MODE-SNR-PWN 2016

Relativistic Star Models with Hot Matter

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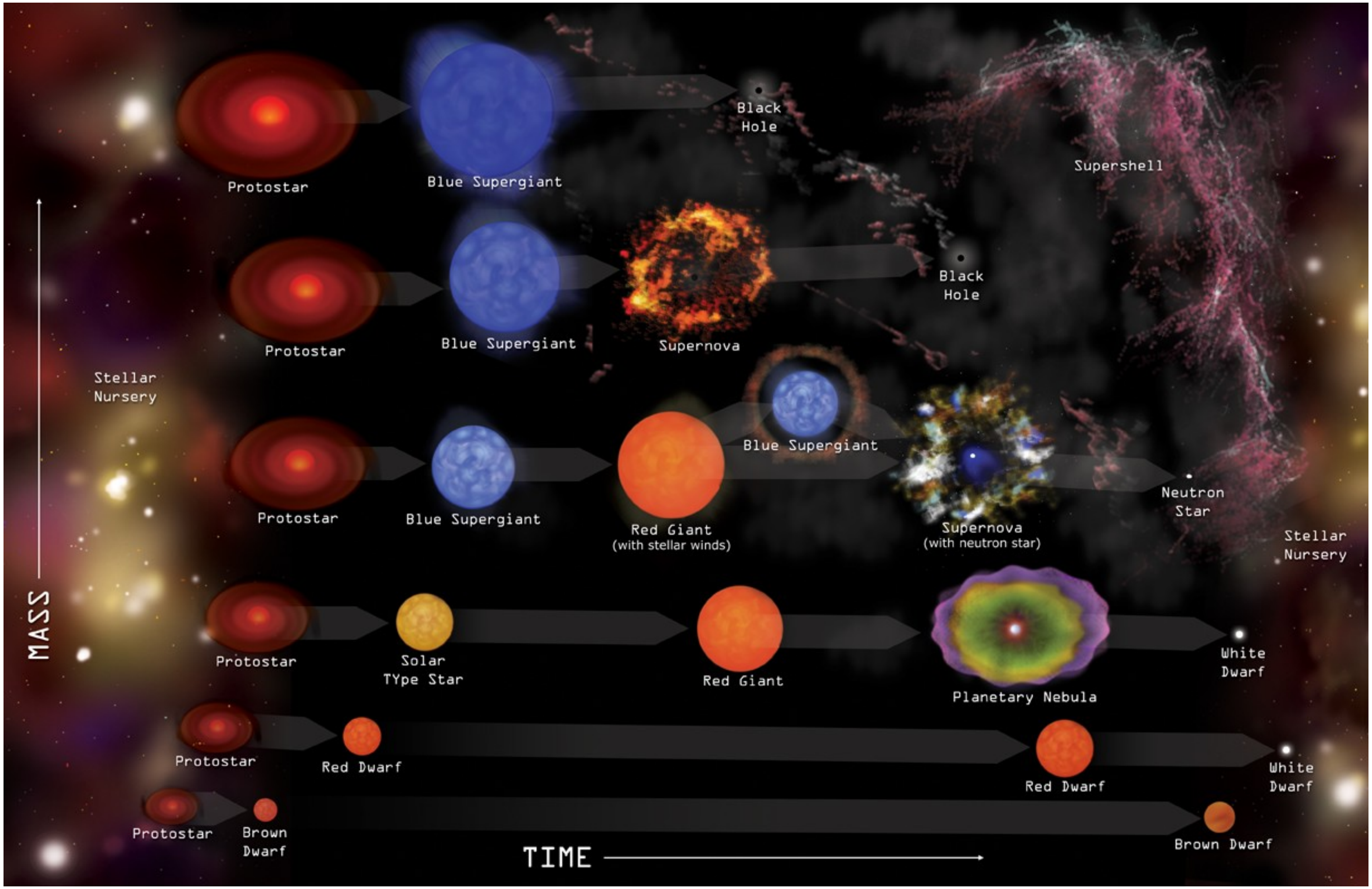
How do Massive Stars Die?

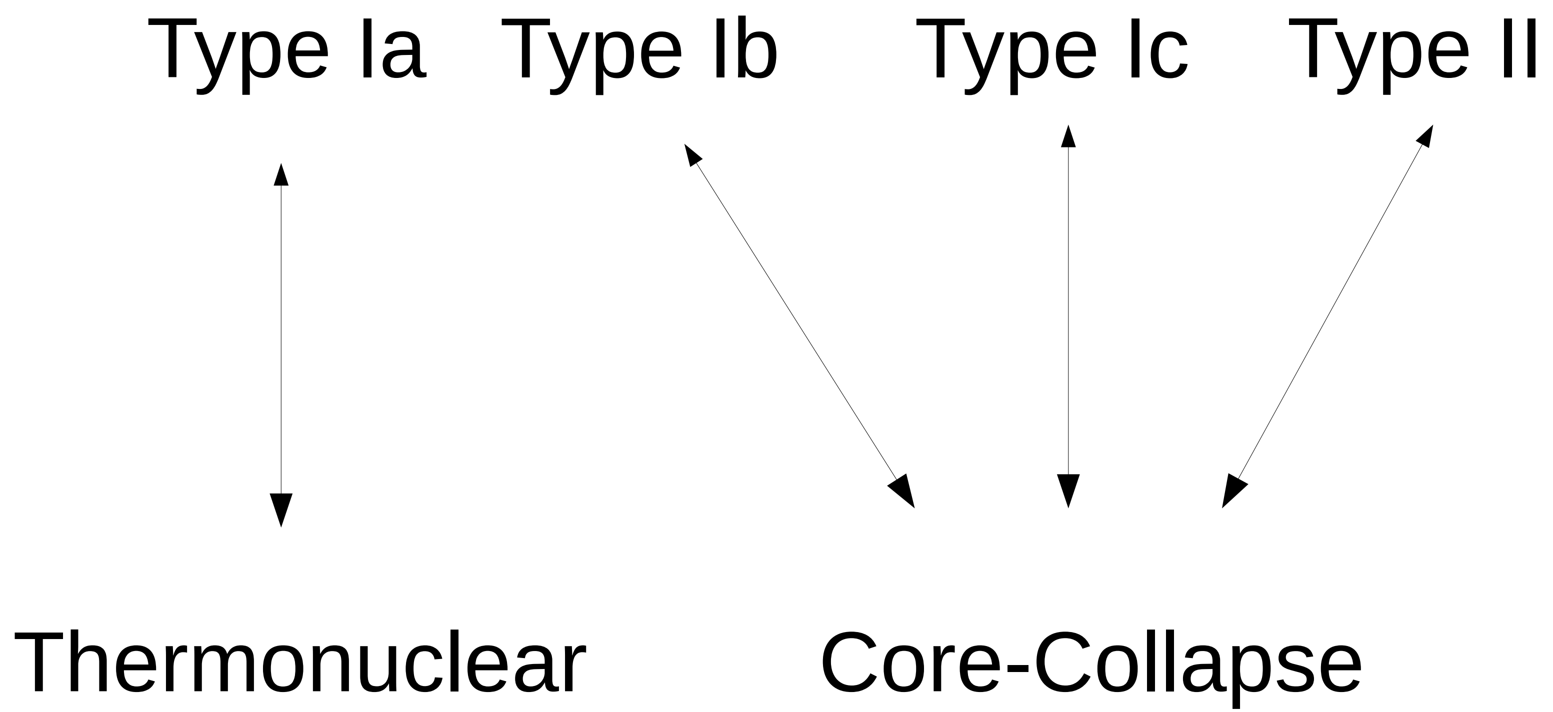
- The final fate of star evolution



Chandra

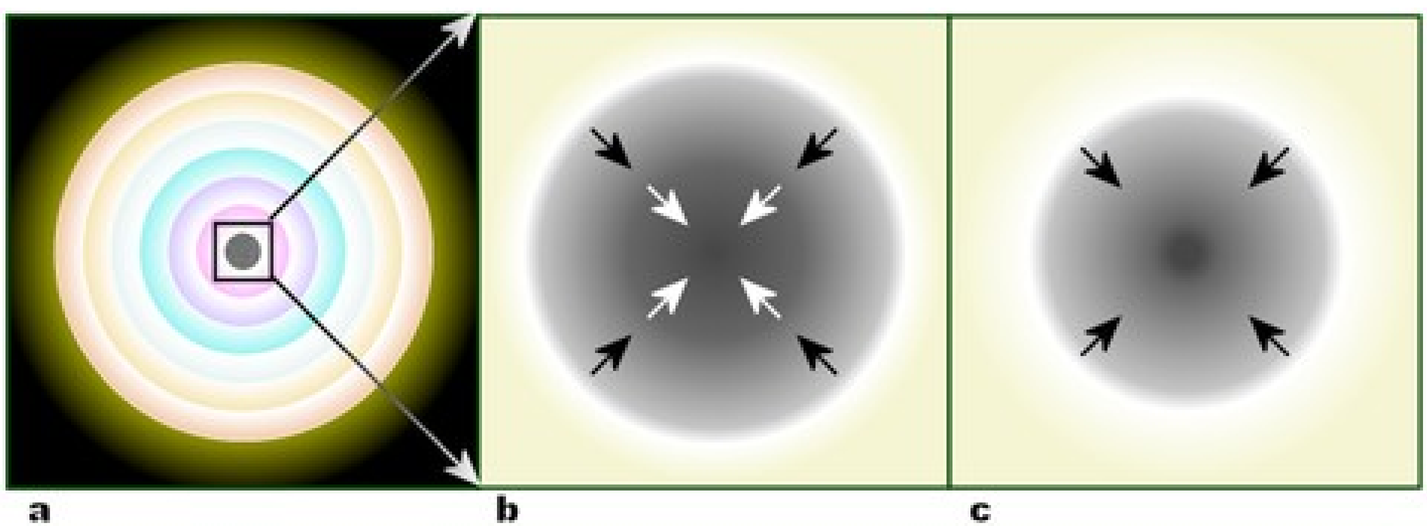
How do Massive Stars Die?

- Which supernovae?



Core-Collapse

- The gravitational collapse of the iron core



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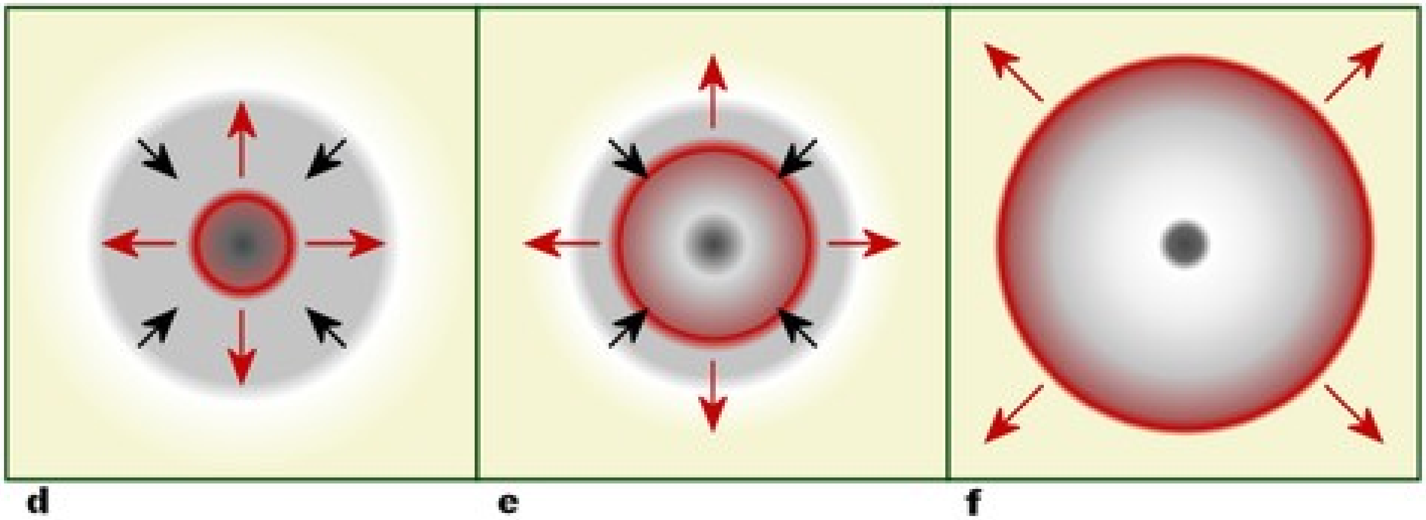
Core-Collapse

- The consequences of the core-collapse



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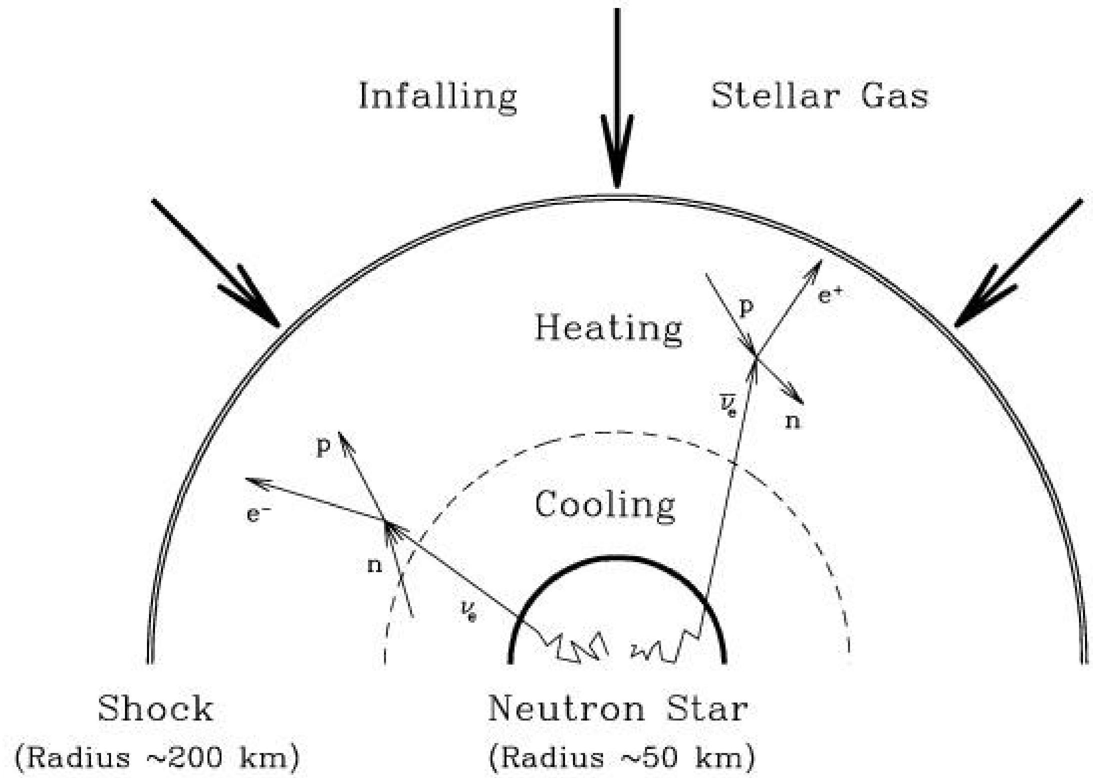
Core Collapse

- The neutrino driven mechanism

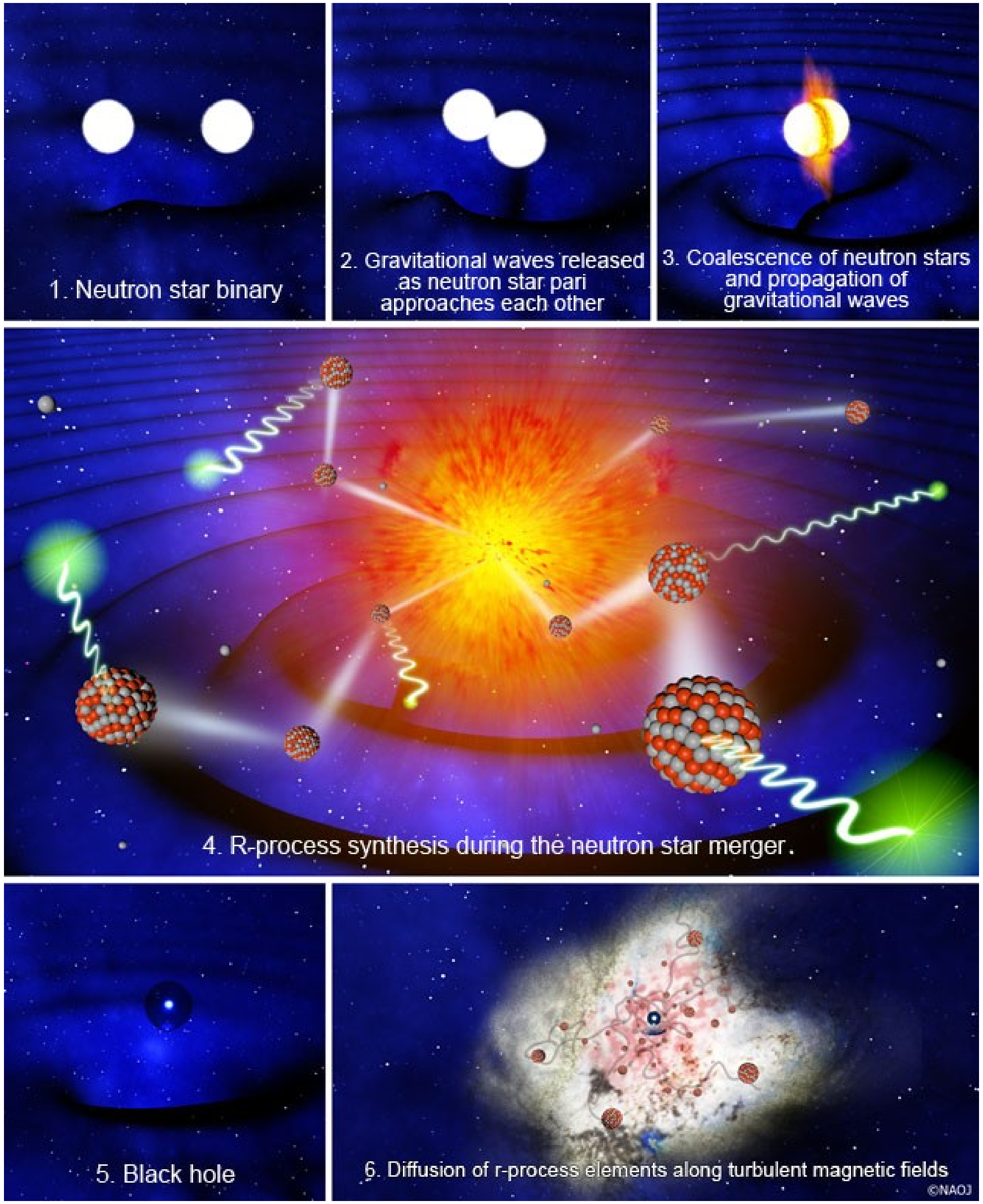
Explosion mechanisms

- The neutrino heating
- Hydrodynamical instabilities (Sasi, convection)
- Magnetorotational instabilities
- ...



H. T. Janka

Neutron star mergers

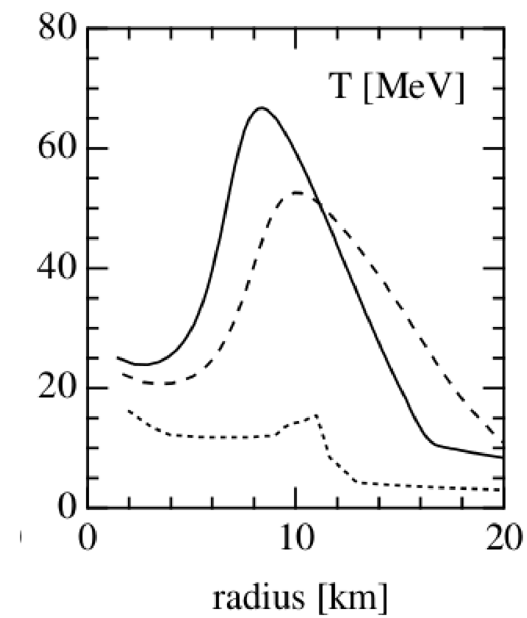


T. Tsujimoto, T.Shigeyama

Finite Temperature in Compact Stars Astrophysics

- Core-Collapse Supernovae

  - T ~ 50 - 100 MeV



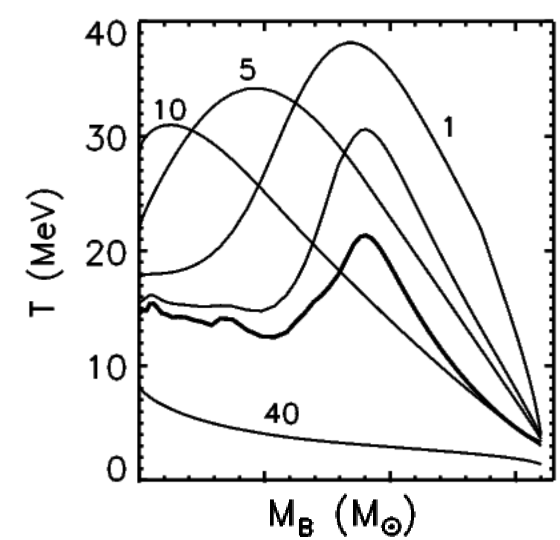
Fischer (2009)

- Neutron Star Mergers

  - T ~ 80 MeV

- Proto-Neutron Stars

  - T ~ 50 MeV



Ferrari et al (2003)

$10\ \mathrm{MeV} \approx 1.16 \times 10^{11}\ \mathrm{K}$

Finite Temperature in Compact Stars Astrophysics

- Core-Collapse Supernovae

  - Temperature effects included since many years in dynamical simulations

- Neutron Star Mergers

  - Kaplan et al (2014), Bauswein et al (2012), Abdikamalov et al (2013), Sekiguchi et al (2011) …

- Proto-Neutron Stars

  - Martinon et al (2014), Villain et al (2004), Pons et al (1999), Goussard et al (1998, 1997) ...

Finite Temperature in Compact Stars

- Finding a general solution for the equilibrium equations of stationary axisymetric spacetimes with finite temperature is difficult. Typical strategies are:

- Effective barotropic EoS

Goussard et al (97,98), ...

$$p = p(n_b, T(n_b), Y_e(n_b, T(n_b)))$$

- Using perturbative approximation methods for the metric

Martinon et al (2014), ...

Our model for hot stars

- General relativistic, stationary axisymmetric solutions of rotating stars

- Perfect fluid

Energy-momentum tensor → $T^{\mu\nu} = (p+\varepsilon)u^\mu u^\nu + p\, g^{\mu\nu}$

- Temperature (or entropy) dependent EoS

$$p := p(n_b, s_b)$$

$$\varepsilon := \varepsilon(n_b, s_b)$$

Spacetime Solution

- Stationary axisymmetric spacetimes

Two killing vector fields:

$$\boldsymbol{\partial_t},\ \boldsymbol{\partial_\phi}$$

We use a conformal metric, defined as:

$$\tilde{\gamma}_{ij} := \Psi^{-4}\gamma_{ij}$$

- Dirac Gauge

Bonazzola et al, (2004)

Lin and Novak, (2006)

$$\mathcal{D}_j h^{ij} = 0,$$

$$h^{ij} := \tilde{\gamma}^{ij} - f^{ij}$$

- Maximal slicing condition

$$K = 0$$

Spacetime Solution

- Metric calculations in Dirac gauge

Bonazzola et al, (2004)
Lin and Novak, (2006)

$$\Delta_1 N = \sigma 1$$
$$\Delta_2 \beta_i = \sigma 2$$
$$\Delta_3 \Psi = \sigma 3$$
$$\Delta_4 h_{ij} = \sigma 4$$

Star equilibrium

- Equilibrium equations

$$(\delta^{\mu}_{\nu} + u^{\mu} u_{\nu}) \nabla_{\mu} T^{\mu\nu} = 0$$

Defining the pseudo-log enthalpy field as

$$H = \ln\left(\frac{\varepsilon + p}{m_b n_b}\right),$$

With the help of the 1st law of thermodynamics, the equilibrium equations can be written as

$$\partial_i \left(H + \nu - \ln \Gamma\right) = T e^{-H} \partial_i s_b - F \partial_i \Omega, \qquad i = r, \theta$$

Star equilibrium

- Equilibrium equations (for rigid rotation)

$$\partial_i \left(H + \nu - \ln \Gamma \right) = T e^{-H} \partial_i s_b, \qquad i = r, \theta$$

No first integral in general! First integral for the barotropic EoS $H(n_b)$.

Star equilibrium

- Equilibrium equations (for rigid rotation)

$$\partial_i \left(H + \nu - \ln\Gamma\right) = Te^{-H}\partial_i s_b, \qquad i = r, \theta$$

Instead of solving an analytical first integral, we propose the following scheme

$$H = -\nu + \ln\Gamma + \int_0^{r_s} Te^{-H}\partial_r s_b \, dr'$$

$$\Delta_{\theta\phi} s_b = \left(\partial^\theta + \frac{1}{\tan\theta}\right)\left[\frac{\partial_\theta (Te^{-H})}{\partial_r (Te^{-H})}\partial_r s_b\right]$$

Where the monopolar part of the s_b has to be specified, whereas the higher multipoles are determined by the equilibrium solver

Star equilibrium

- Equilibrium equations

$$\partial_i \left(H + \nu - \ln \Gamma\right) = Te^{-H}\partial_i s_b - F\partial_i \Omega, \qquad i = r, \theta$$

Assuming that we have a rotation law such that $F := F(\Omega)$

$$H = -\nu + \ln \Gamma + \int_0^{r_s} \left(Te^{-H}\partial_r s_b - F(\Omega)\partial_r \Omega\right)\, dr'$$

$$\Delta_{\theta\phi} s_b = \left(\partial^\theta + \frac{1}{\tan\theta}\right)\left[\frac{\partial_\theta(Te^{-H})}{\partial_r(Te^{-H})}\partial_r s_b\right]$$

Results

- To test the code, we use the relativistic ideal gas EoS:

$$p(H, s_b) = k\, n_b(H, s_b)^\gamma e^{(\gamma-1)s_b}$$

$$\varepsilon(H, s_b) = \frac{k}{\gamma - 1} n_b(H, s_b)^\gamma e^{(\gamma-1)s_b} + m_b n_b(H, s_b)$$

$$T(H) = m_b \frac{\gamma - 1}{\gamma\, k_b} \left(e^H - 1\right)$$

$$n_b(H, s_b) = \left(m_b \frac{\gamma - 1}{\gamma\, k} \left(e^H - 1\right) \right)^{\frac{1}{\gamma-1}} e^{-s_b}$$

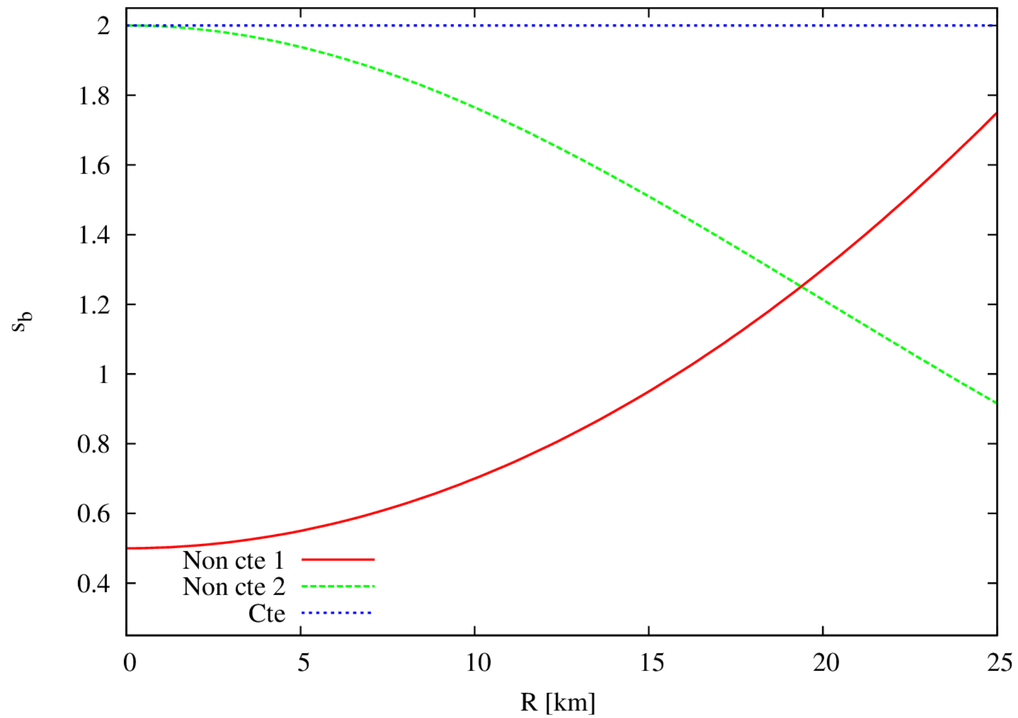
with $\gamma = 2$ and $k = 0.025\, \rho_{nuc}\, c^2 / n_{nuc}$

Results

- We will consider two (ad hoc) non-constant entropy per baryon radial profiles

non cte 1 → $s_b = 0.5 + \frac{r^2}{5}$

non cte 2 → $s_b = 2 \times e^{-\frac{r^2}{2^3}}$



Results
(rigid rotation)

cte sb profile

$$M_g = 4.73\, M_\odot$$

$$R_{eq} = 47.37\, Km$$

$$R_p/R_{eq} = 0.58$$

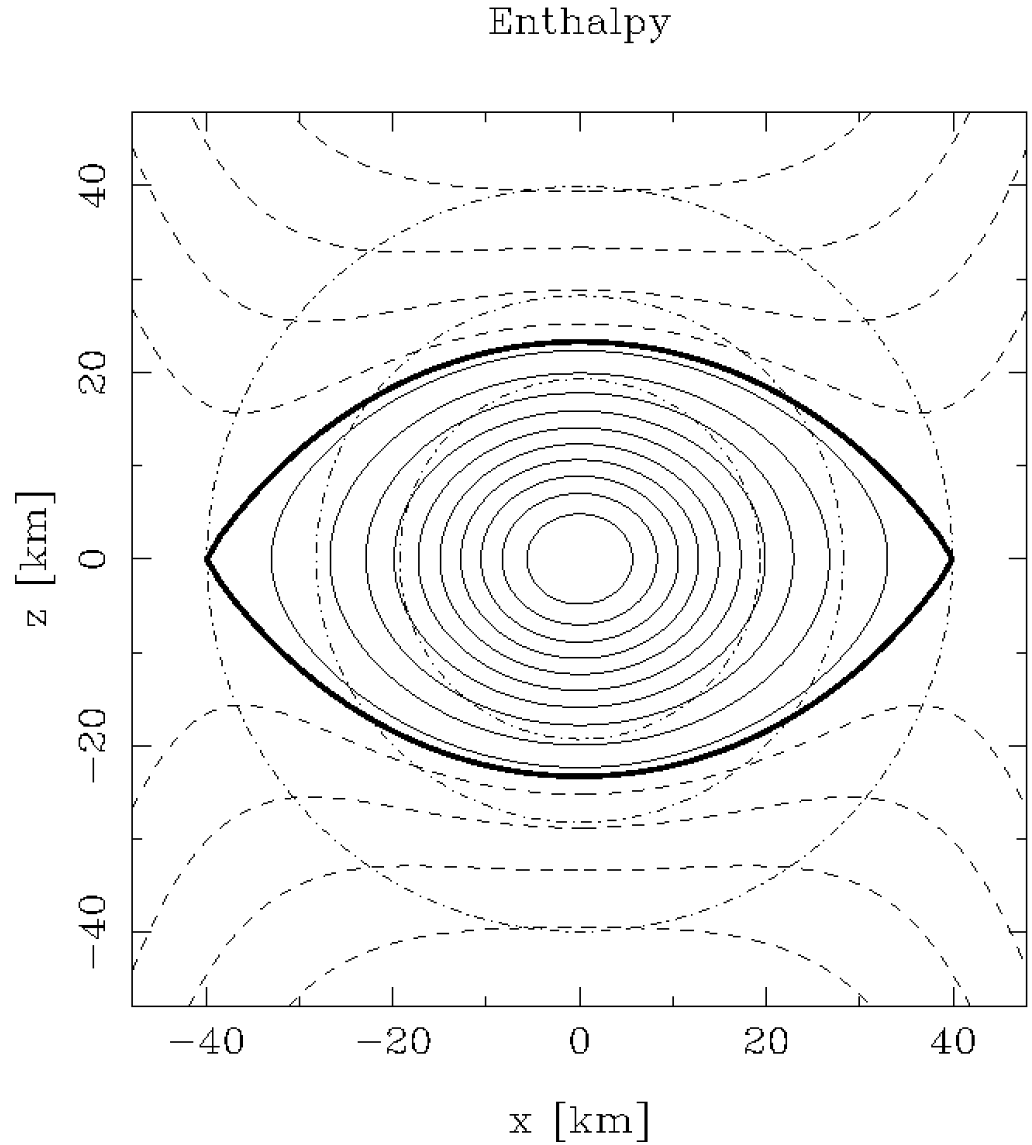
$$f_{rot} = 382.84\, Hz$$

$$H_c = 0.3$$

$$T_c = 162.89\, MeV$$

$$GRV2 = -7.43\, e-7$$

$$GRV3 = 1.32\, e-6$$



Results

(rigid rotation)

non-cte sb profile 1

$M_g = 2.26\, M_\odot$

$R_{eq} = 22.65\, Km$

$R_p/R_{eq} = 0.64$

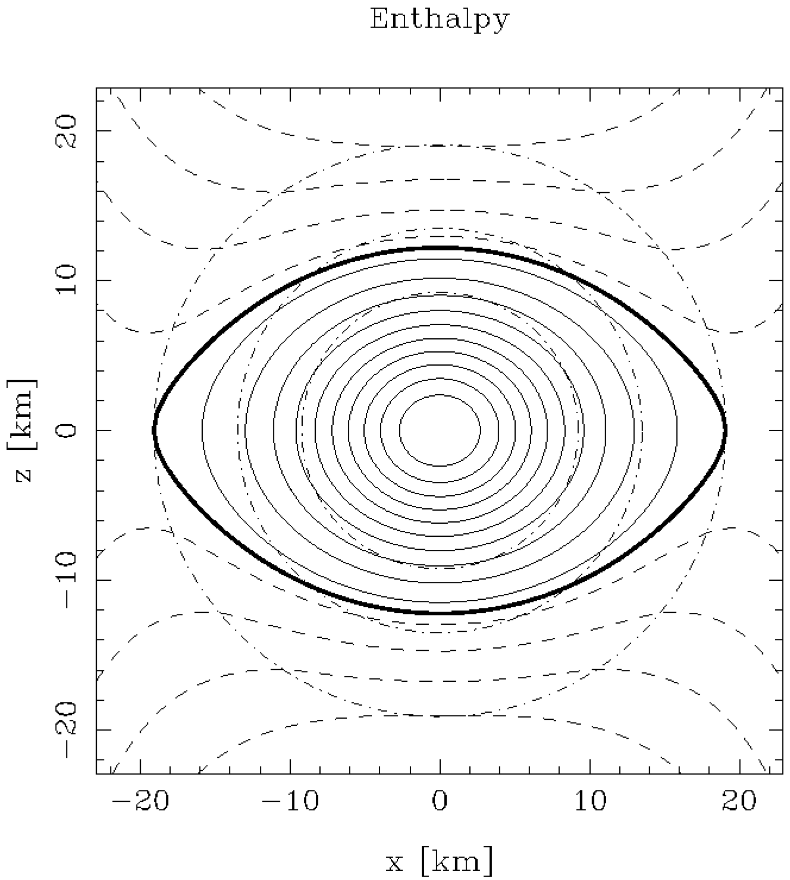
$f_{rot} = 721.85\, Hz$

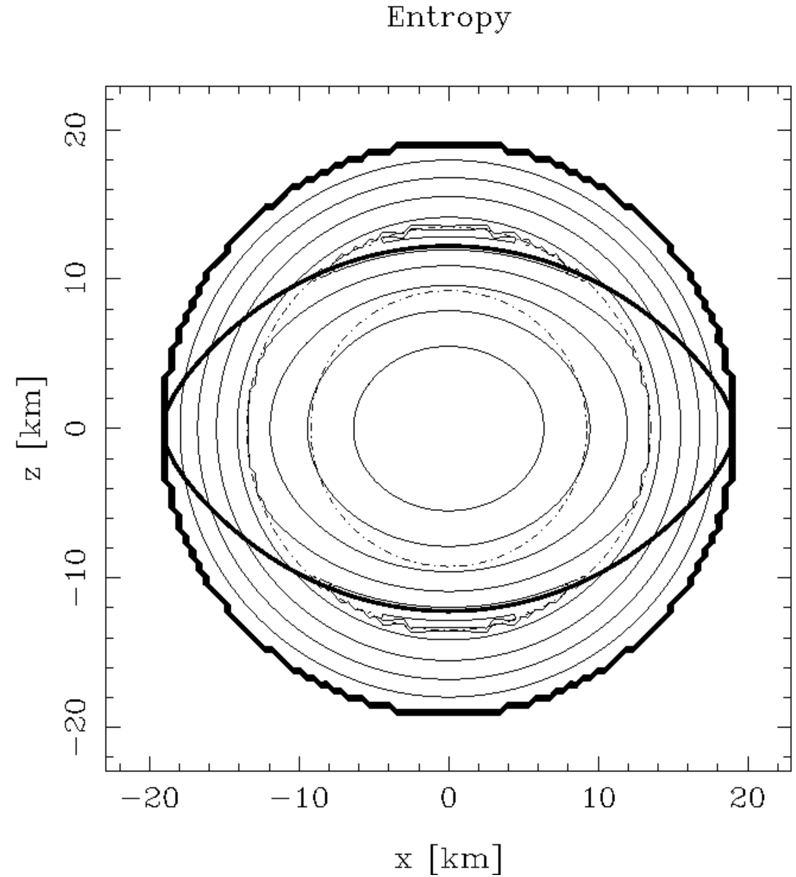
$H_c = 0.3$

$T_c = 162.89\, MeV$

$GRV2 = -9.45\, e-4$

$GRV3 = 9.35\, e-4$





Results

(rigid rotation)

non-cte sb profile 1

$M_g = 2.18\, M_\odot$

$R_{eq} = 20.26\, Km$

$R_p/R_{eq} = 0.75$

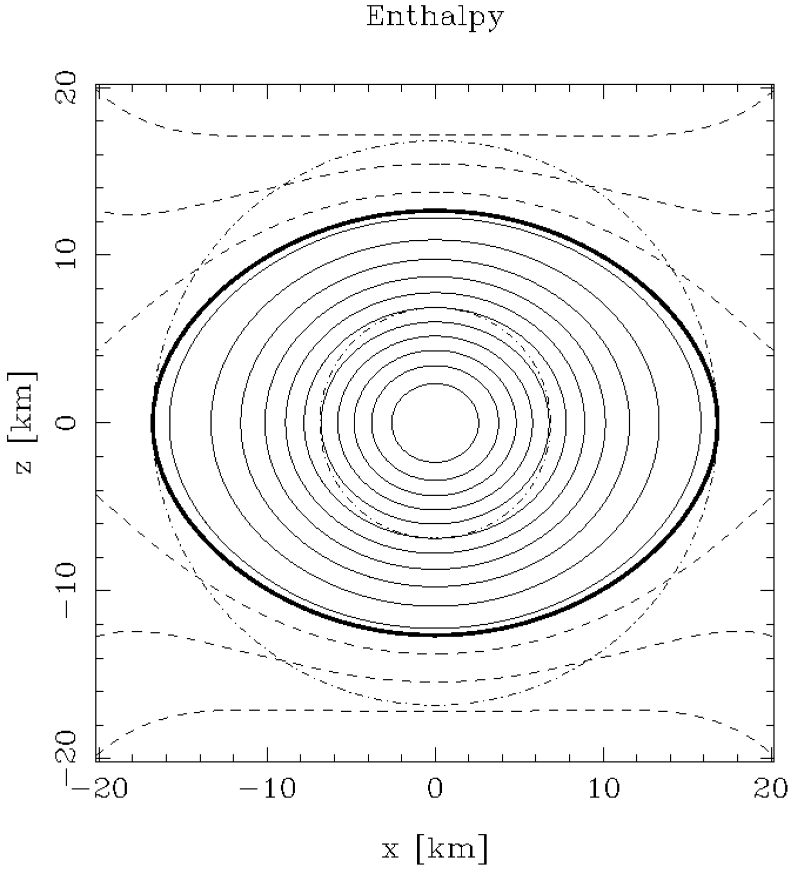
$f_{rot} = 650\, Hz$

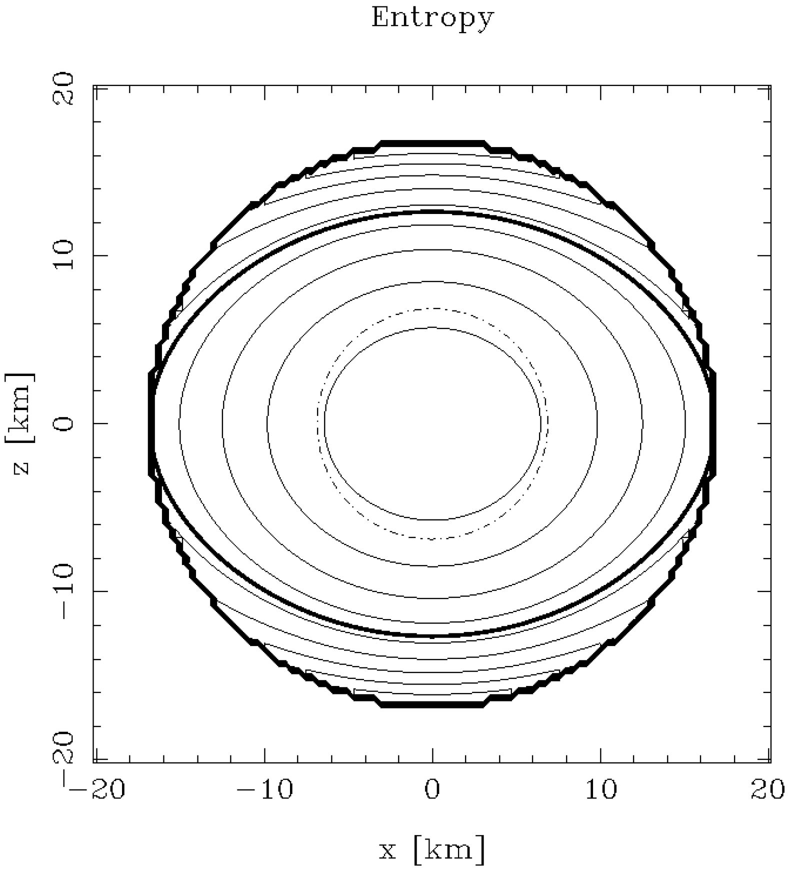
$H_c = 0.3$

$T_c = 162.89\, MeV$

$GRV2 = -3.34\, e - 6$

$GRV3 = 2.38\, e - 5$





Results

(rigid rotation)

non-cte sb profile 2

$M_g = 4.40\, M_\odot$

$R_{eq} = 37.65\, Km$

$R_p/R_{eq} = 0.57$

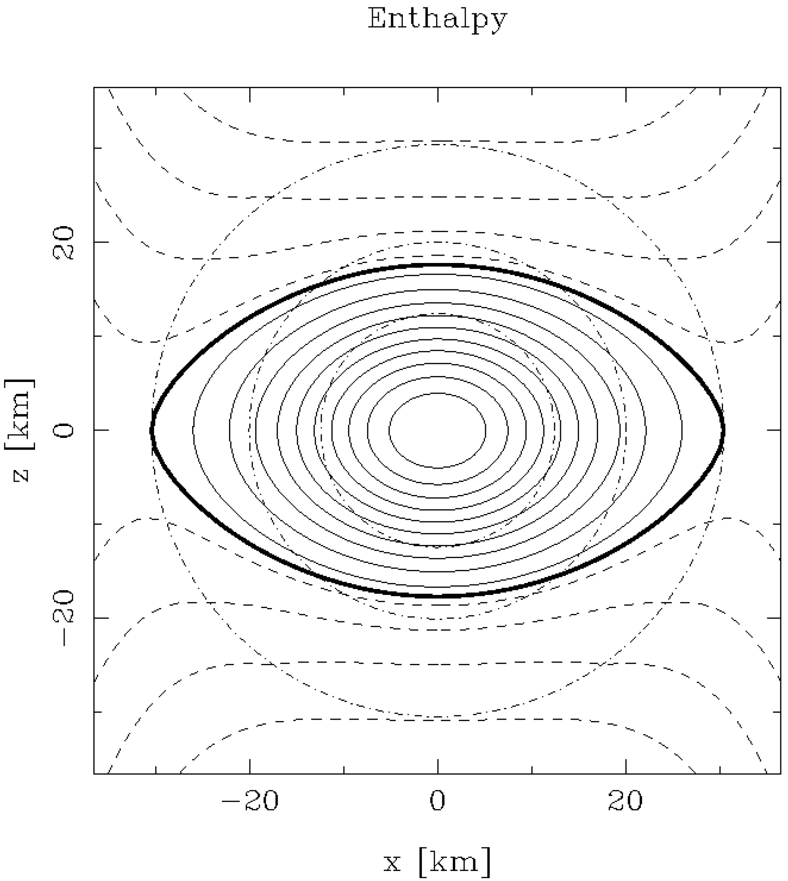
$f_{rot} = 490\, Hz$

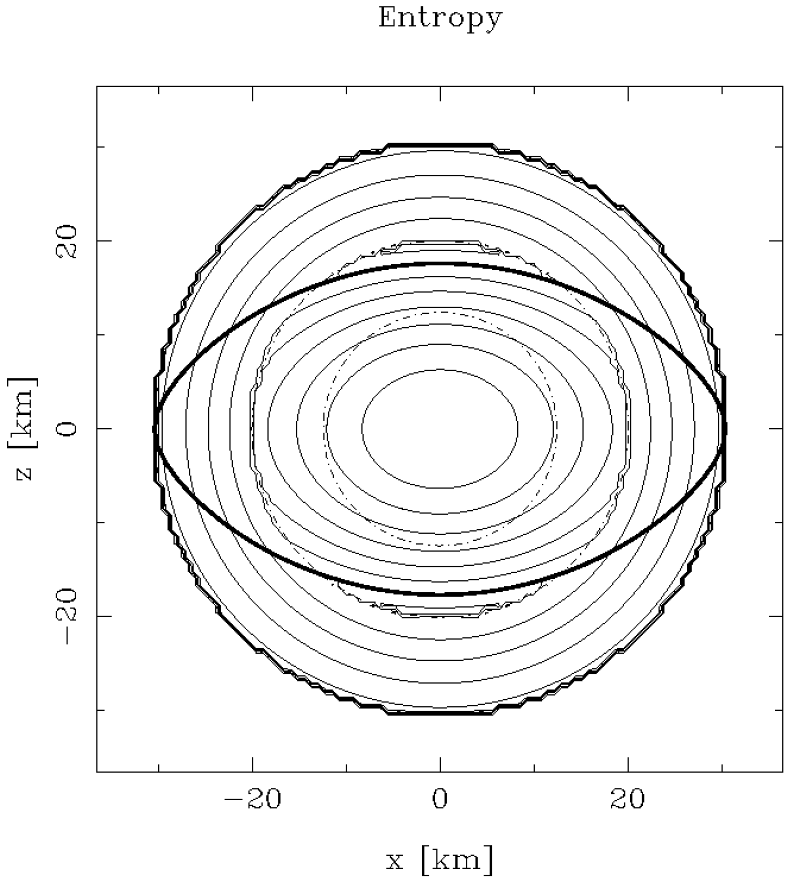
$H_c = 0.3$

$T_c = 162.89\, MeV$

$GRV2 = 1.01\, e - 2$

$GRV3 = 5.73\, e - 3$





Results

(rigid rotation)

non-cte sb profile 2

$M_g = 3.79\, M_\odot$

$R_{eq} = 28.72\, Km$

$R_p/R_{eq} = 0.76$

$f_{rot} = 395\, Hz$

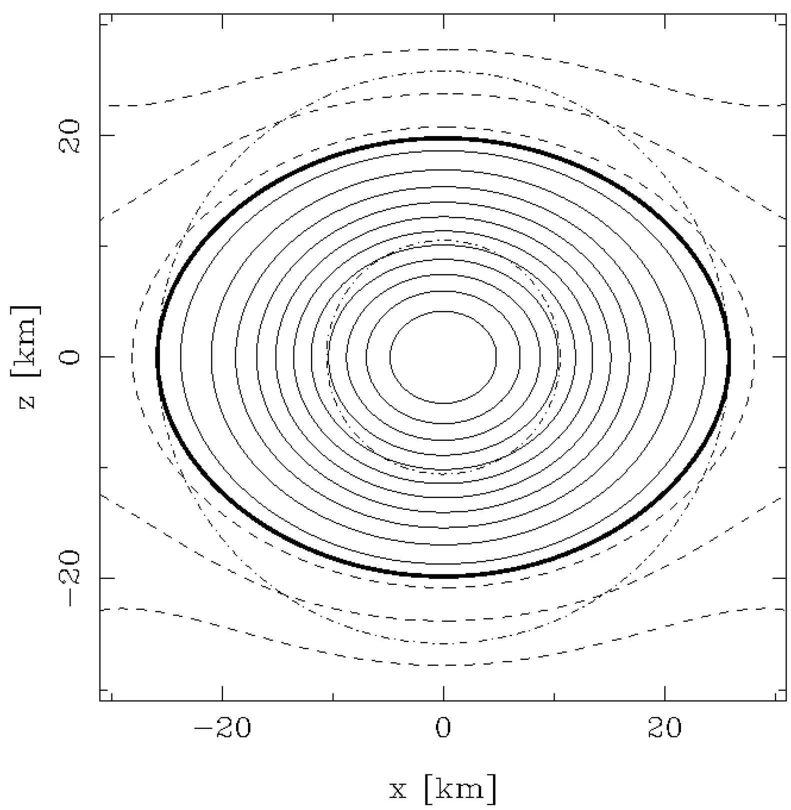
$H_c = 0.3$

$T_c = 162.89\, MeV$

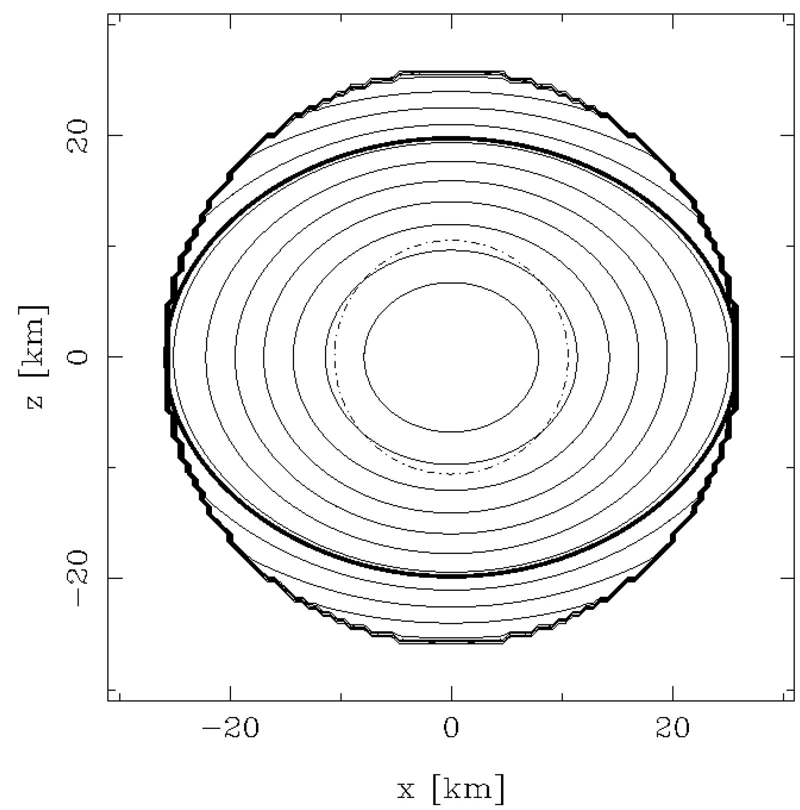
$GRV2 = 2.35\, e - 5$

$GRV3 = -4.69\, e - 5$

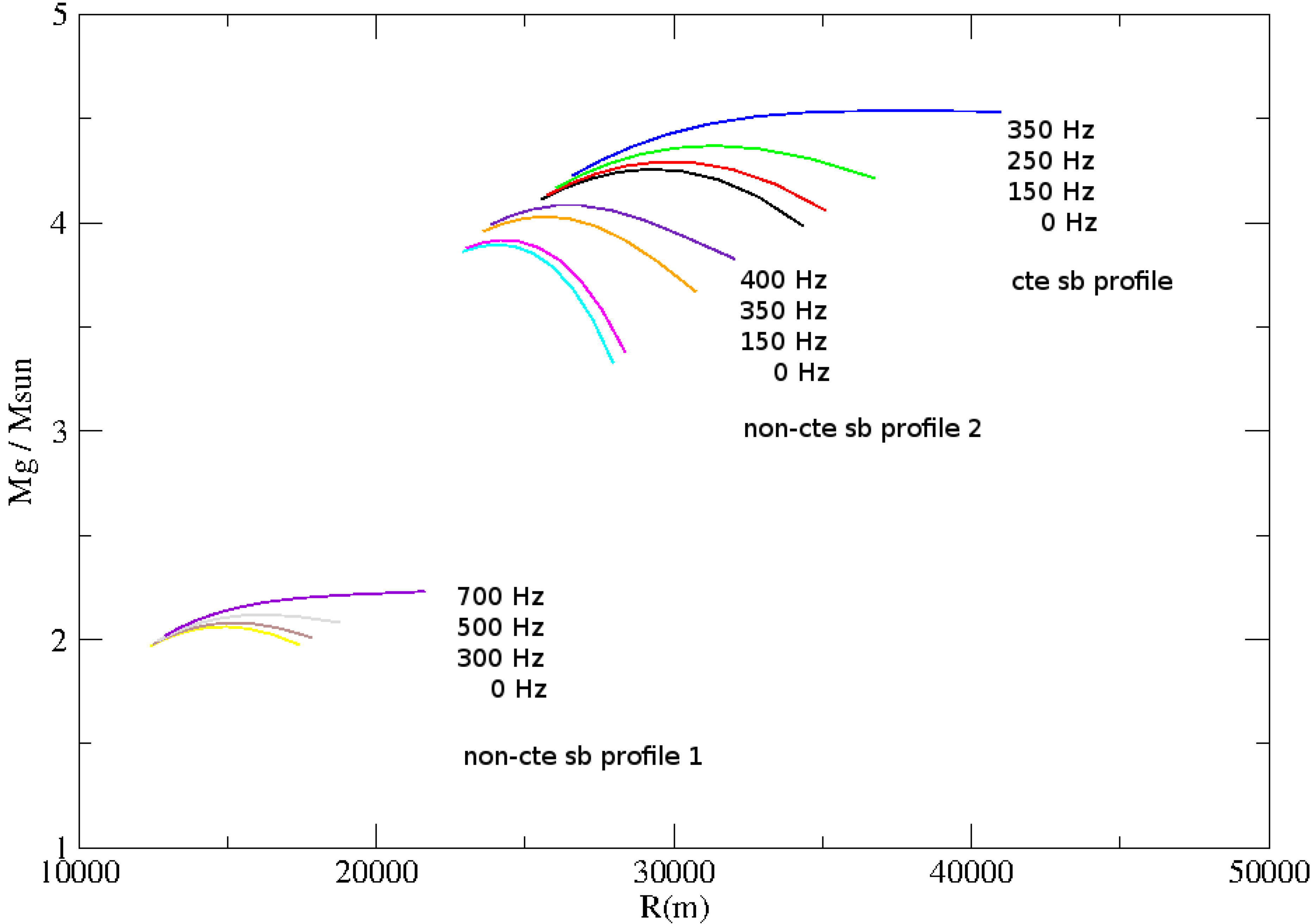
Enthalpy



Entropy



Results



5
4
3
2
1
10000
20000
30000
40000
50000
R(m)
Mg / Msun
350 Hz
250 Hz
150 Hz
0 Hz
cte sb profile
400 Hz
350 Hz
150 Hz
0 Hz
non-cte sb profile 2
700 Hz
500 Hz
300 Hz
0 Hz
non-cte sb profile 1

Results
(differential rotation)

- We will consider a simple linear rotation law, of the form

Baumgarte et al (2000)

$$F(\Omega) = \left(\frac{R_{eq}}{a}\right)^2 (\Omega_c - \Omega)$$

Results
(differential rotation)

cte sb profile

$M_g = 7.18\,M_\odot$

$R_{eq} = 47.50\,Km$

$R_p/R_{eq} = 0.35$

$f_c = 1000\,Hz$

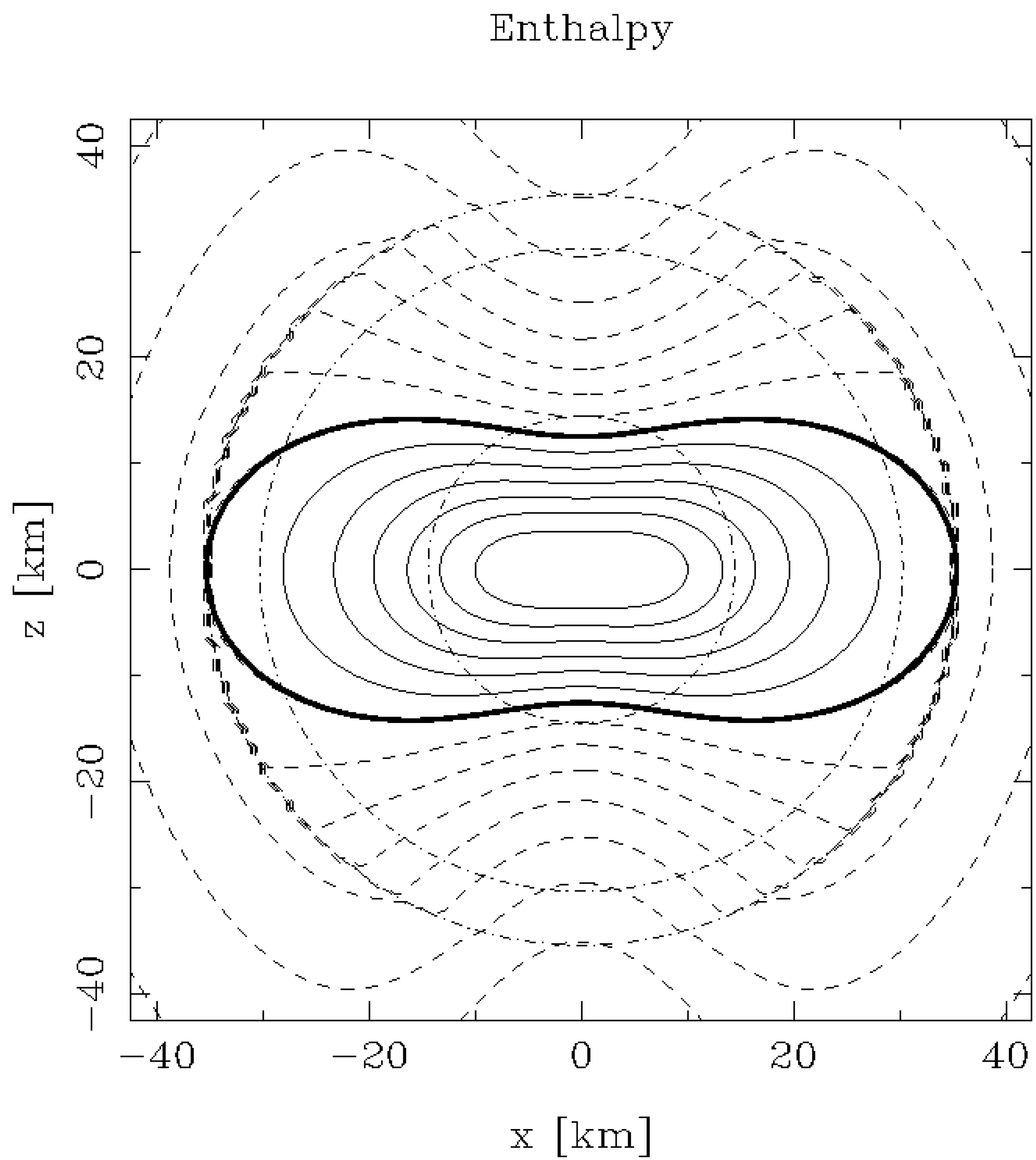
$a = 0.75$

$H_c = 0.3$

$T_c = 162.89\,MeV$

$GRV2 = -1.67\,e-5$

$GRV3 = 2.14\,e-5$



Results

(differential rotation)

non-cte sb profile 1

$M_g = 2.31\, M_\odot$

$R_{eq} = 20.23\, Km$

$R_p/R_{eq} = 0.69$

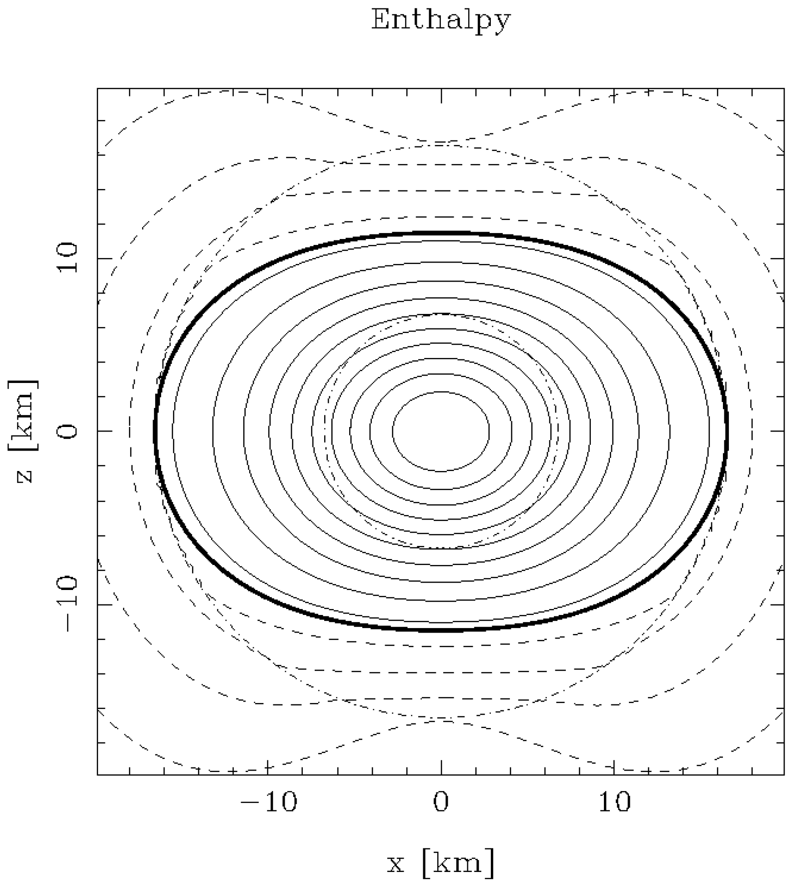
$f_{rot} = 1000\, Hz$

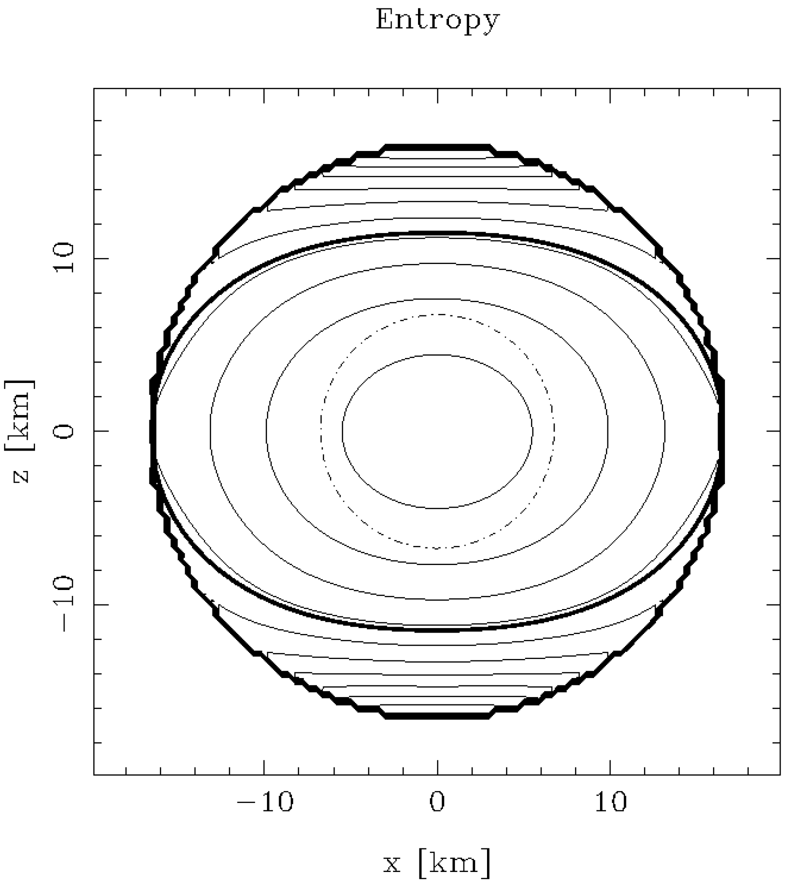
$a = 0.60$

$H_c = 0.3$

$GRV2 = -5.23\, e - 06$

$GRV3 = 4.14\, e - 06$





Results

(differential rotation)

non-cte sb profile 2

$M_g = 4.43\, M_\odot$

$R_{eq} = 32\, Km$

$R_p/R_{eq} = 0.76$

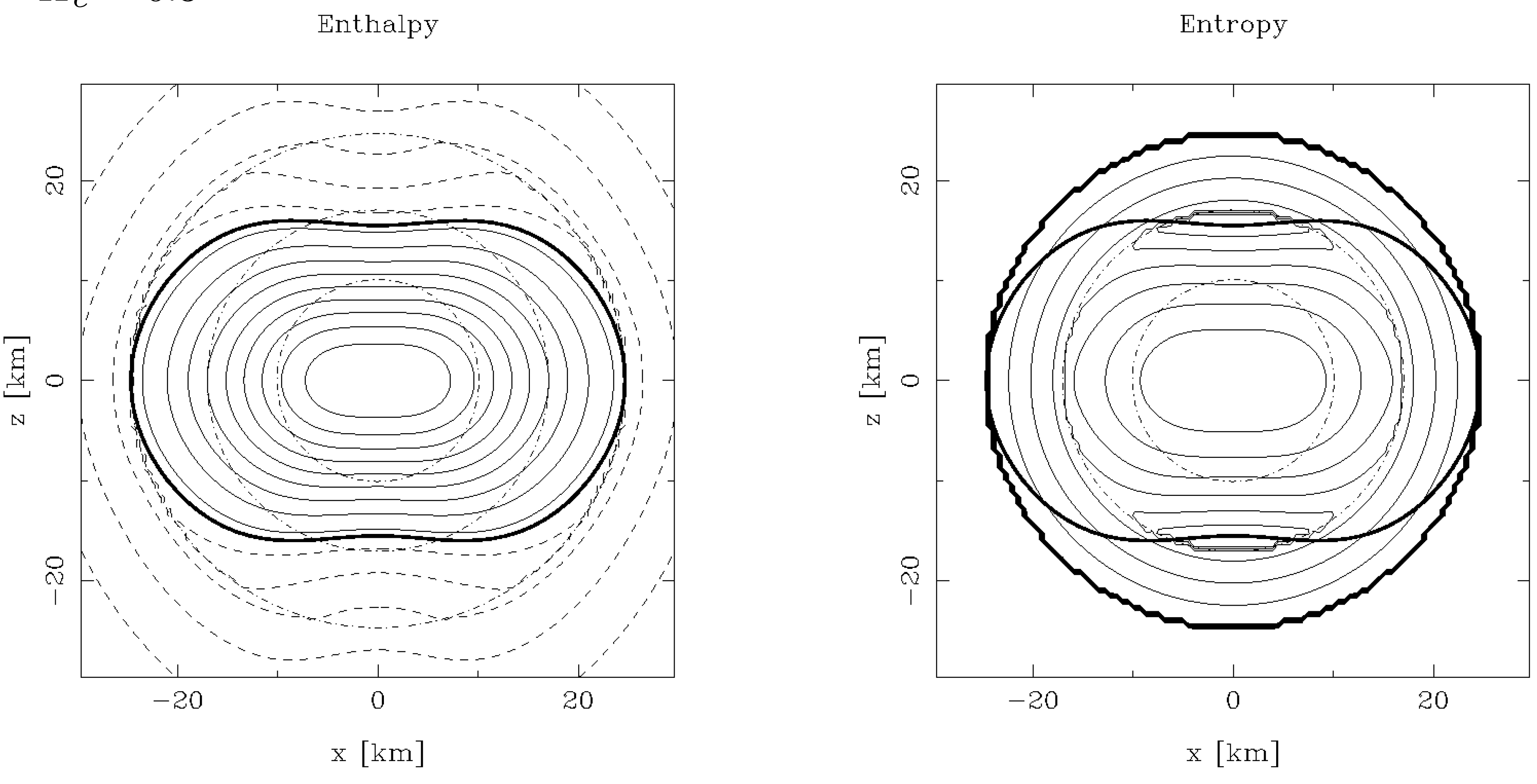
$f_c = 1000\, Hz$

$a = 1$

$H_c = 0.3$

$GRV2 = 2.04\, e-3$

$GRV3 = 1.15\, e-3$



Results

- A more realistic EoS: Banik et al, (2014)
  - density dependent relativistic mean field model
    (parameter set DD2 – Typel et al, (2013))
  - Λ hyperons and its interactions via Φ mesons included
  - β-equilibrium

Results
(rigid rotation)

cte sb profile

$M_g = 2.11\, M_\odot$

$R_{eq} = 20.30\, Km$

$R_p/R_{eq} = 0.58$

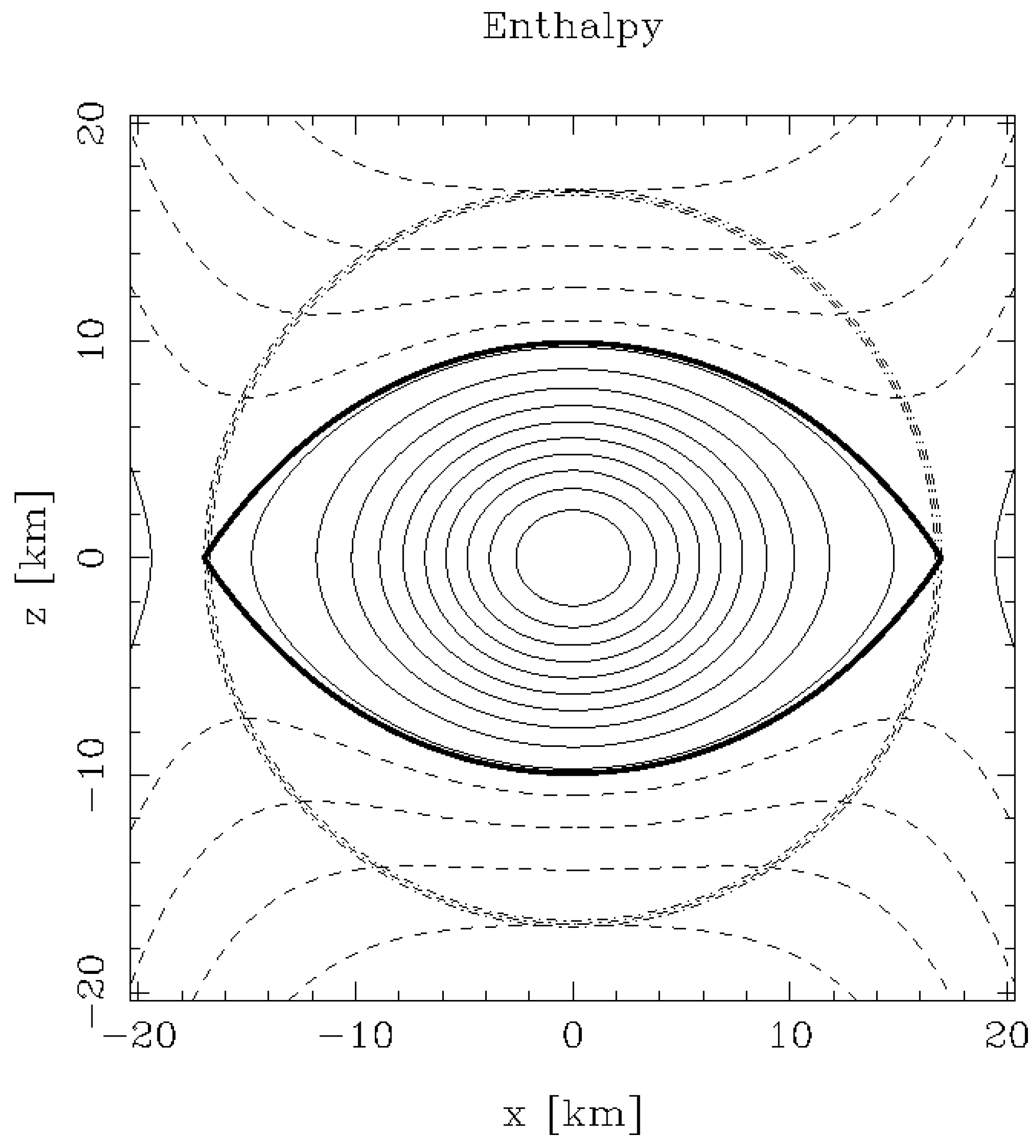
$f_{rot} = 913.63\, Hz$

$H_c = 0.3$

$T_c = 47.35\, MeV$

$GRV2 = 2.31\, e-4$

$GRV3 = -1.96\, e-4$



Results
(rigid rotation)

non-cte sb profile 2

$M_g = 2.13\, M_\odot$

$R_{eq} = 19.92\, Km$

$R_p/R_{eq} = 0.58$

$f_{rot} = 945.5\, Hz$

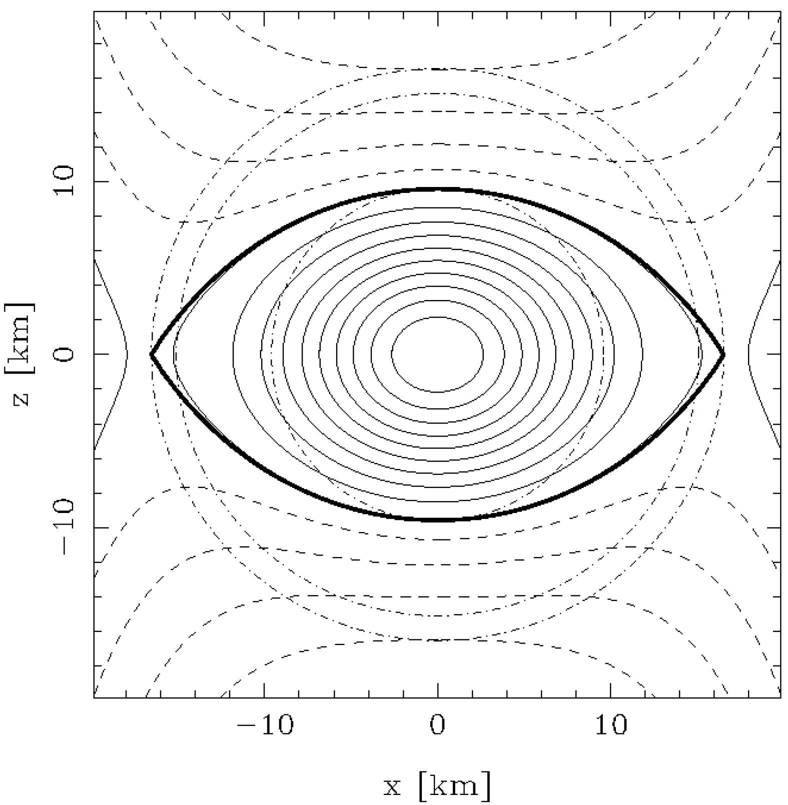
$H_c = 0.3$

$T_c = 47.43\, MeV$

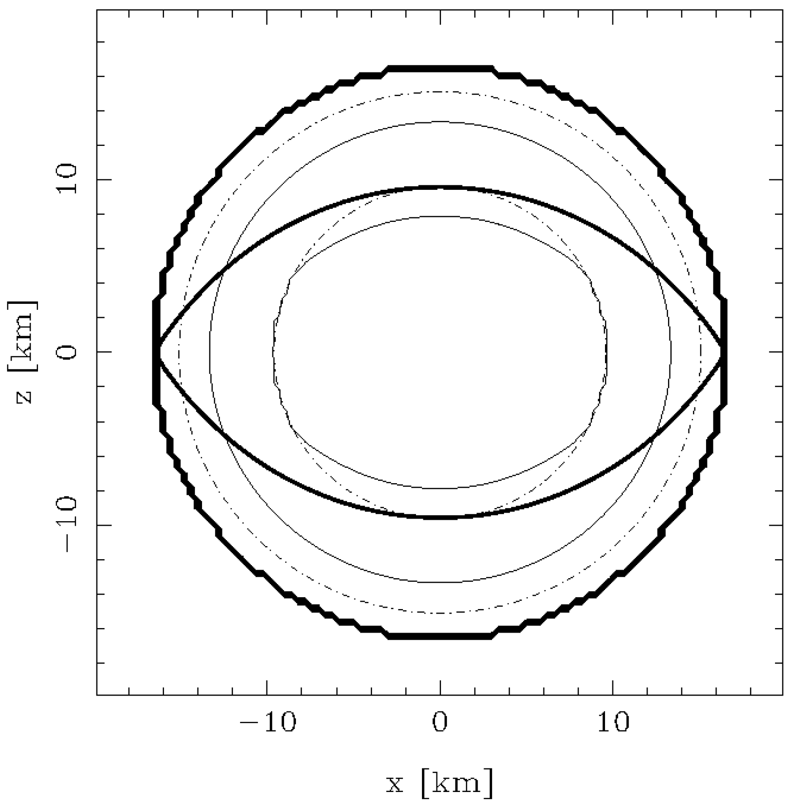
$GRV2 = 1.36\, e-3$

$GRV3 = -7.99\, e-4$

Enthalpy



Entropy



Results
(rigid rotation)

non-cte sb profile 2

$M_g = 1.91\, M_\odot$

$R_{eq} = 15.71\, Km$

$R_p/R_{eq} = 0.85$

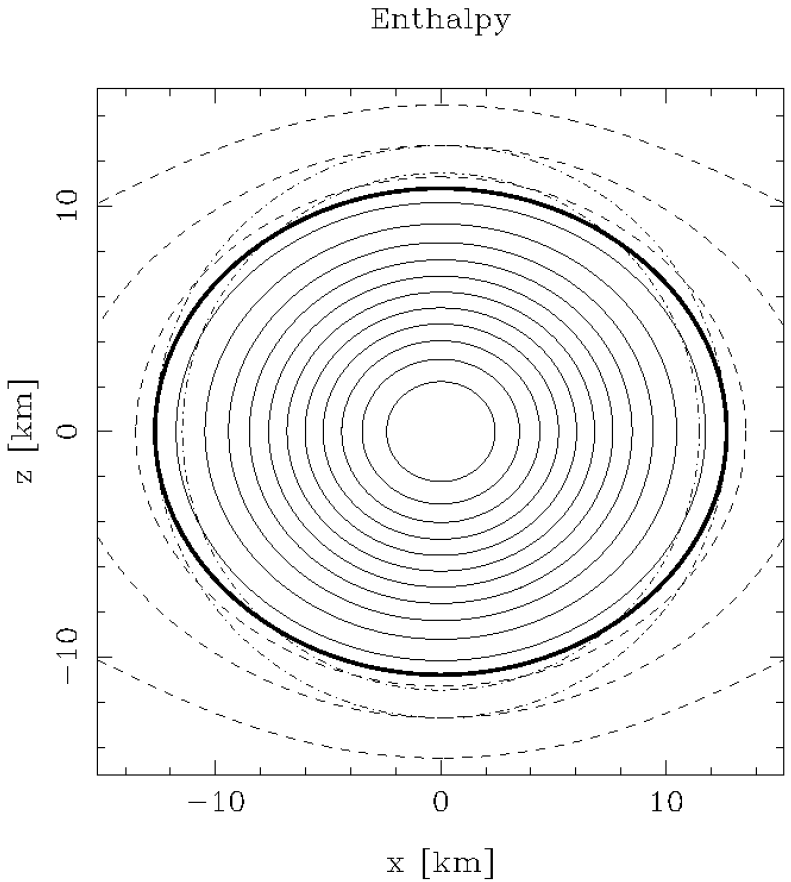
$f_{rot} = 650\, Hz$

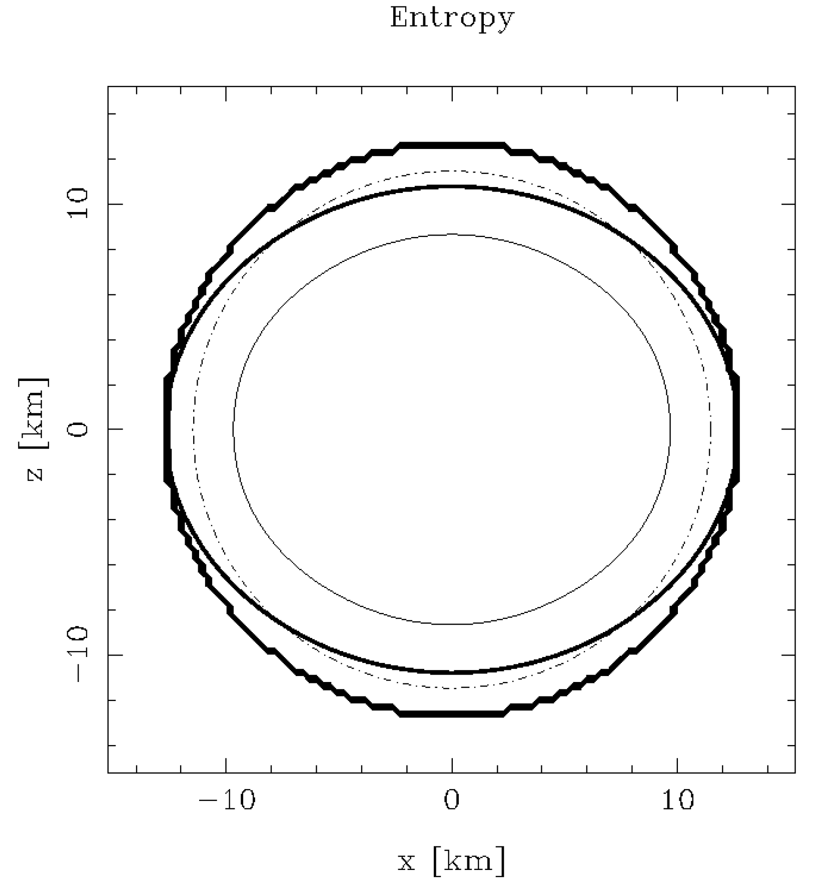
$H_c = 0.3$

$T_c = 47.27\, MeV$

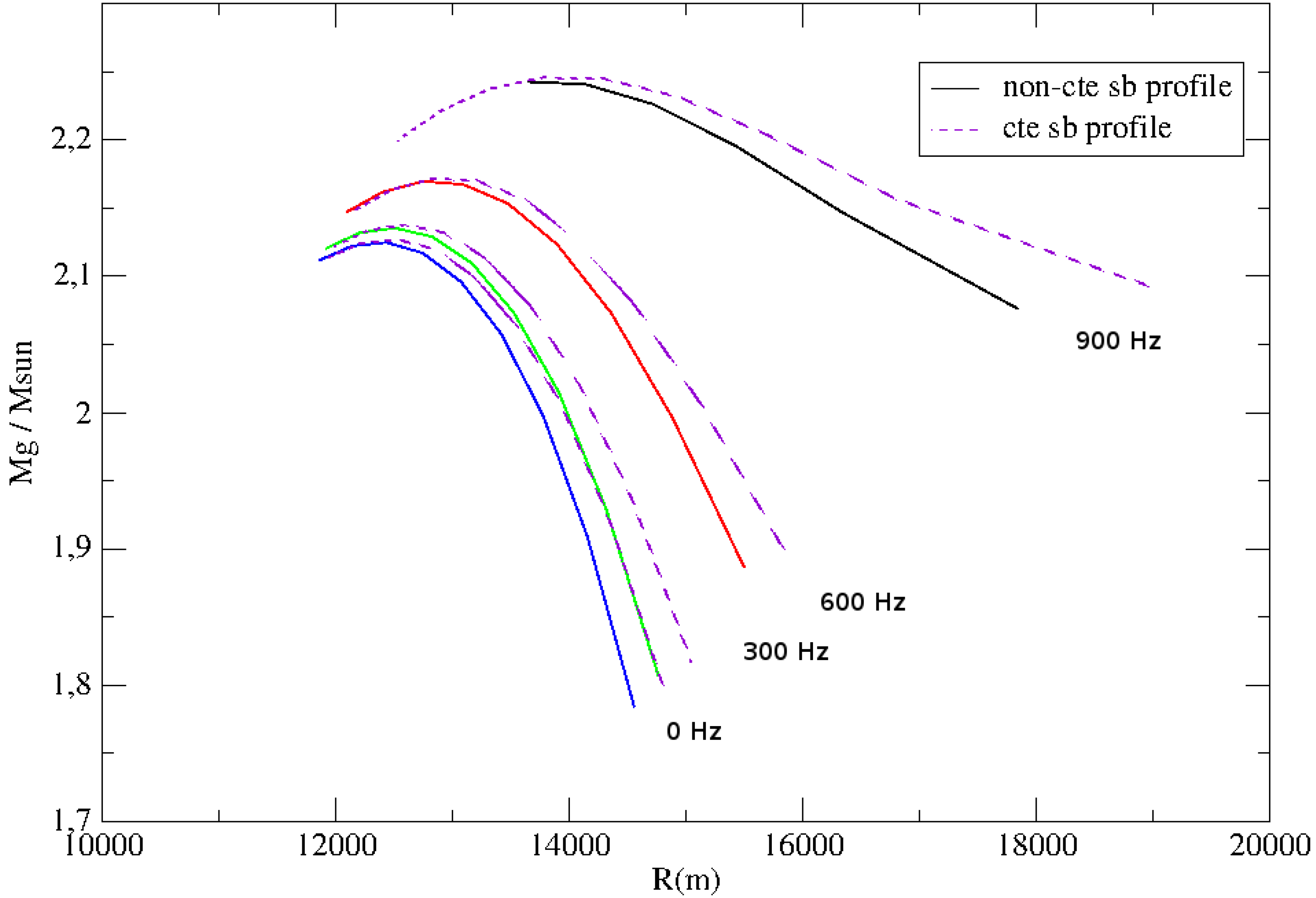
$GRV2 = -7.18\, e - 5$

$GRV3 = 2.01\, e - 4$





Results



non-cte sb profile
cte sb profile
900 Hz
600 Hz
300 Hz
0 Hz
Mg / Msun
R(m)
2,2
2,1
2
1,9
1,8
1,7
10000
12000
14000
16000
18000
20000

Conclusions and future perspectives

- We propose a numerical scheme to consistently make use of not necessarily barotropic EoS capable of modeling appropriately the finite temperature effects on a stationary axisymmetric star, using general entropy profiles
- Having in mind the use of realistic EoS's out of β-equilibrium, we intend to extend the code for EoS's with electron fraction as a third parameter
- We plan to use this for developing quasi-stationary models of proto-neutron stars

Thank You!