0943 fm^{-3}

ρ_{OUT} = 0.0528 fm^{-3}



$\mathbf{v_p} = v_p\,\mathbf{e_1}$

Colors → Norm of *v*
White → Streamlines

$\mathbf{v_p} = v_p\,\mathbf{e_2}$

“Spaghetti” phase – Potential Φ

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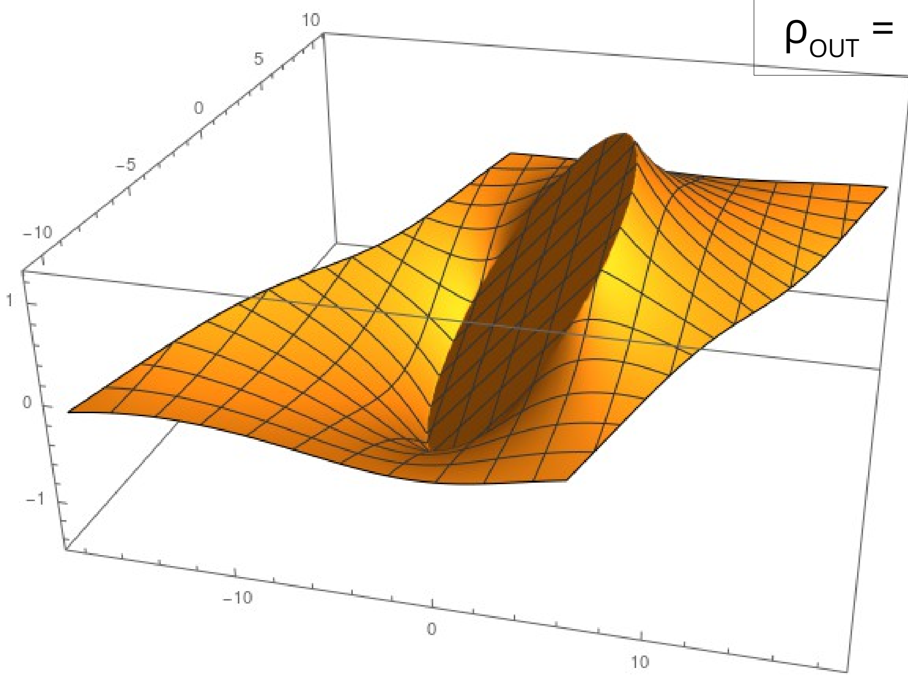
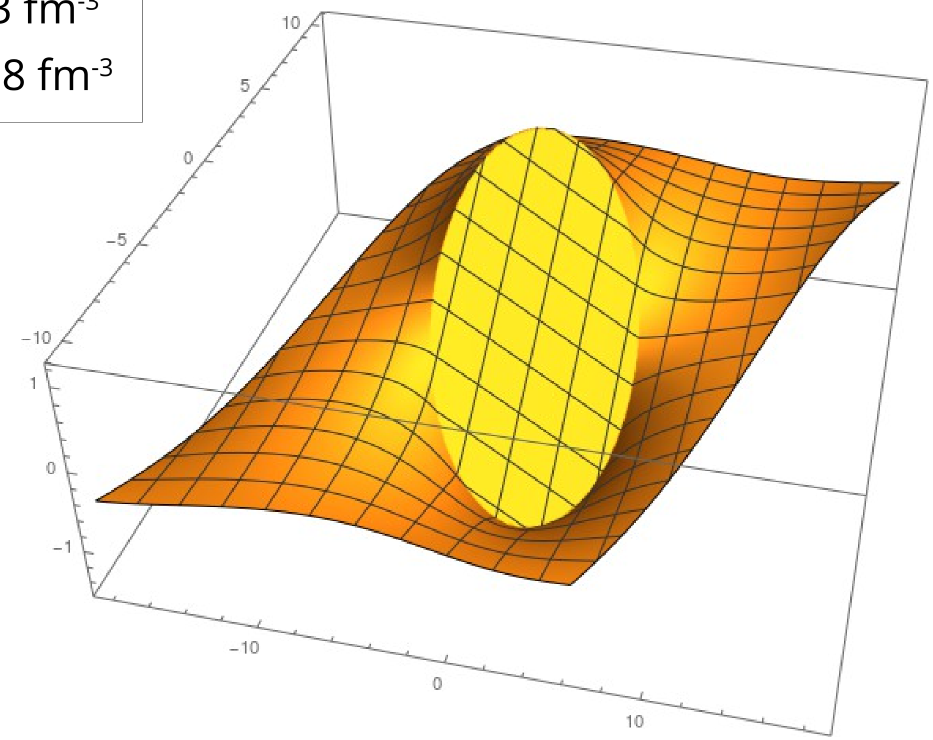


$\mathbf{v_p} = v_p\,\mathbf{e_1}$

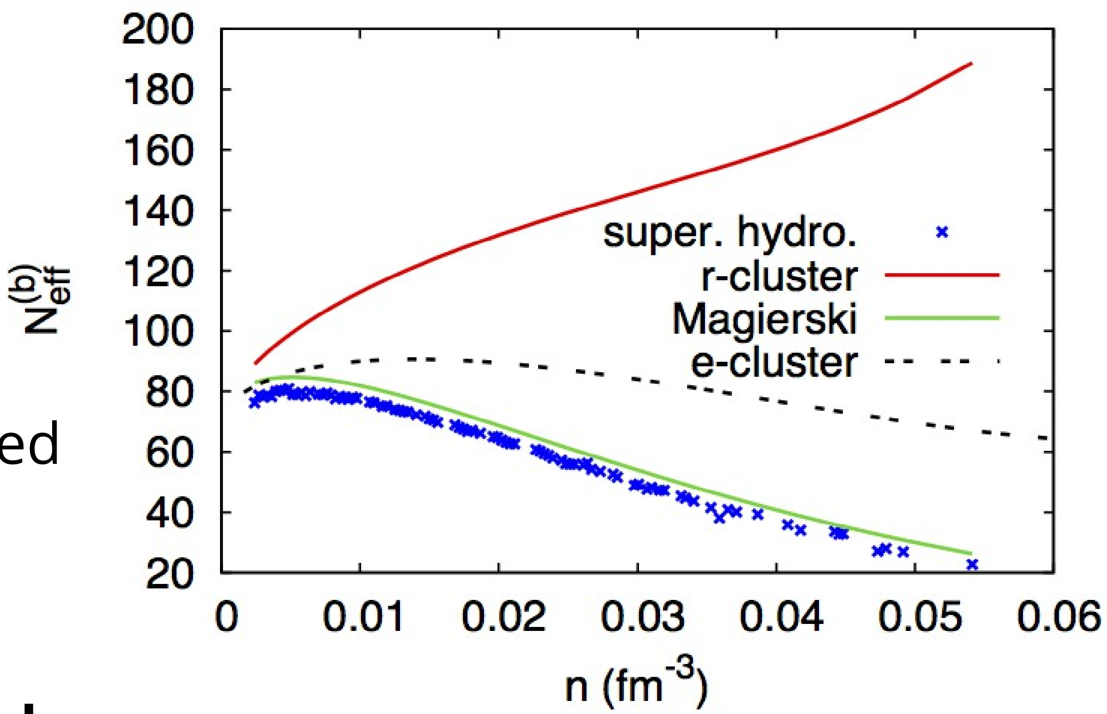


$\mathbf{v_p} = v_p\,\mathbf{e_2}$

Cluster properties obtained from ETF calculation

“Bound” neutrons

- **r-cluster**: number of neutrons in the cluster.
- **Magierski-Bulgac**: analytical solution for a **cluster in an infinite gas** *[NPA 738 (2004) 143]*
- **Numerical:** the present calculations.
- **e-cluster:** neutron kind defined with respect to their energy.

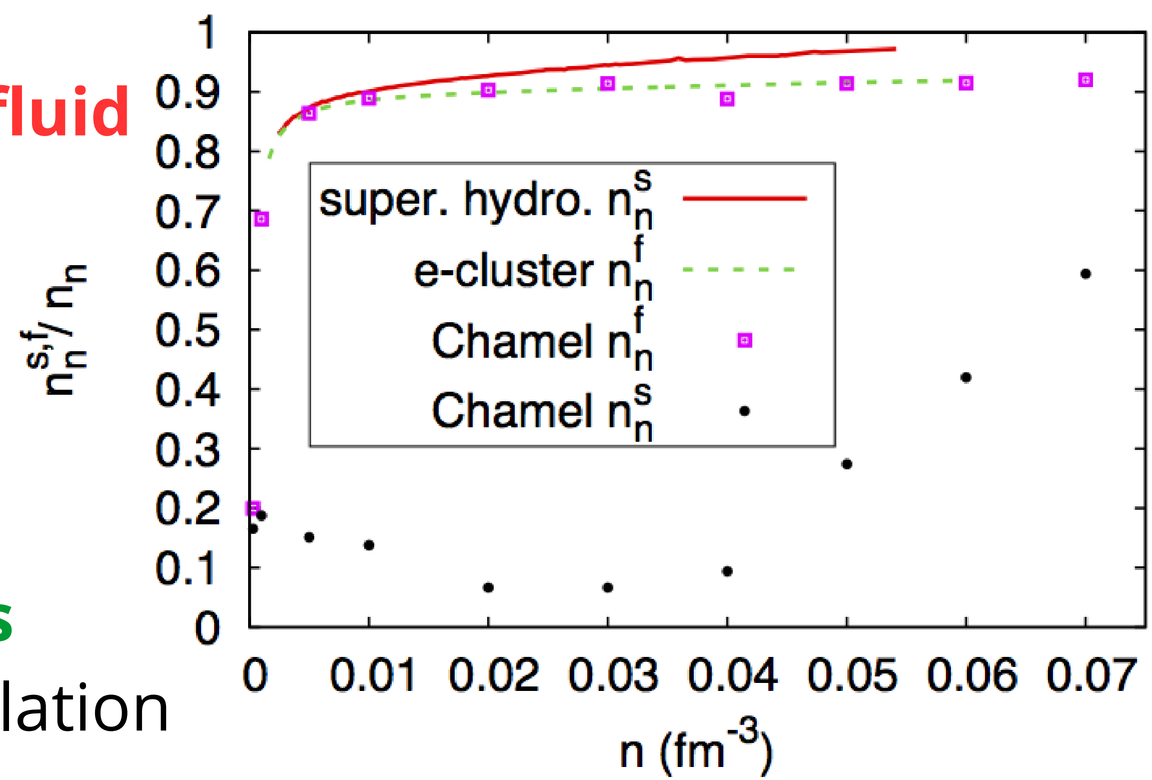


<u>**Comments**</u>

- The three **approaches coincide** at low densities.
- **Periodicity has a small effect**.

Superfluid neutrons

- **Superfluid density,** obtained with **superfluid hydrodynamics.**
- Results larger than Chamel from **band structure.**
- The **e-cluster results** agrees with the calculation of **n^f of Chamel.**
- Here, one observes that **n^s > n^f**. *Which impact on astrophysics?*



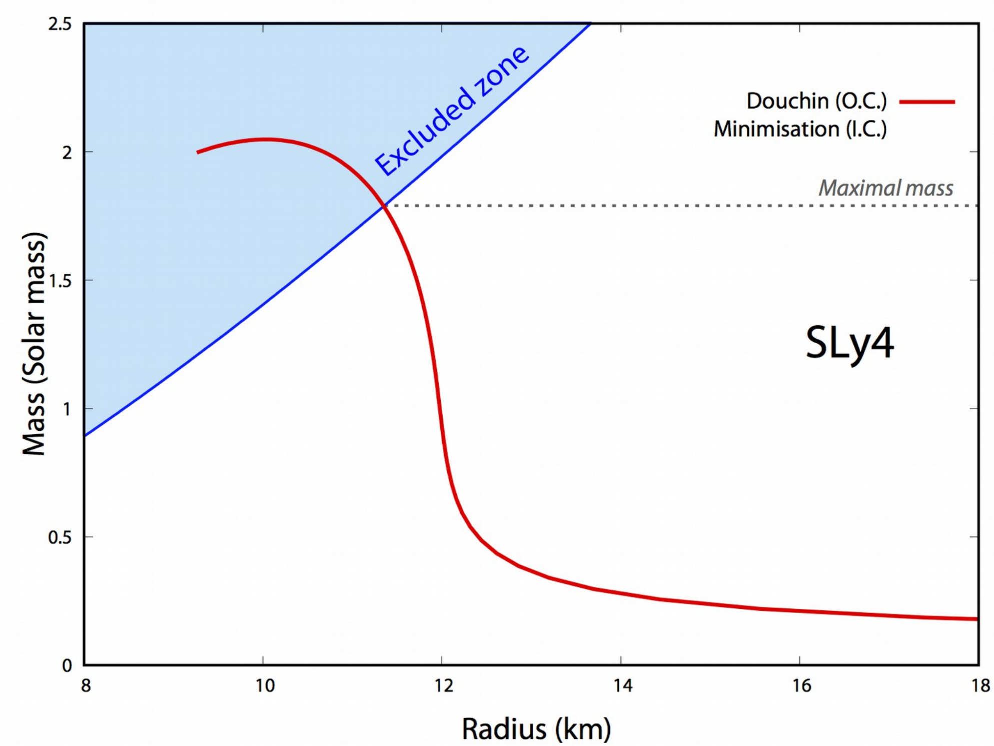
Possible reason for discrepancy?

- <u>In condensed matter physics:</u>
  band gap >> pairing gap
- <u>Neutron star crust:</u>
  band gap < pairing gap

Impact on glitches

Glitch: sudden increase of the rotation velocity

- **Transfer of angular momentum** from superfluid to rigid parts of the star.
- **Strongly affected by the superfluid properties.**
- Calculated with: **superfluid** properties (**entrainment**) and **clusters.**
- <u>**Constraint on possible maximal mass and radius for *Vela*:**</u>
  - <u>**Super. Hydro.:**</u> **~1.8 $M_\odot$**
  - <u>Band theory:</u> ~ 0.7 $M_\odot$ [Chamel PRL 110, 011101 (2013)]



Better agreement of our constraint on the maximal mass and radius with the **glitch activity observations.**

Summary

- **Inner crust description** done with **ETF approximation**.

- **Superfluid hydrodynamics** description of neutron **entrainment** within **2D and 3D geometries.**
  → **1D calculations** (analytical) already done *[M. Urban, M. Oertel and L. Di-Gallo]*

- Effect of **periodicity in the lattice.**

- Impact on the **astrophysical properties** of the neutron star.

Outlook

- **Phonon velocity.**
- Which **impact** on **neutron star cooling ?**
- Adding **surface tension** and **neutron skin** to the hydrodynamics.
- **Time dependent** hydrodynamic model: **density waves.**