MODE, Mai, 19th, 2016

Empirical nuclear equation of state and impact on NS and CC-SN

Jérôme Margueron, IPN Lyon

Empirical EoS → guided by data, with minimal theoretical assumption.

Work in collaboration with R. Casali, F. Gulminelli, A. Fantina and P. Blottiau

Does NS observation data could provide the nuclear EoS?

Recent attempts to deduce the nuclear EoS from:

- thermal X-rays,
- X-ray pulsations,
- Photospheric expansion,
- Etc…

Hebeler 2013
Ozel 2010, 2012, 2014
Steiner 2010, 2013
Etc…

… NS merging (see talk of Nick)

EQUATION OF STATE AND NEUTRON STAR PROPERTIES CONSTRAINED BY NUCLEAR PHYSICS AND OBSERVATION

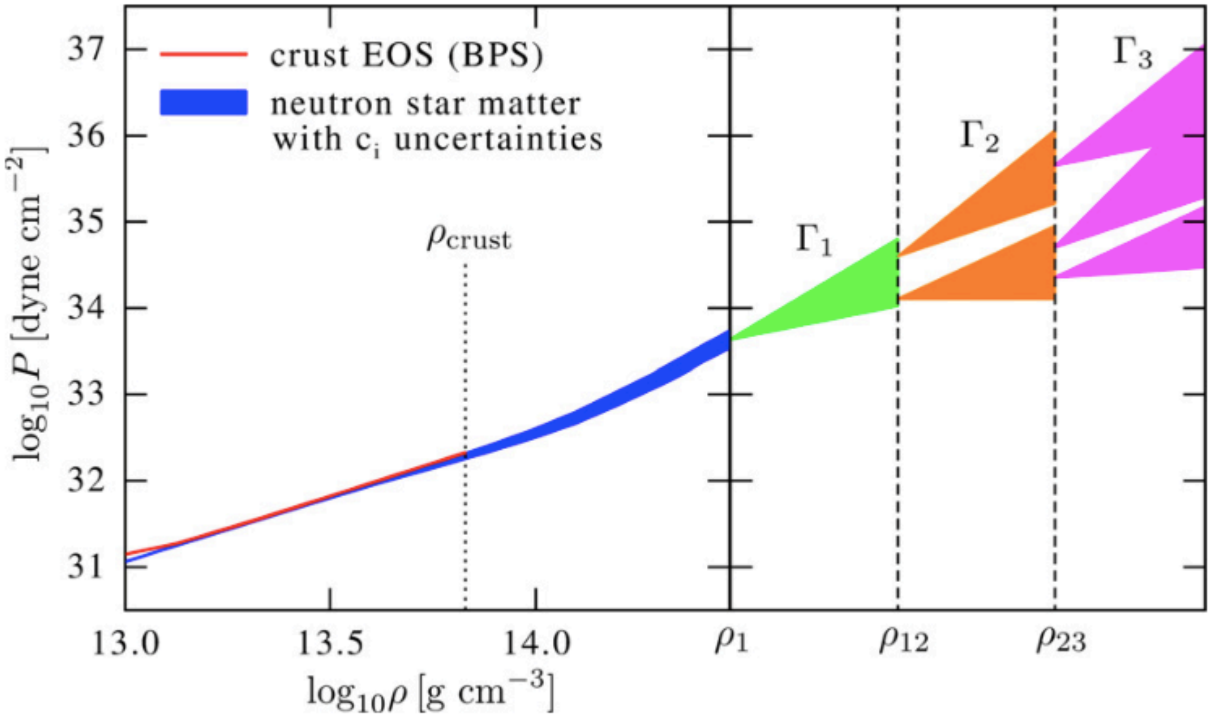
Astro. J. 773 (2013)

K. HEBELER[1], J. M. LATTIMER[2], C. J. PETHICK[3,4], AND A. SCHWENK[5,6]

For $\rho<\rho_1$

$$\epsilon(\bar{n}, x)T_0 = \frac{3}{5}[x^{5/3} + (1-x)^{5/3}](2\bar{n})^{2/3} - [(2\alpha - 4\alpha_L) \times x(1-x) + \alpha_L]\bar{n} + [(2\eta - 4\eta_L)x(1-x) + \eta_L]\bar{n}^{\gamma} ,$$

For $\rho>\rho_1$ Piecewise polytropes.



- Fractional power of the density
- Polytropes → non-analytic

Needs for simple parameterization of the EoS

- For the analysis of NS observation (thermal X-rays, X-ray pulsations, etc…)
  - Extraction of NS radius
- For implementation in hydro-dynamical codes (CCSN, NS mergers, etc…)
- For Heavy Ion Collisions and analysis of the results

Empirical parameters

The **empirical parameters** code the **bulk** properties of the nuclear EoS.

Taylor expansion of the EoS around ρ_0 (theory):

$$\frac{E}{A} = (E_0 + E_{sym}\delta^2) + L_{sym}x\delta^2 + \frac{1}{2}(K_0 + K_{sym}\delta^2)x^2 + \dots$$

$$x = \frac{\rho - \rho_0}{3\rho_0} \qquad I = \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

Phenomenological mass formulae (experiment):

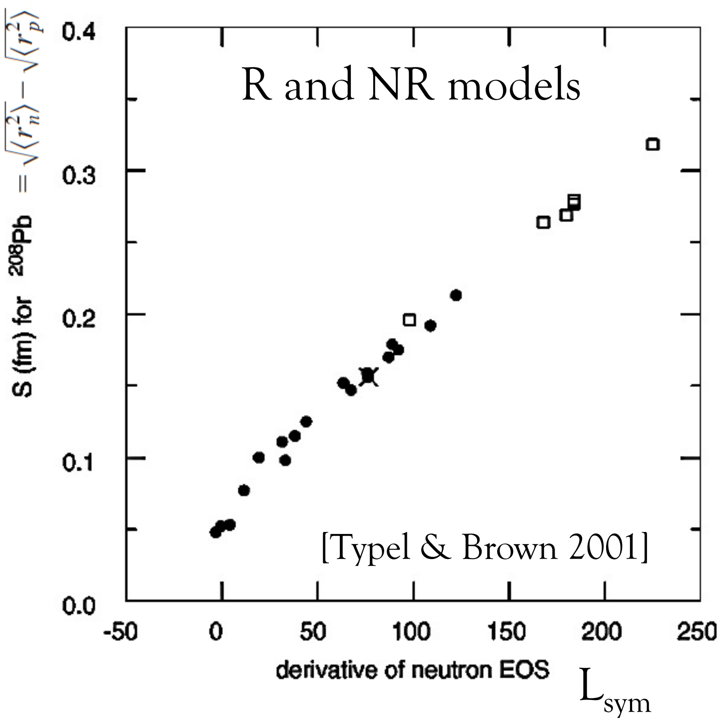
- LD formula: $B(Z,A) = \underbrace{E_0 + E_{sym}I^2}_{\text{bulk}} + \underbrace{E_{surf}A^{-1/3}}_{\text{surface}} + \dots$
- Droplet formula: including skin contribution
- Compressible LD formula: $B(Z,A) = \frac{E}{A}(\rho, I) + E_{surf}A^{-1/3} + \dots$

$$\rho(A) = \rho_0\left(1 - \frac{3L_{sym}}{K_0}I^2\right)$$

Experimental determination of the empirical parameters

- E_0, E_{sym}: From fit of LDM through the nuclear chart, or from DFT adjustment.
- K_0: from ISGMR [Blaizot, 1980]

  → better correlated to M_c [Khan, J.M. 2012]
- L_{sym}, K_{sym}: more difficult
  - Neutron skin in Pb,
  - ISGMR in neutron rich nuclei (Ksym, K_τ) [Garg+2010]



Measurement of neutron skin:

With strong probes:

- p-N elastic diffusion
- π, α, d scattering
- π photoproduction
- Heavy-ion collisions
- Electric dipole polarizability

With weak probes:

- PREX / C-REX

Empirical parameters from various effective approaches

Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
		fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV
Skyrme	Average	0.1586	-15.91	251.68	-300.20	1178.35	31.22	53.52	-130.15	316.68	-1890.99
	σ	0.0040	0.21	45.42	157.81	848.47	2.03	31.06	132.03	218.23	1191.23
RMF	Average	0.1494	-16.24	267.99	-1.94	5058.30	35.11	90.20	-4.58	271.07	-3671.83
	σ	0.0025	0.06	33.52	392.51	2294.07	2.63	29.56	87.66	357.13	1582.34
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	σ	0.0035	0.08	11.63	350.44	838.41	1.37	11.06	51.11	236.80	4155.74
Average		0.1540	-16.04	255.91	29.01	3835.24	33.43	77.92	-2.19	370.34	-5172.77
σ		0.0051	0.20	34.39	424.59	2401.14	2.64	30.84	142.71	298.54	4362.35

$$e_{IS}(\rho) = E_0 + \frac{K_0}{2}x(\rho)^2 + \frac{Q_0}{6}x(\rho)^3 + \dots,$$

$$e_{IV}(\rho) = E_{sym} + L_{sym}x(\rho) + \frac{K_{sym}}{2}x(\rho)^2 + \frac{Q_{sym}}{6}x(\rho)^3 + \dots,$$

$$\begin{cases} \rho = \rho_n + \rho_p \\ \delta = (\rho_n - \rho_p)/\rho \end{cases}$$

Empirical parameters from various effective approaches

1 % accuracy

Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
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Empirical parameters from various effective approaches

10-20 % accuracy

Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
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Empirical parameters from various effective approaches

50 % accuracy

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Empirical parameters from various effective approaches

Very large inaccuracy

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Empirical parameters from various effective approaches

fixed | Explore inside small interval | Consider large interval

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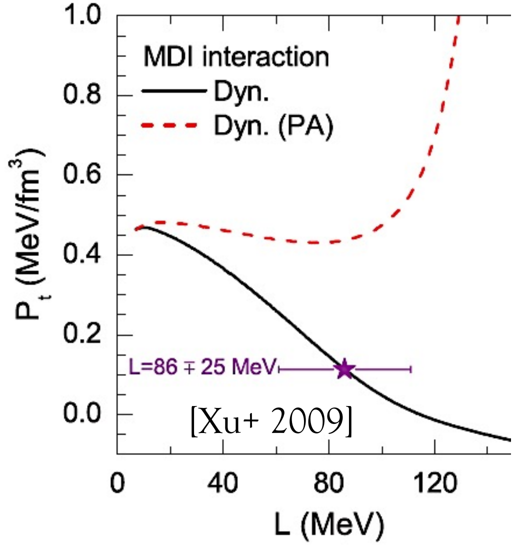
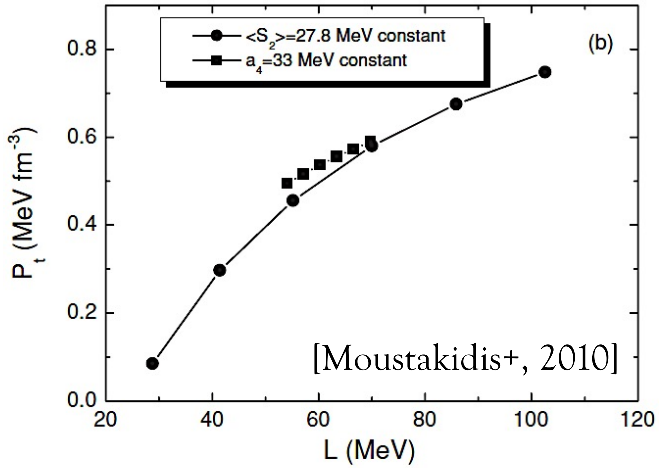
Another approach is possible: apply the eEOS directly in nuclei and fix the uncertainties directly from the experimental data (work in process).

Few remarks about correlations

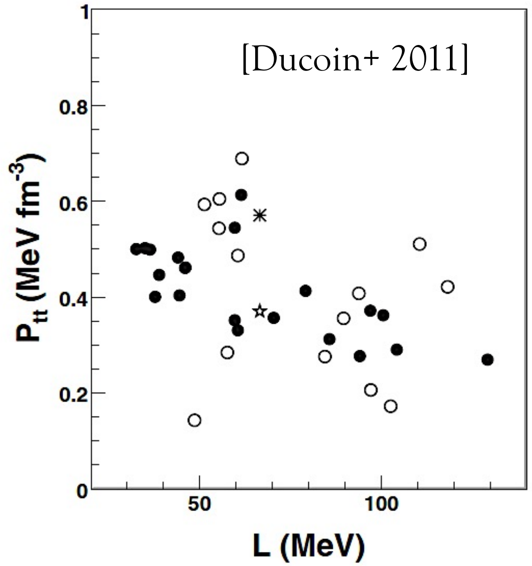
Correlation between P_t and L_{sym}?





Comparing 40 different EoS models

→ We do not find any correlation.

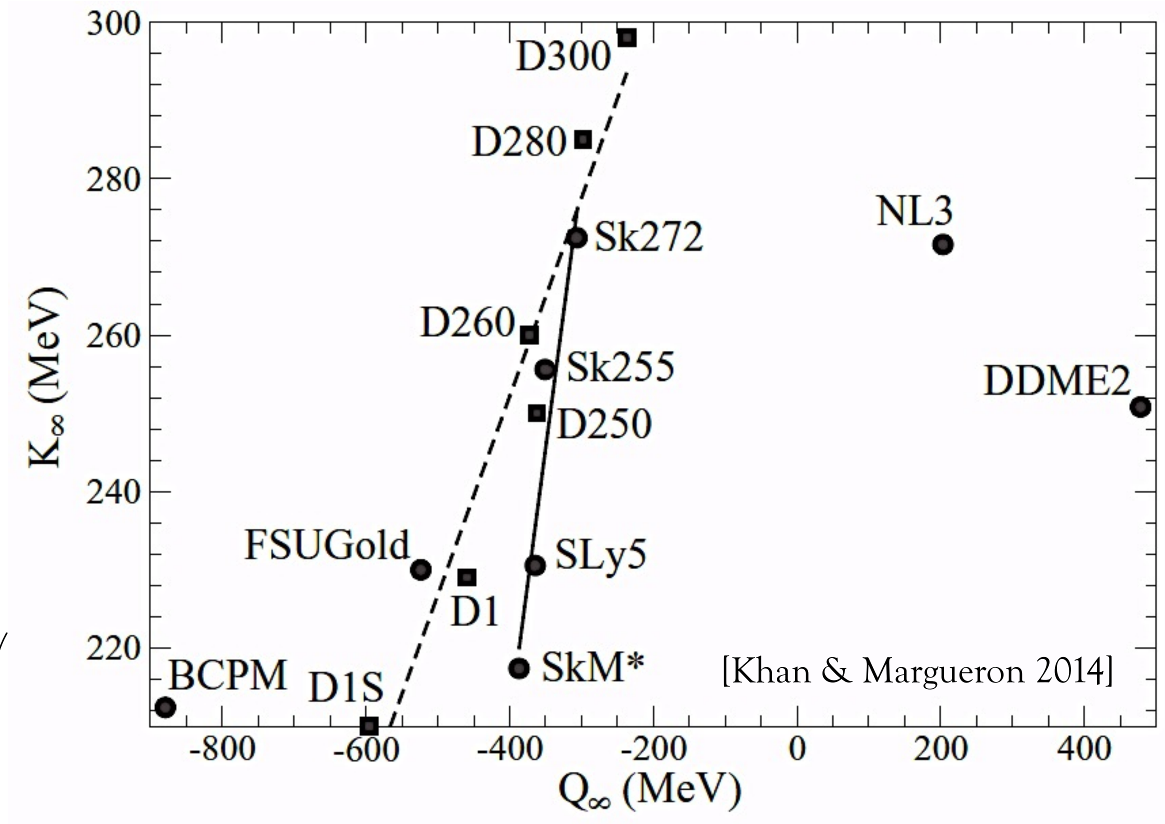


There is a correlation between P_t and, at least, 2 parameters [Ducoin+ 2012]

It is difficult to perform statistical analysis for more than 2 varibales (needs a lot of models)

Few remarks about correlations

Spurious correlations with effective interactions:



These correlations are induced by the term: $t_3\rho^\gamma$

The fractional power γ make this term contributing to all derivatives of the EoS.

The limited number of parameters of effective models is:

- An advantage – a simplification,
- A limitation in their flexibility.

An empirical model for the nuclear EoS

Hypothesis:
1) Matter is non-relativitic (→ E=T+V),
2) Nuclear potential quadratic in δ,
3) The EoS is analytic in x (→ polynomial expansion possible),
4) lim e(ρ,δ) → 0 for ρ → 0.

Kinetic energy: $$t^{eff}(\rho,\delta)=\frac{1}{2}t_0^{FG}\left(\frac{\rho}{\rho_0}\right)^{2/3}\left[f^{FG}(\delta)+\frac{\rho}{\rho_0}f^{eff}(\delta)\right]$$

Binding energy: $$e^N(\rho,\delta)=t^{eff}(\rho,\delta)+\sum_{\alpha\geq 0}^{N}\left[v_\alpha^{s,is}+v_\alpha^{s,iv}\delta^2\right]\frac{x^\alpha}{\alpha!}u_\alpha(\rho)$$

satisfy the limit ρ→0 (for $u_\alpha(\rho)$)

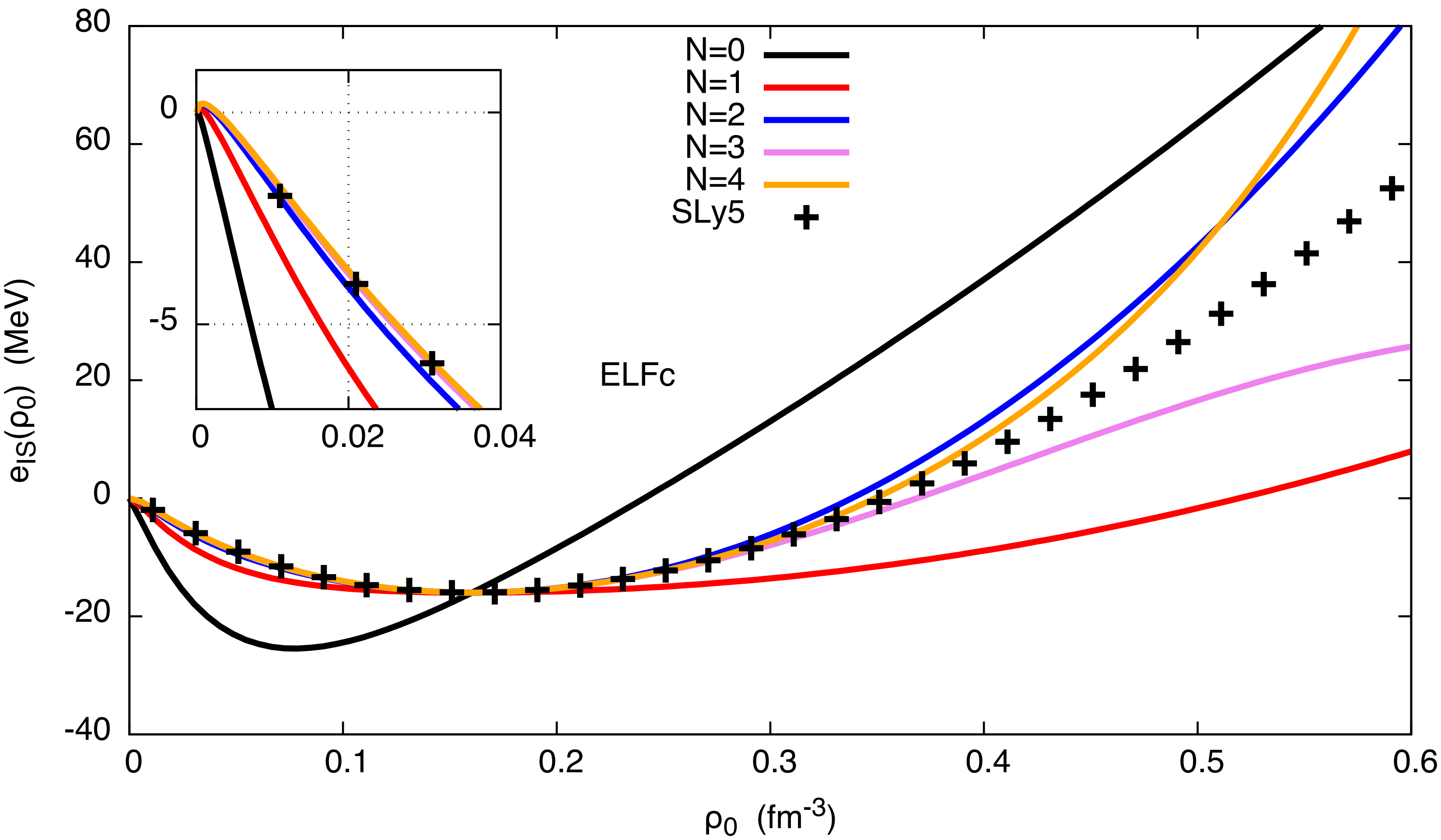
→ One-to-one correspondence between model parameters and empirical quantities:

$$v_{\alpha=0}^{s,is}=E_0-t_0^{FG}(1+\bar{M})$$
$$v_{\alpha=1}^{s,is}=-t_0^{FG}(2+5\bar{M})$$
$$v_{\alpha=2}^{s,is}=K_0-2t_0^{FG}(-1+5\bar{M})$$
$$v_{\alpha=3}^{s,is}=Q_0-2t_0^{FG}(4-5\bar{M})$$
$$v_{\alpha=4}^{s,is}=Z_0-8t_0^{FG}(-7+5\bar{M})$$

→ Flexible model with no hidden correlations among parameters.

Test of the convergence of the eEOS

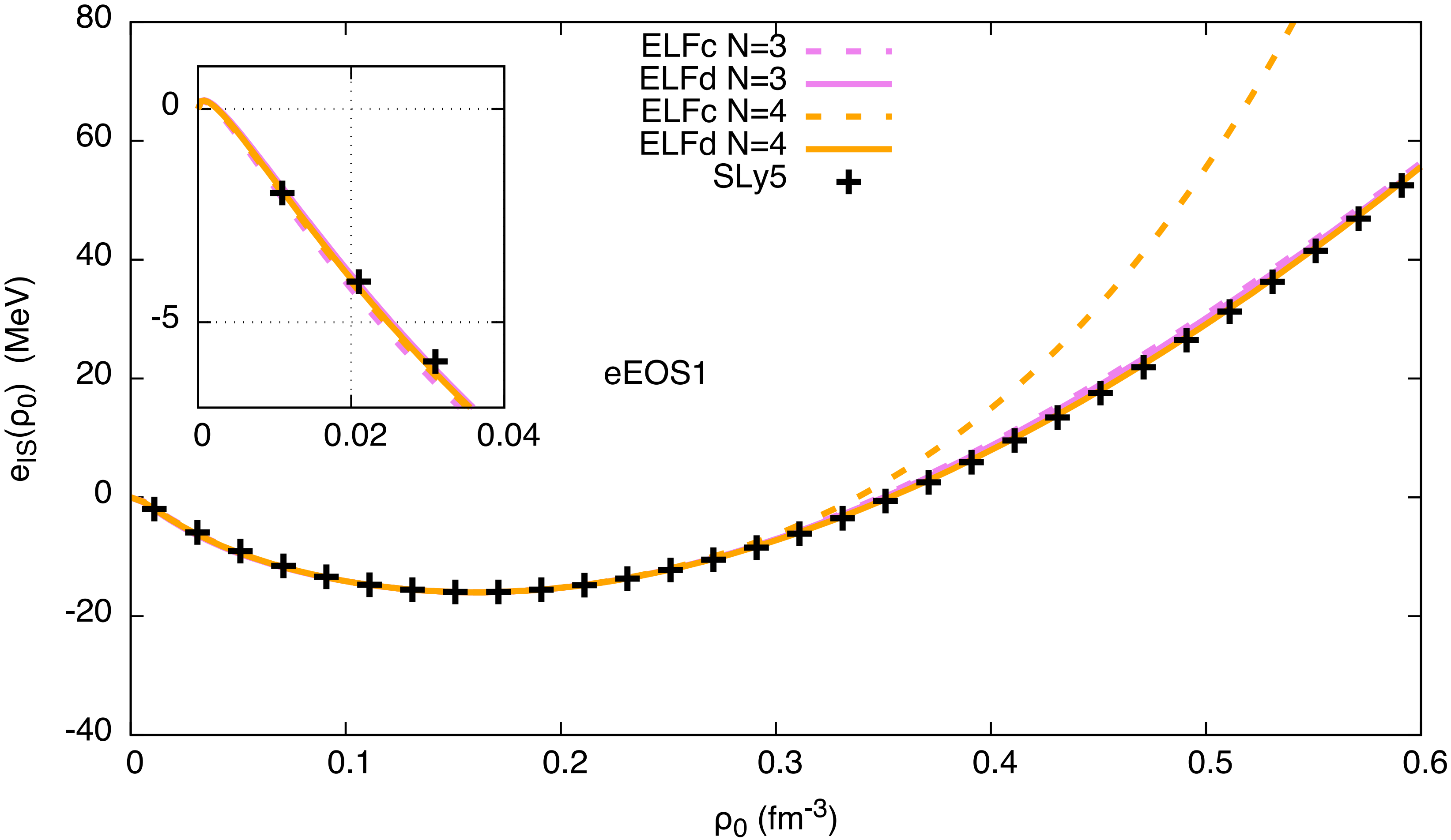
The EoS takes as inputs the empirical parameters of the considered model.



Test of the convergence of the eEOS

Consider additional constraints at $\rho=4\rho_0$ (e and p) for order 3 and 4.

→ Needs of data beyond saturation density



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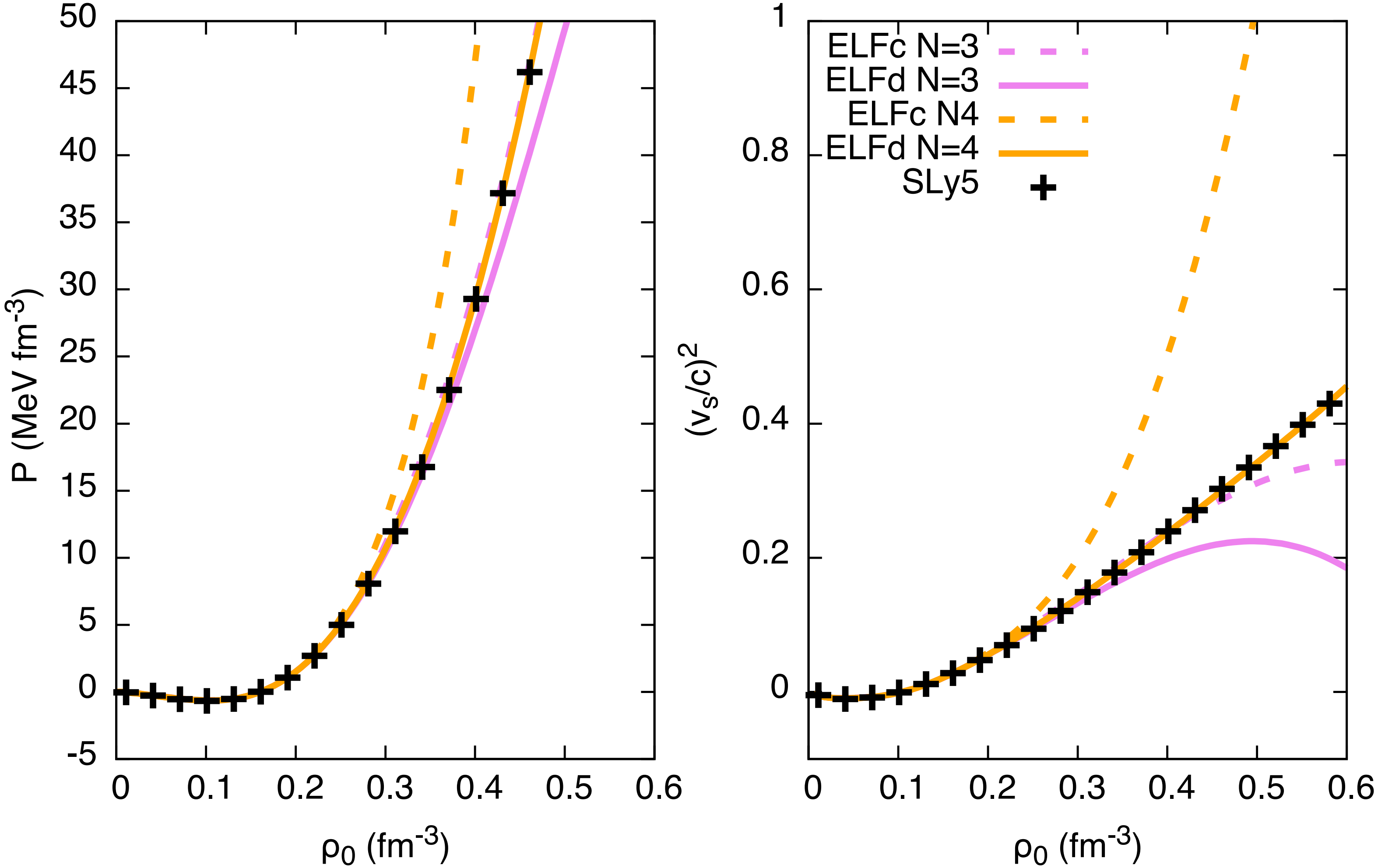

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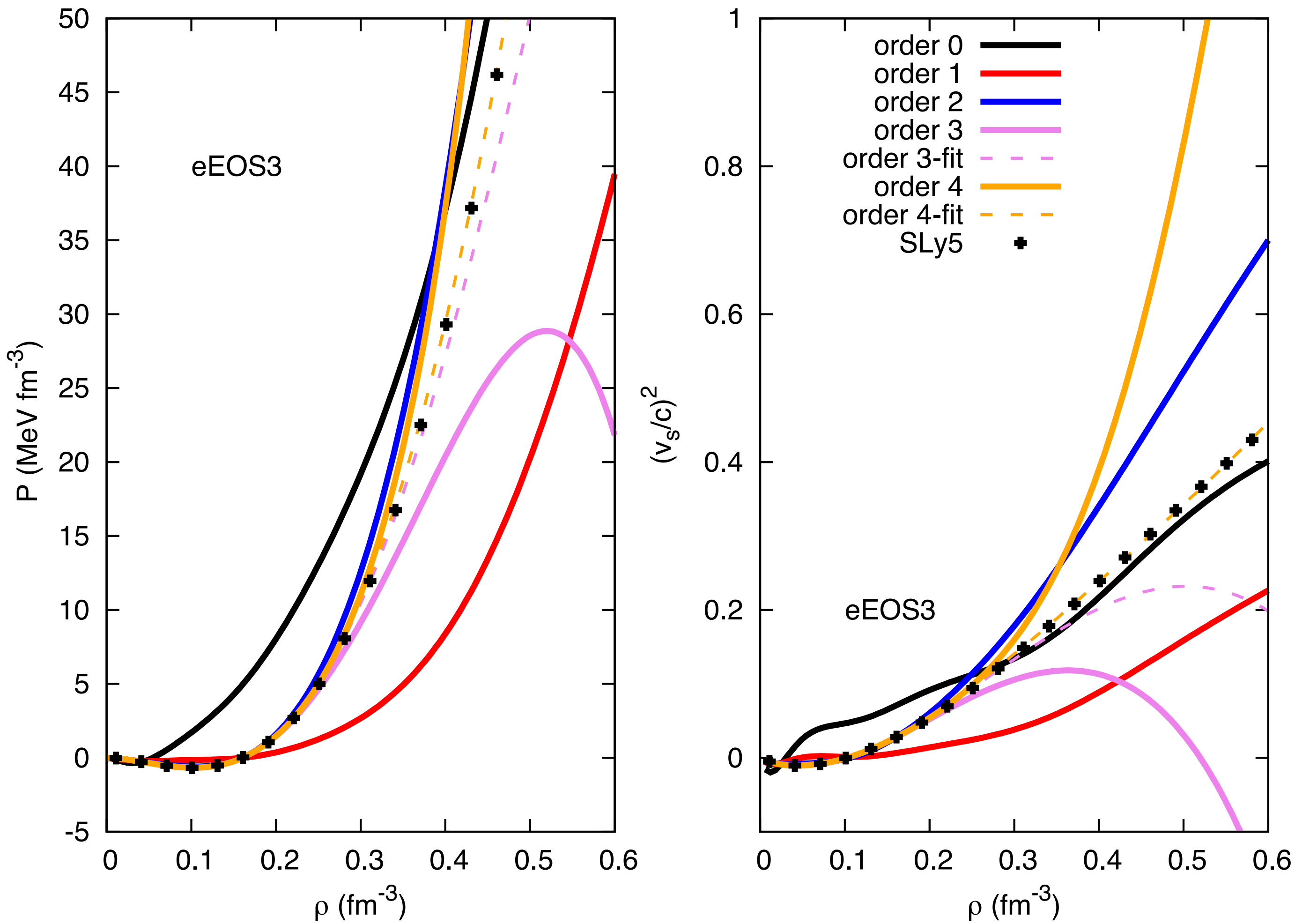

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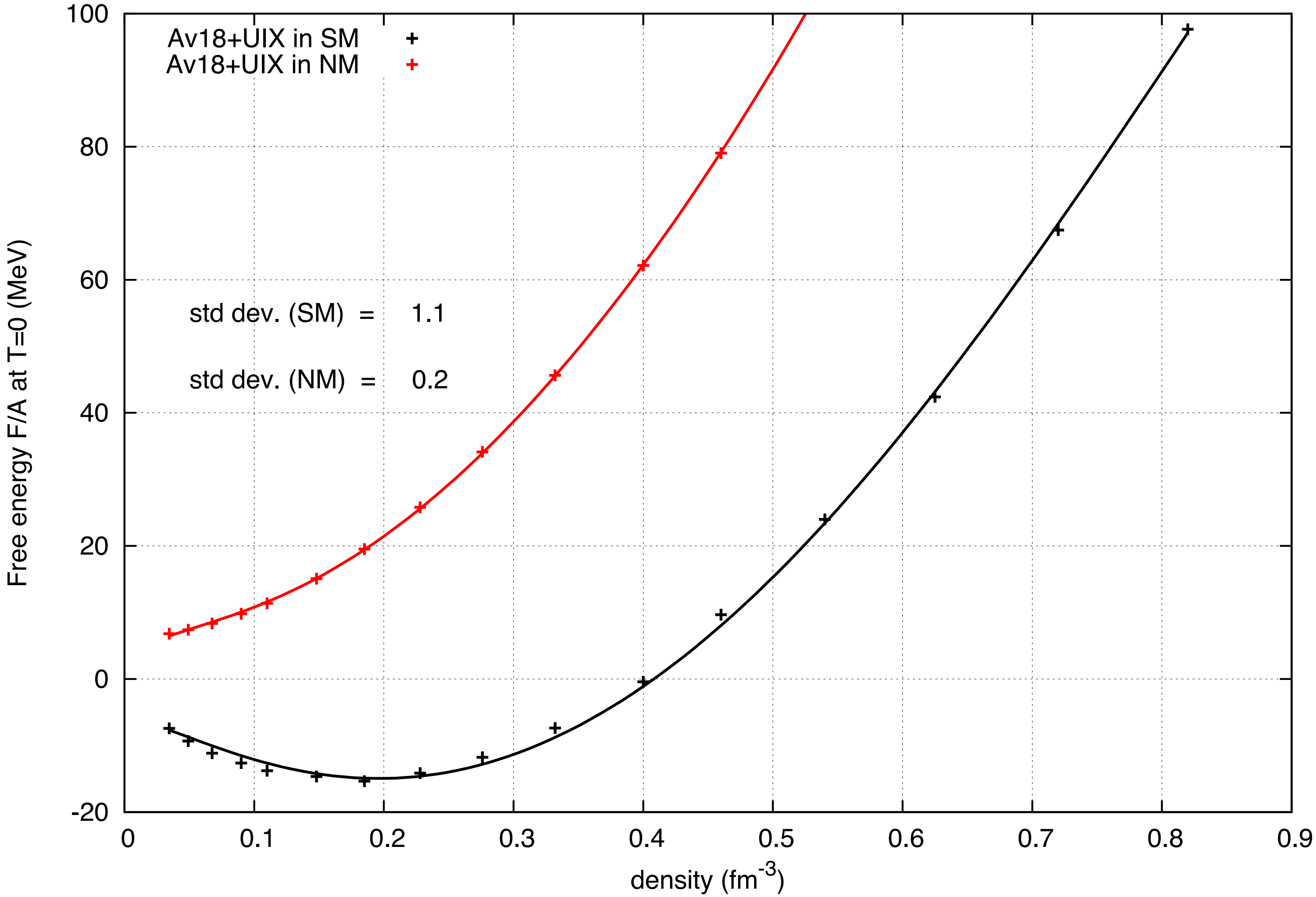
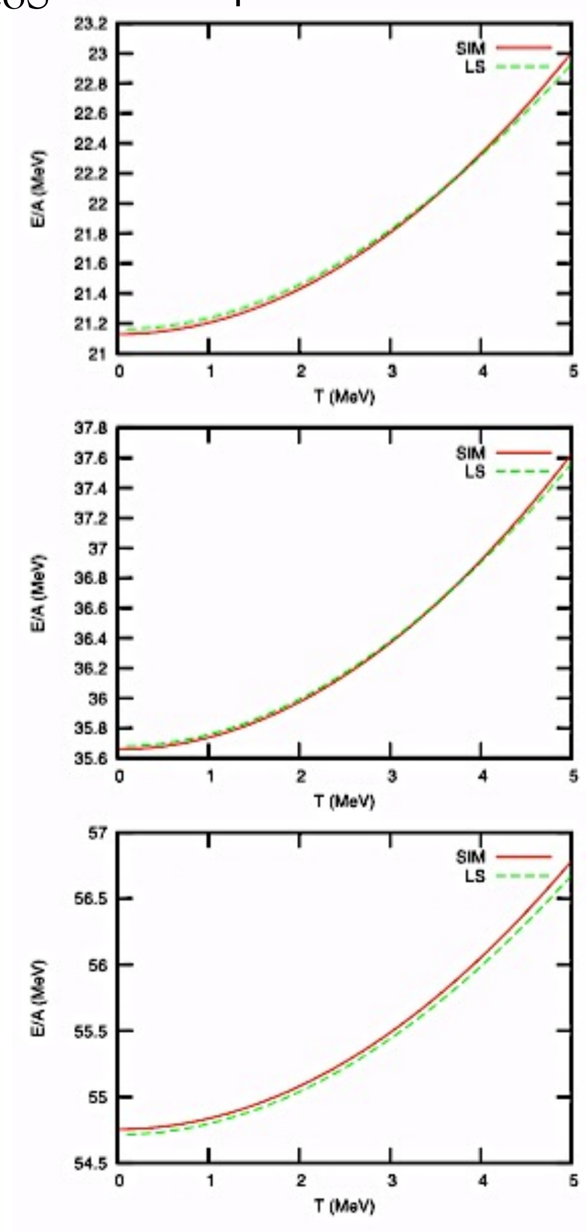
Reproduction of other EoS (Av18+UIX BHF)



Reproduction of other EoS (Lattimer-Swesty LS220)

Finite T EoS

$\rho=0.14\ fm^{-3}$

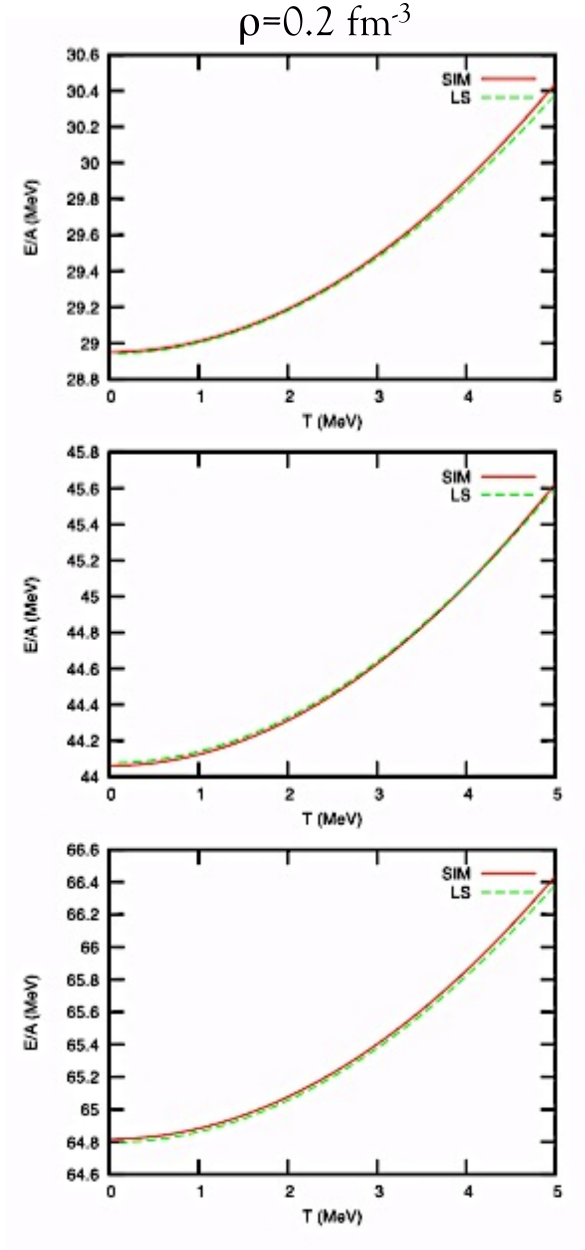


$y_e=0.0$

$y_e=0.25$

$y_e=0.5$

$\rho=0.2\ fm^{-3}$



Our present knowledge on the empirical parameters

The set of "*experimental*" empirical quantities represents our knowledge and uncertainties on the properties of the EoS.

fixed | Explore inside small interval | Consider large interval

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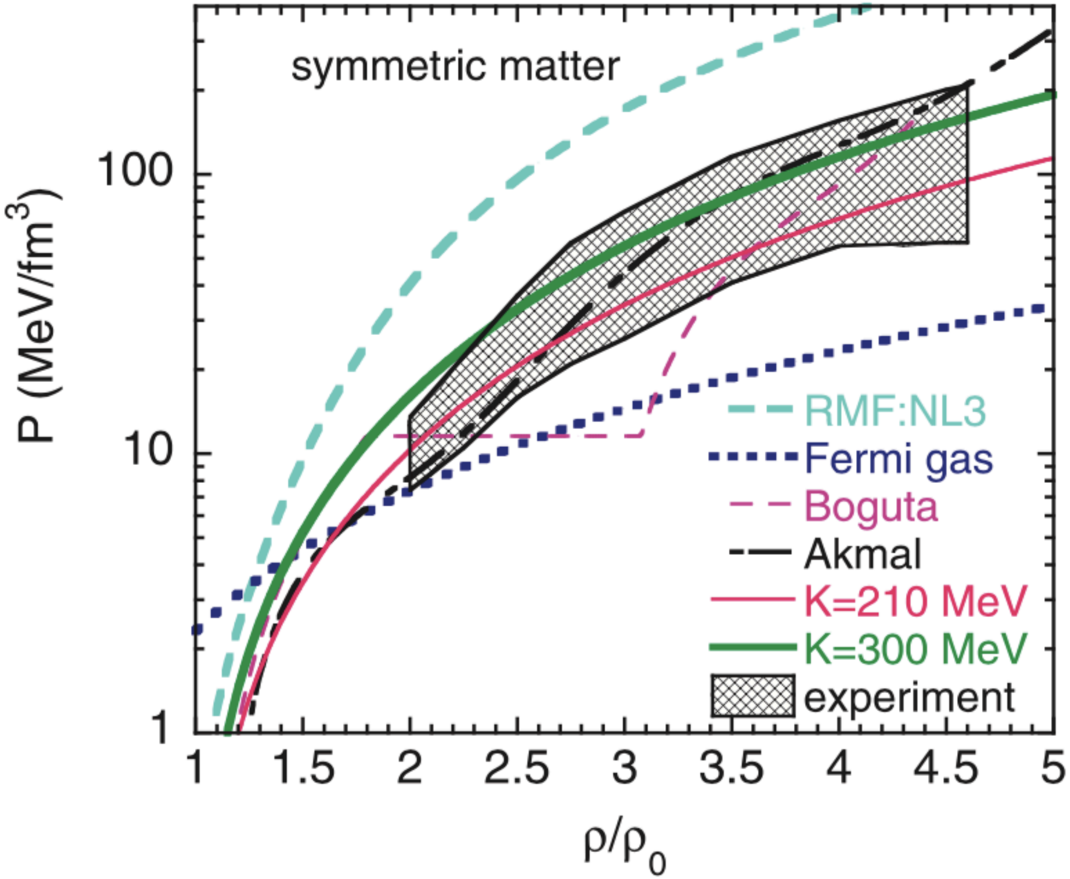
Could these uncertainties be reduced by imposing some general constraints to the EOS?

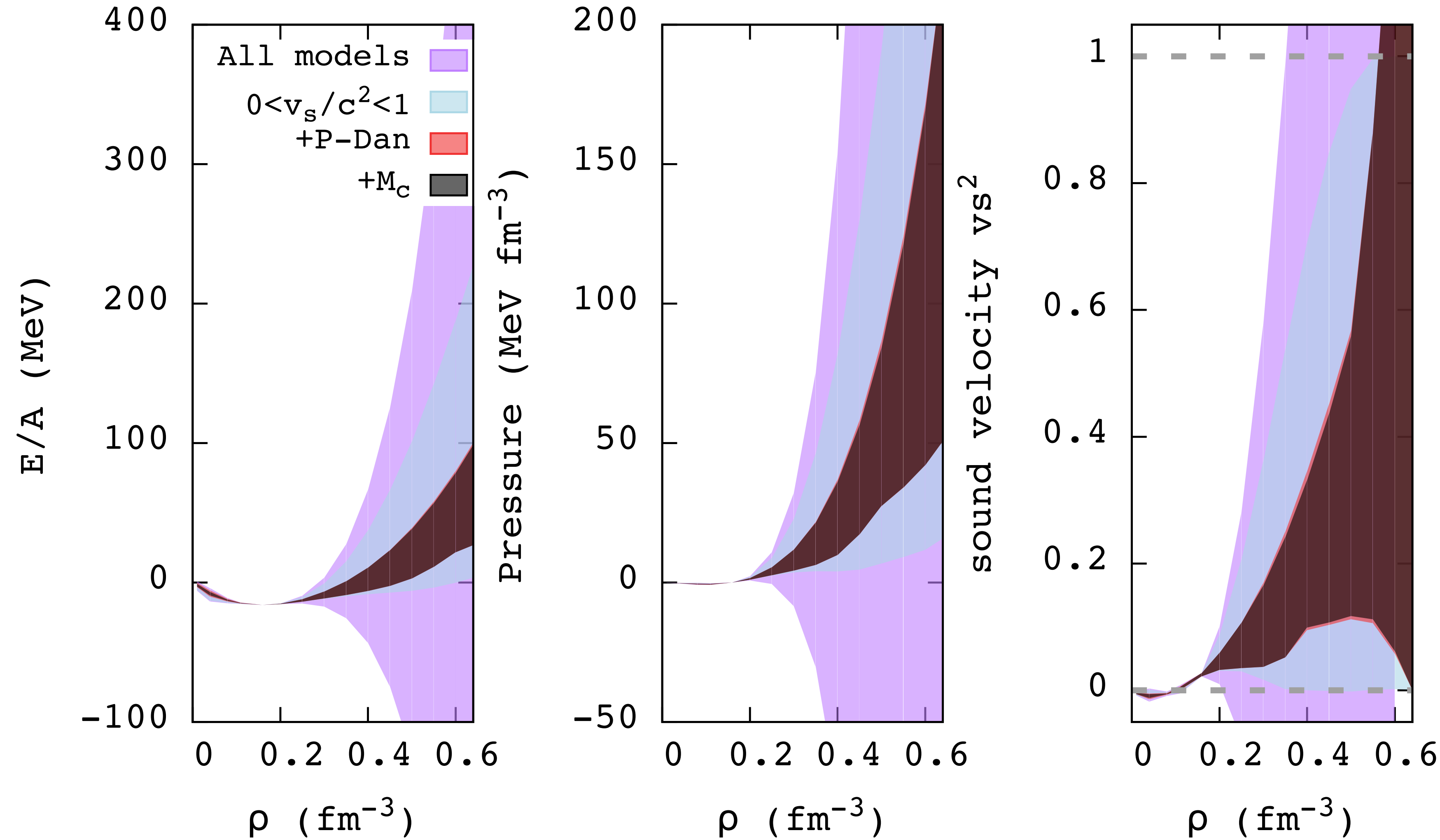
Constraints on the nuclear properties in symmetric matter

Pressure deduced from analysis of the elliptic flow in Heavy Ion Collisions (HIC).

Danielewicz nature 2002



The best known EoS considering our unknown



# of all models: 363 609
Compatible with Vs : 28 433
P-Dan: 137 44
Mc : 108 252

With P-Dan and Vs: 12 863
Mc and Vs : 62 652

With P-Dan+Vs+Mc: 7 366

→ need additional constraints.

iPNL

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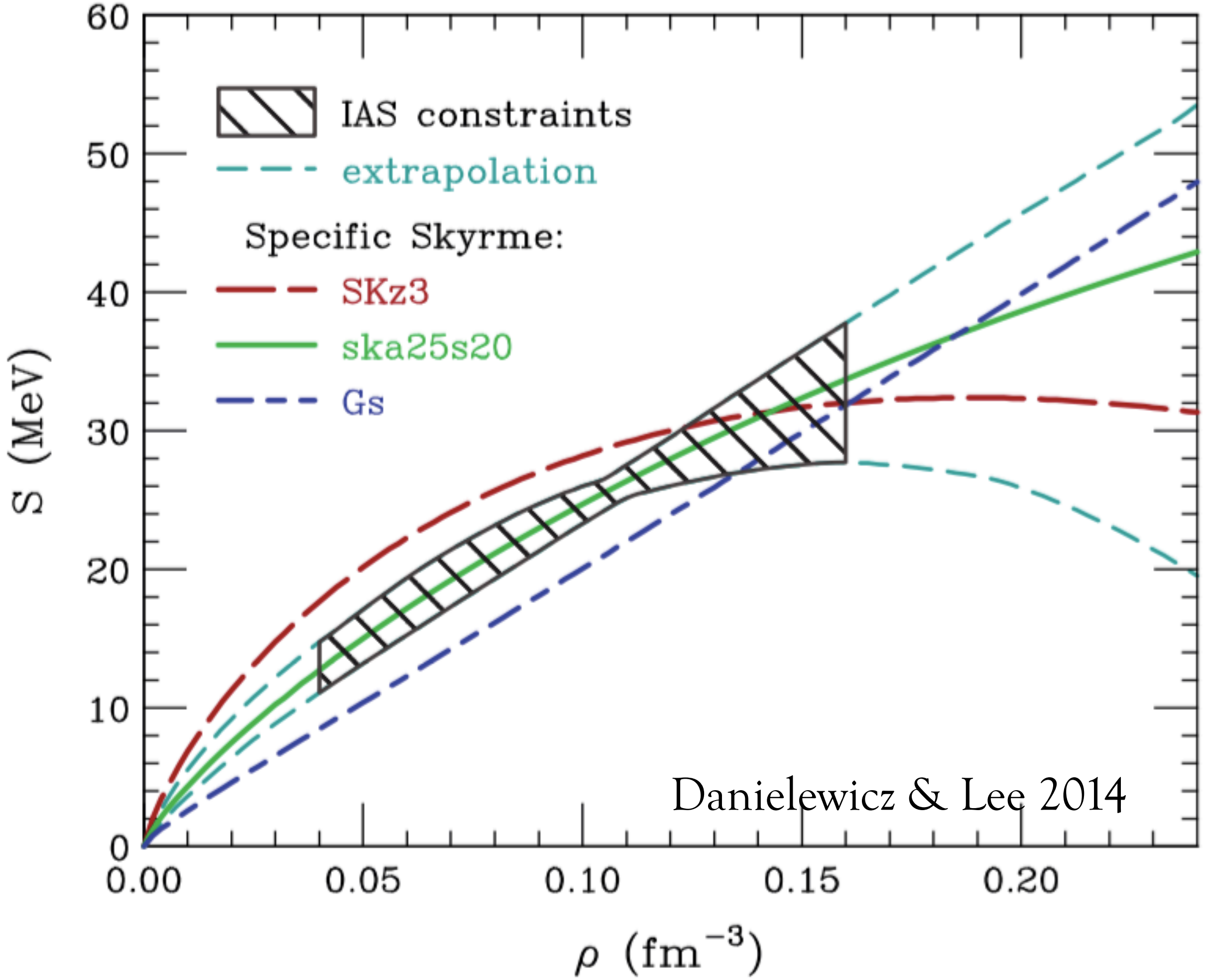
Additional constraints in asymmetric matter

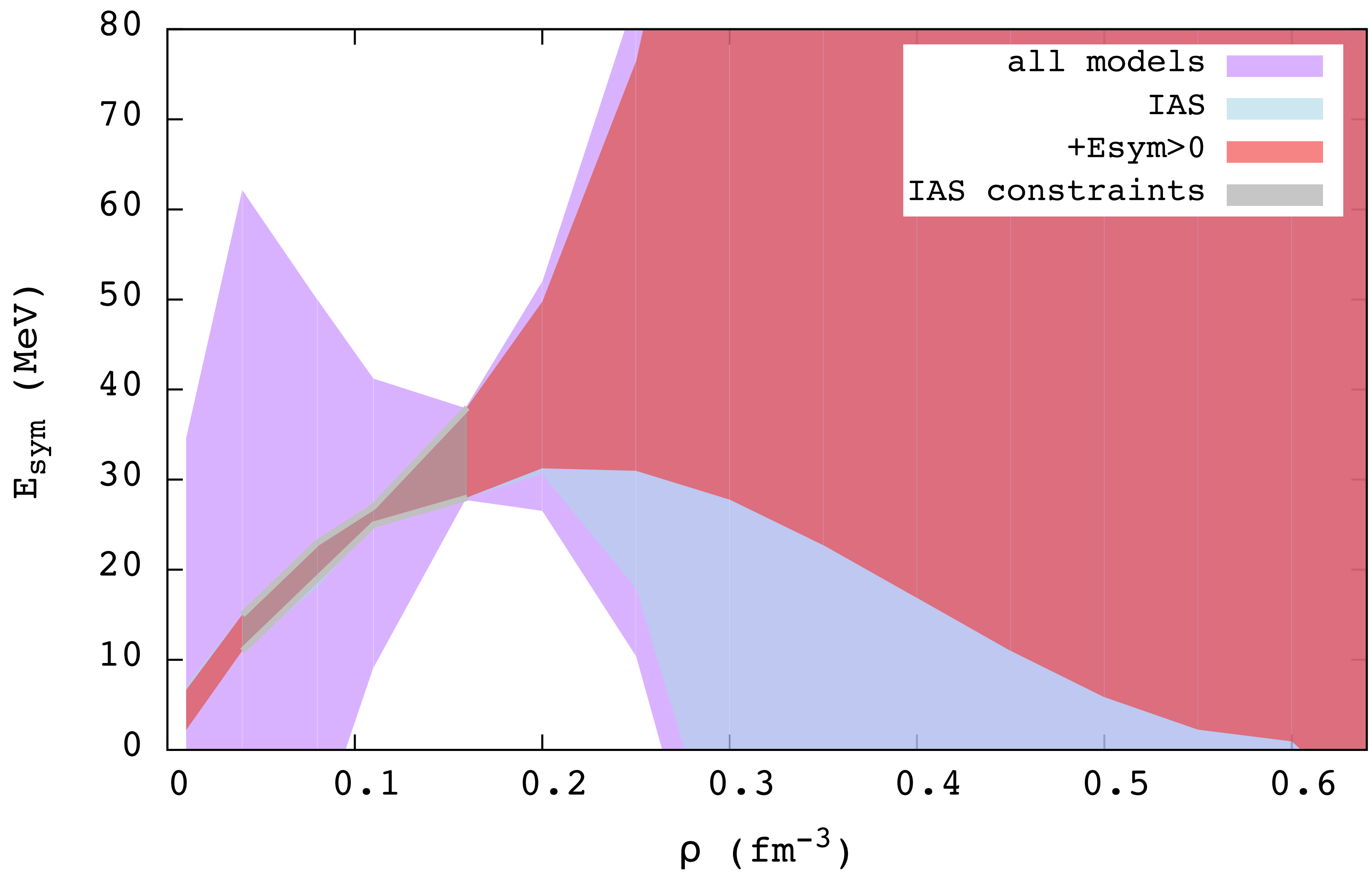
For $\rho < 4\rho_0$ Symmetry energy: Density-dependence from IAS (below ρ_0)



Constraints on the symmetry energy

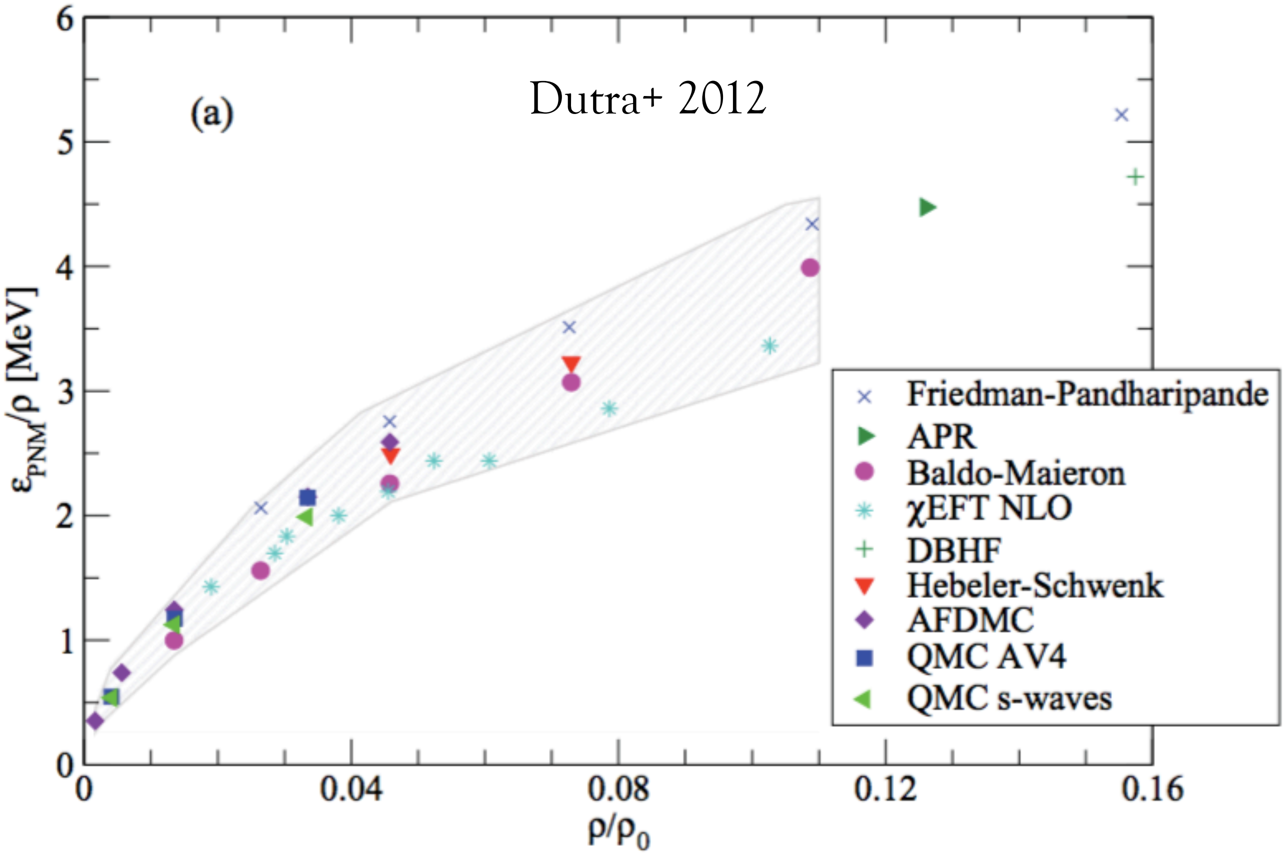
IAS: Danielewicz & Lee 2014

Symmetry energy $E_{sym}(\rho) > 0$



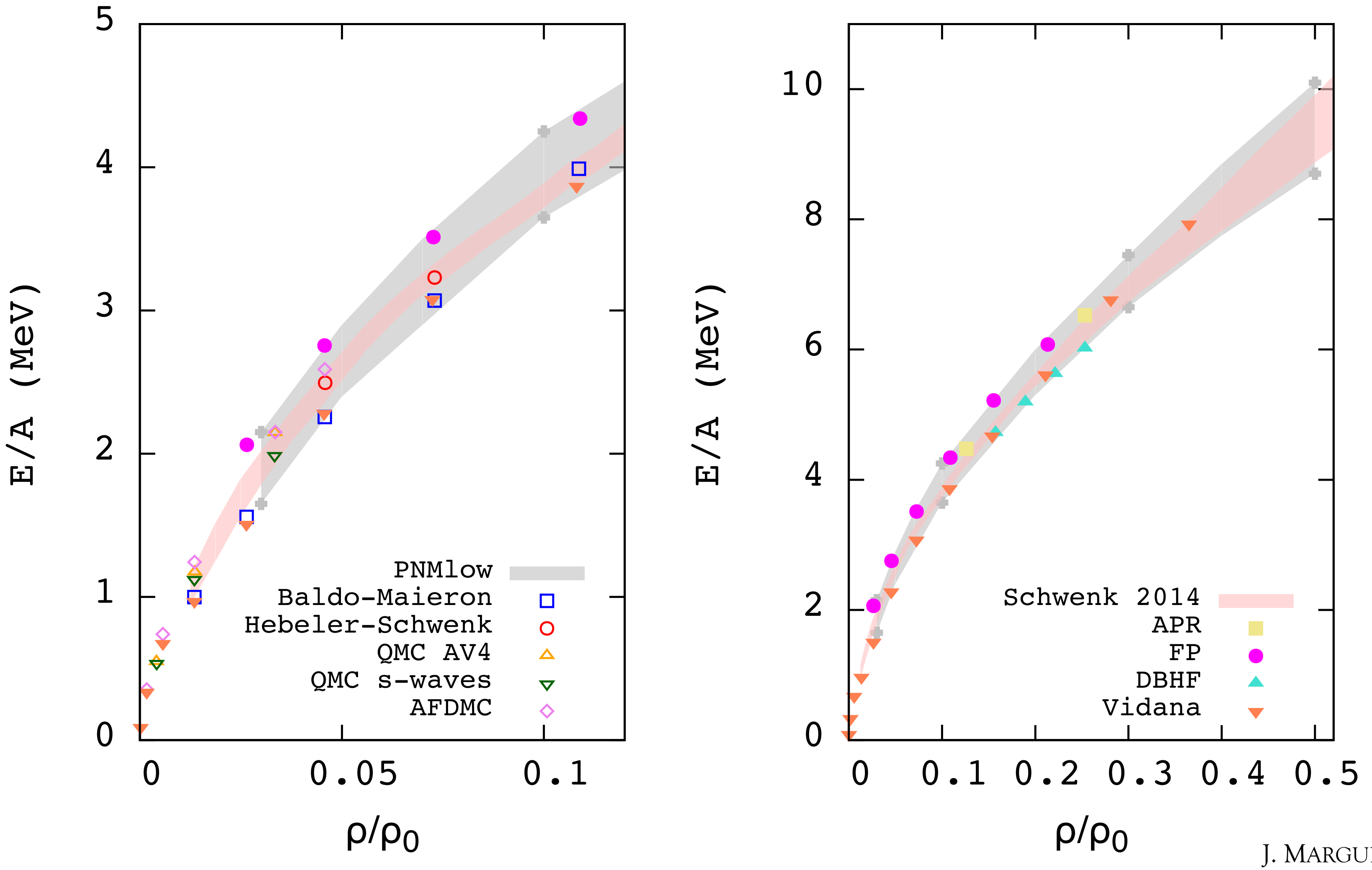
Additional constraints in neutron matter

Low density neutron matter: universal behaviour



Constraints on the nuclear properties in neutron matter

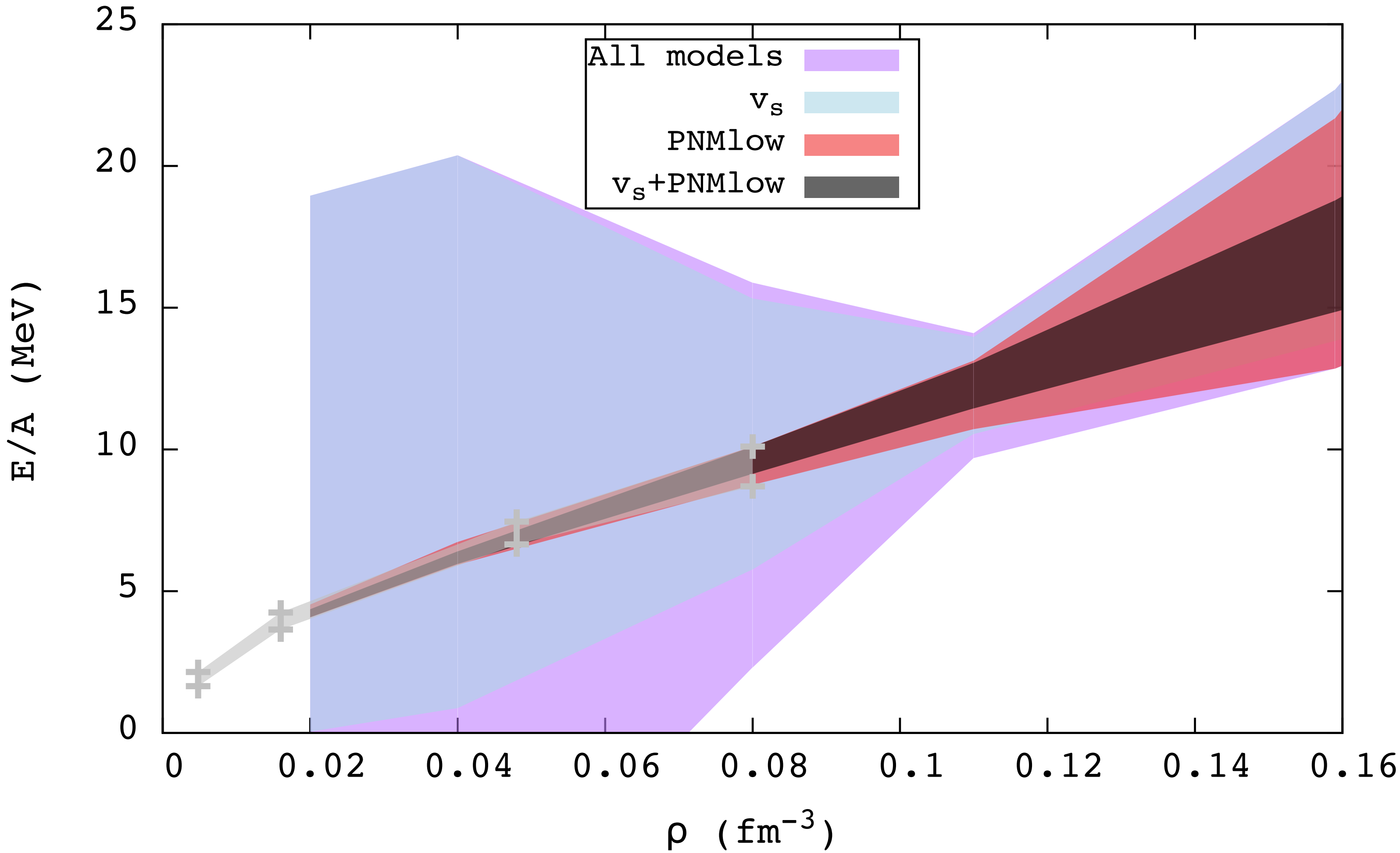
PNMLow: universal behaviour of low density neutron matter (a new analysis)



Constraints on the nuclear properties in NM

Sound velocity: $0 \leq (c_s/c)^2 < 1$

PNMLow: universal behaviour of low density neutron matter



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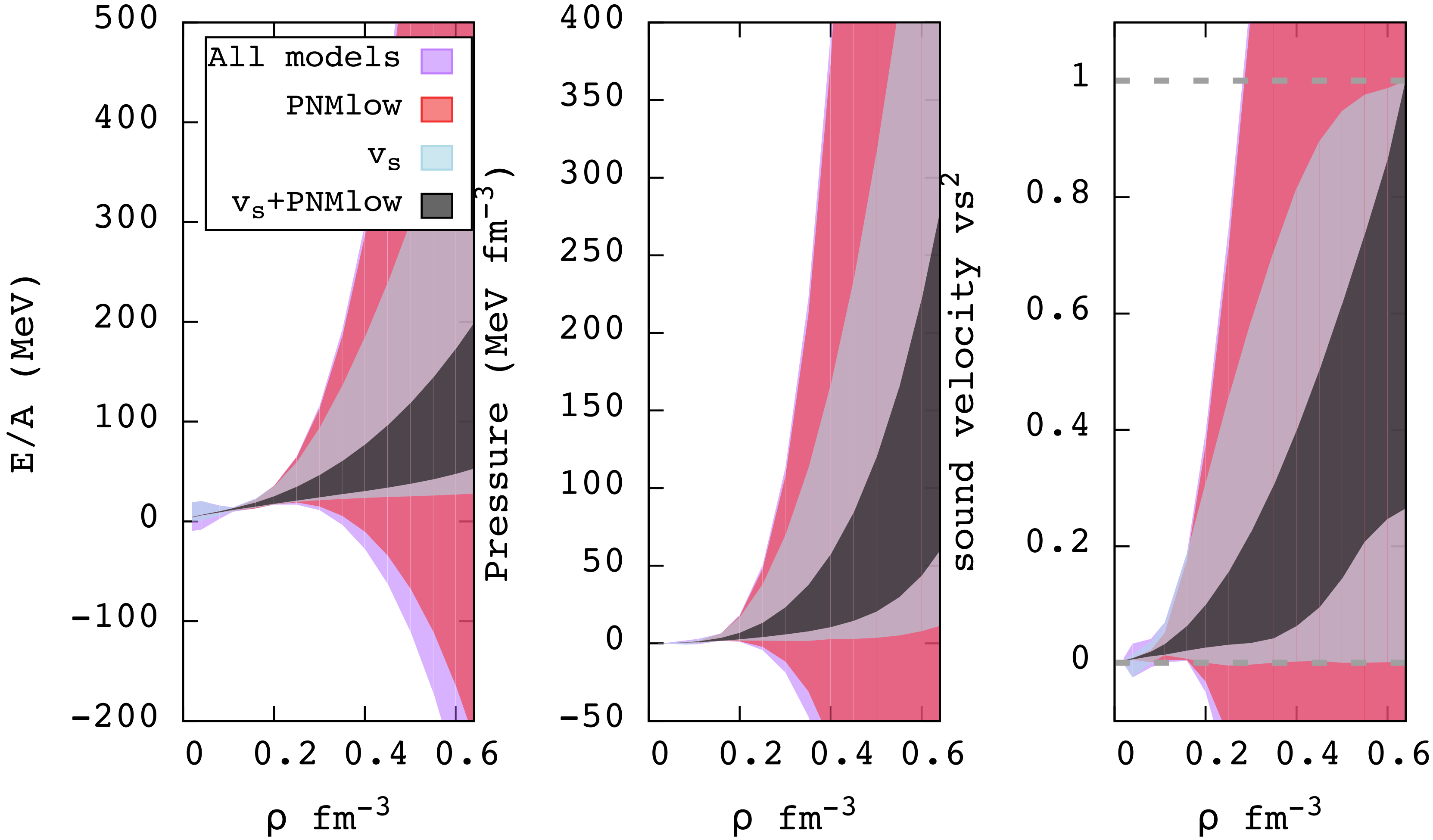
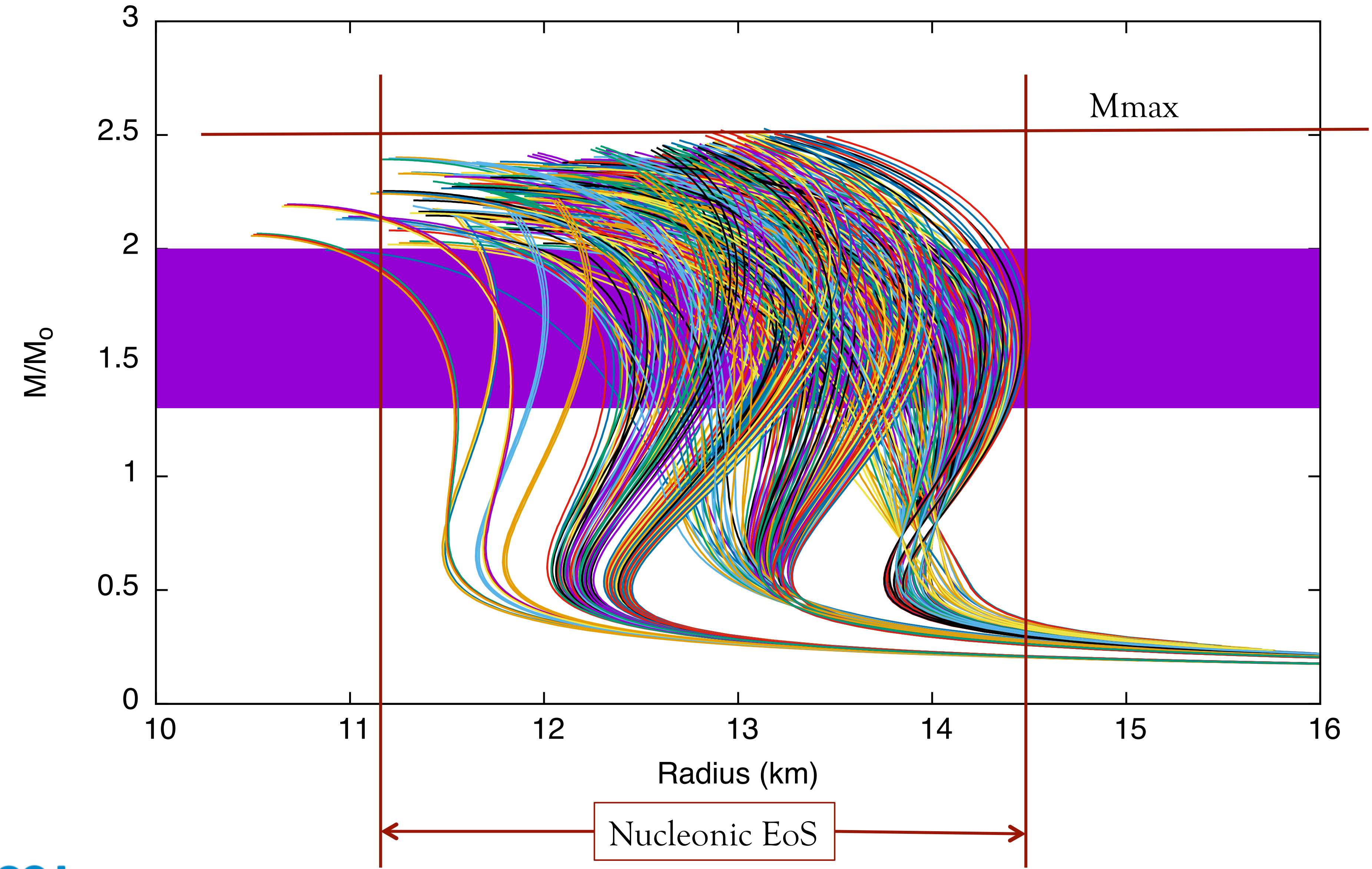
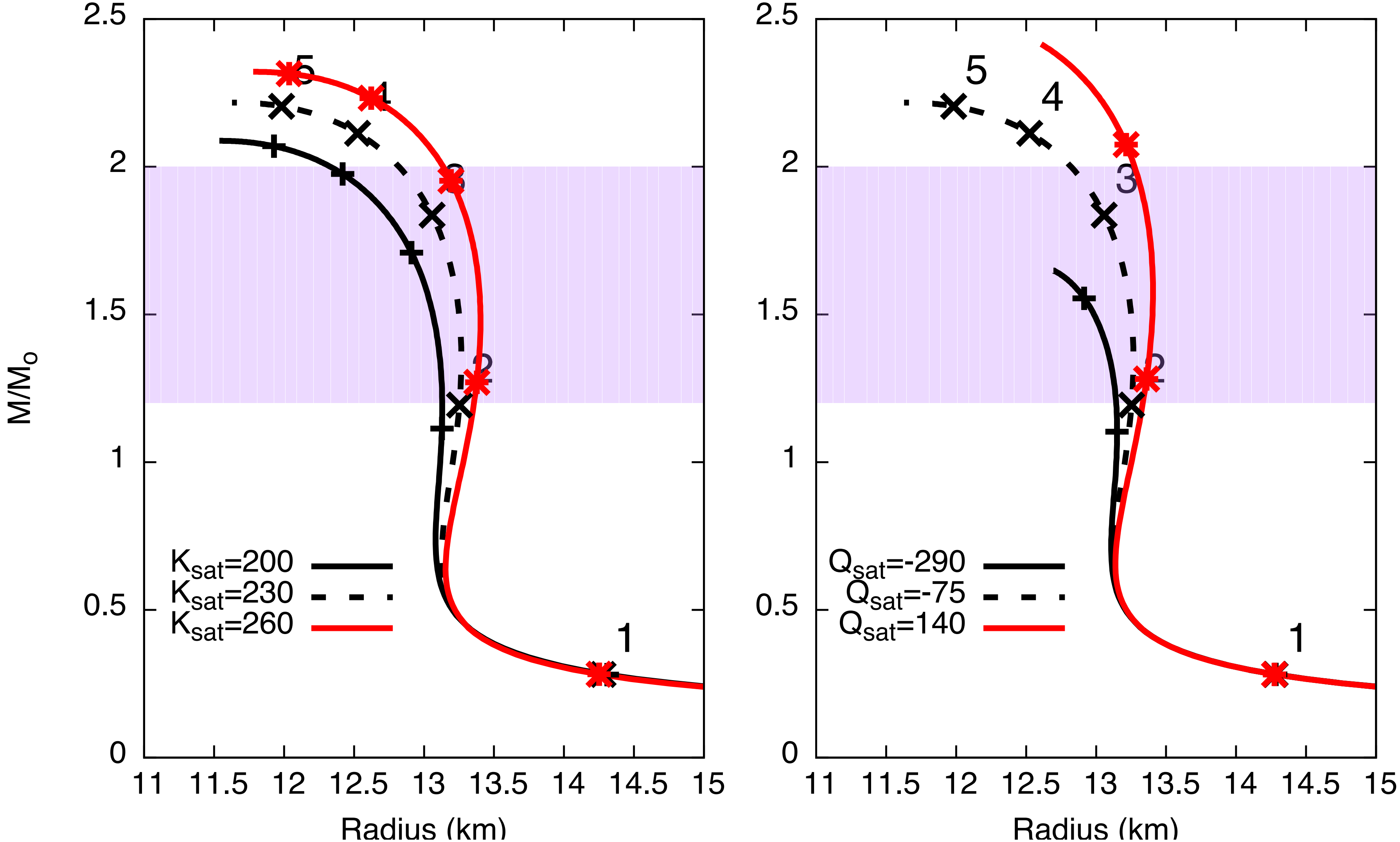


Application to Neutron Stars

Impact of the "exp" unknown on the Mass/Radius relation



Impact of the isoscalar empirical parameters



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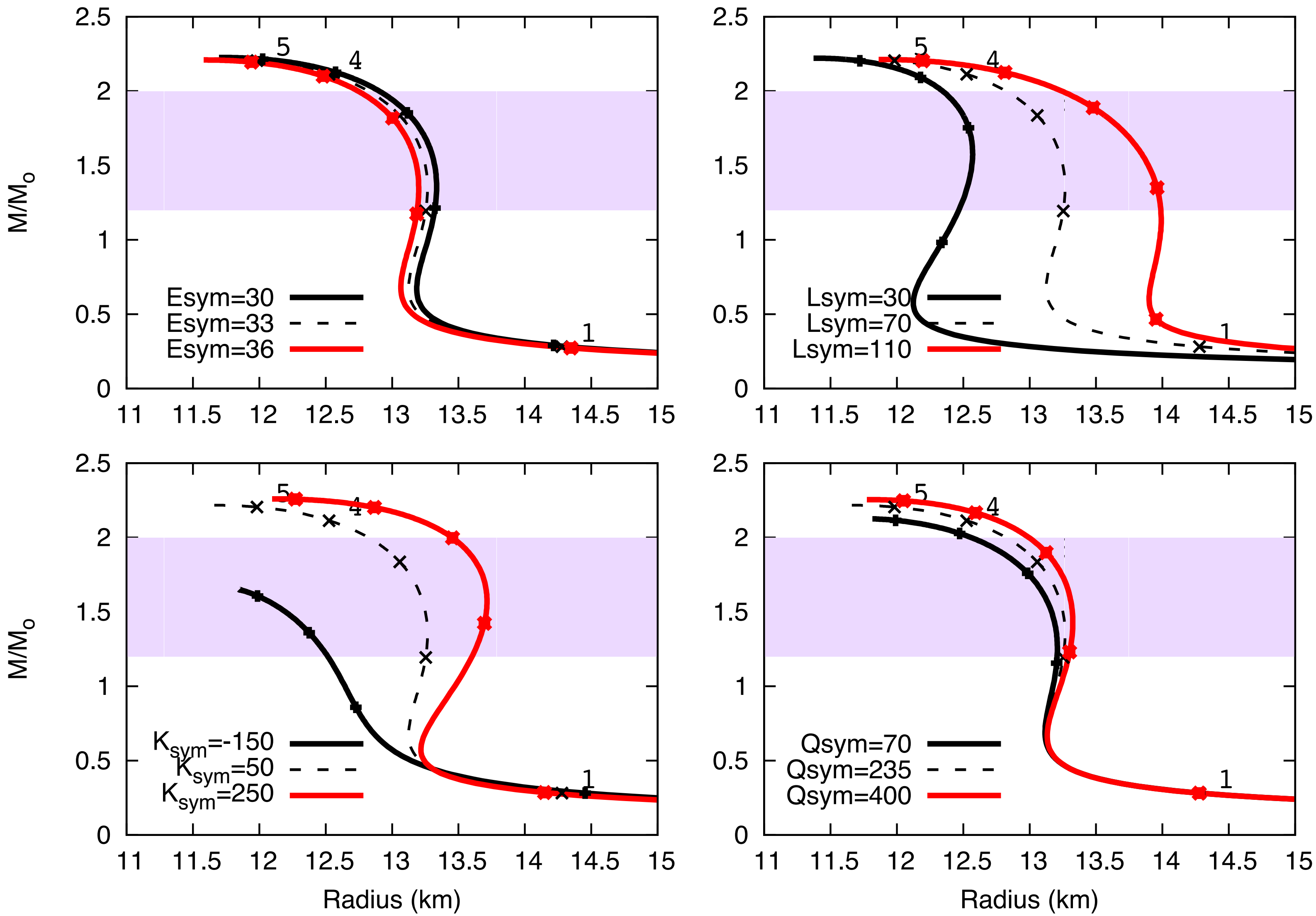
Impact of the isovector empirical parameters



Largest source of uncertainty: Lsym and Ksym

Application to CC-SN

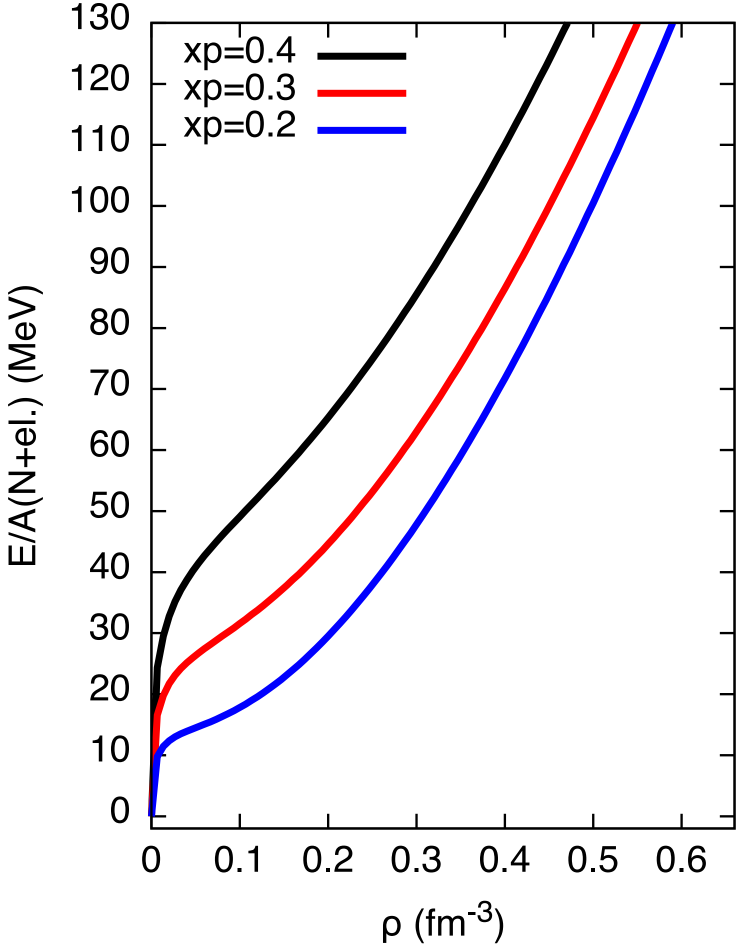
Can we better constrain the nuclear EOS with CC SN properties at bounce ?

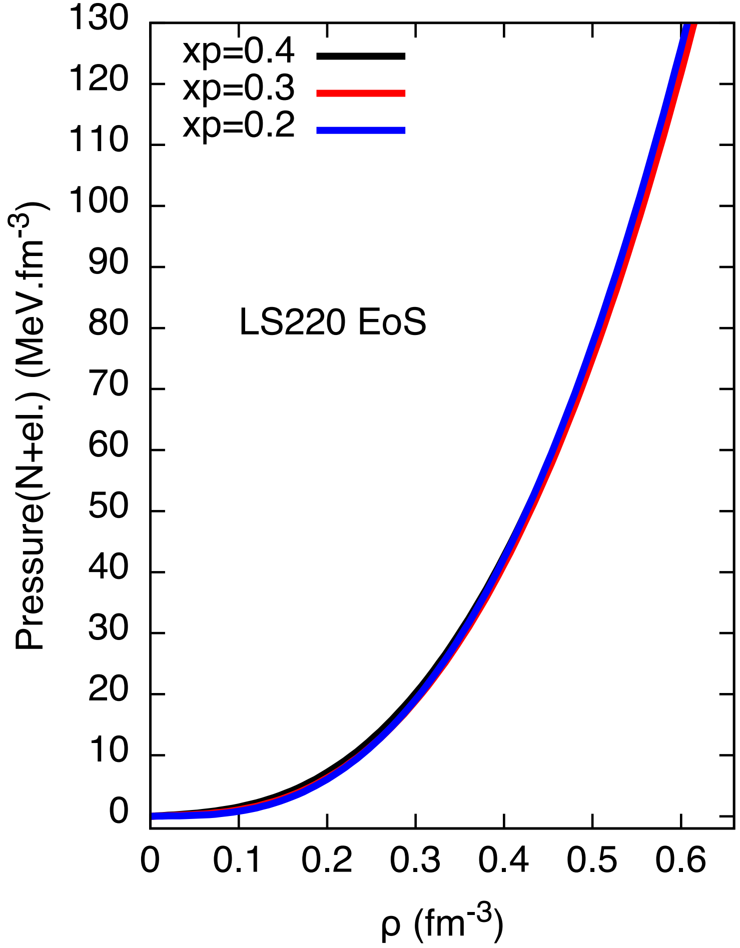
Here, we probe only the dense and uniform matter EoS.

The low density properties (non-uniform matter) will be presented by D. Chatterjee

Core-collapse supernovae

We start with LS-220 EOS, and modify the isoscalar (Ksat) and isovector (Esym) channels.





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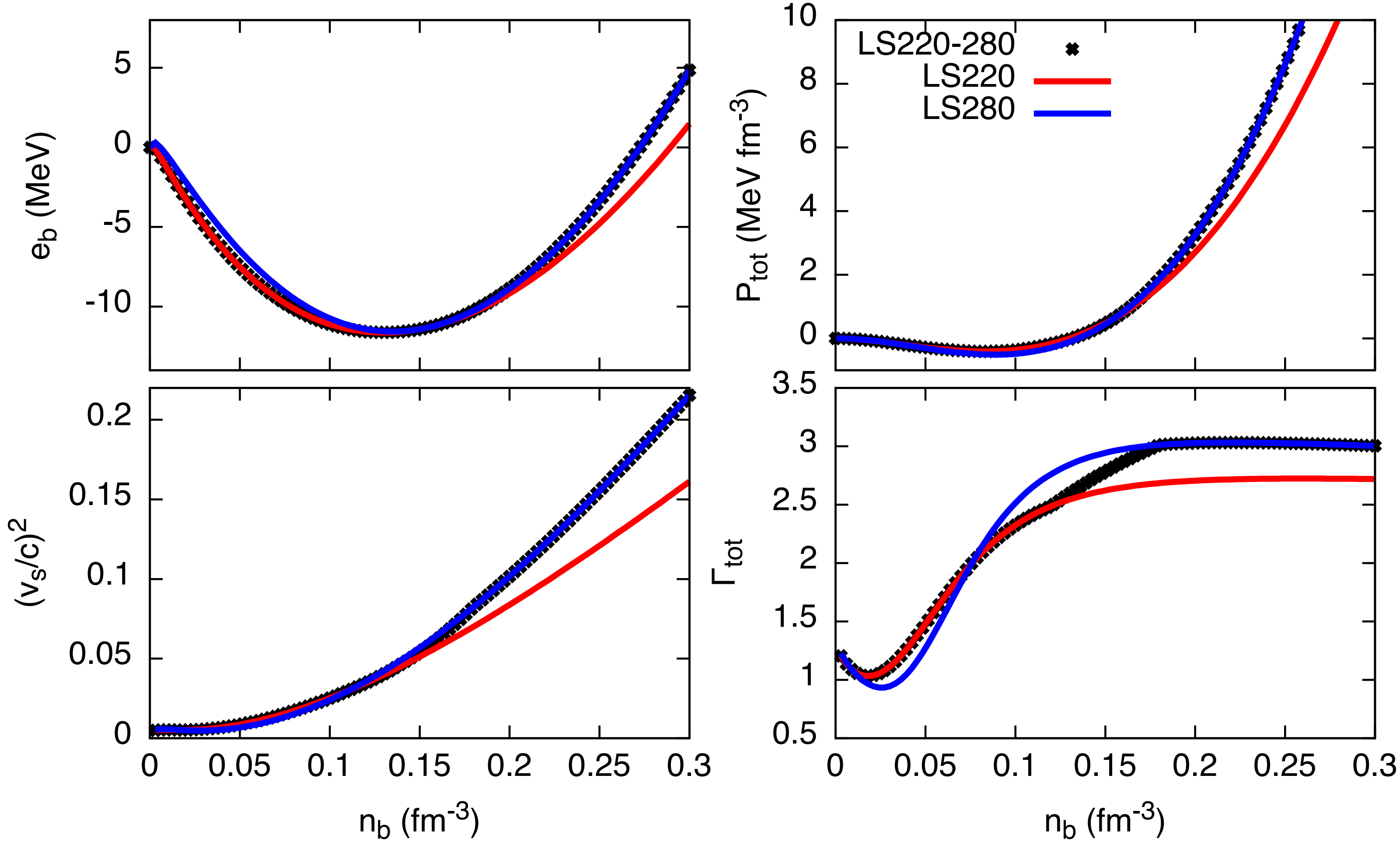

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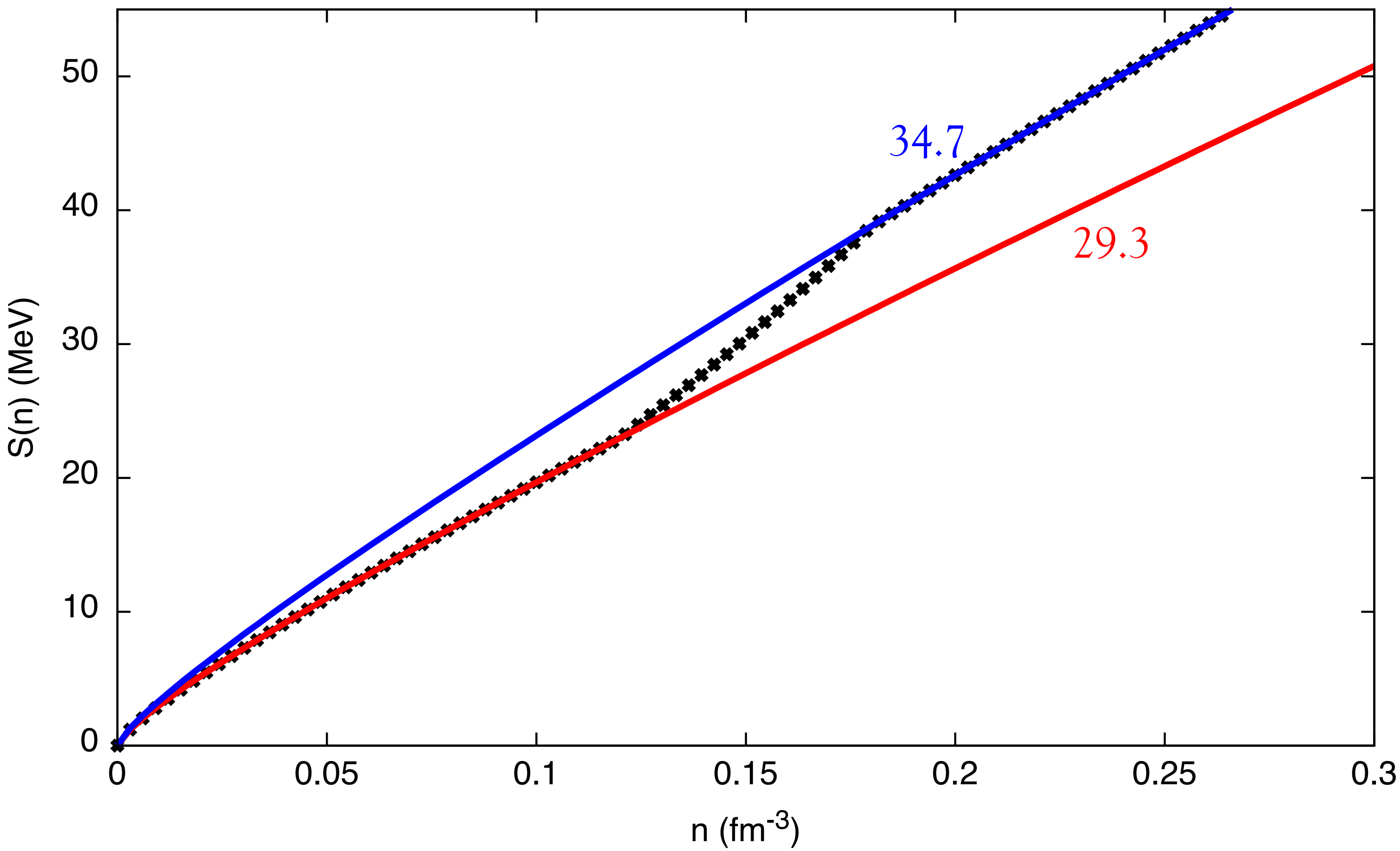
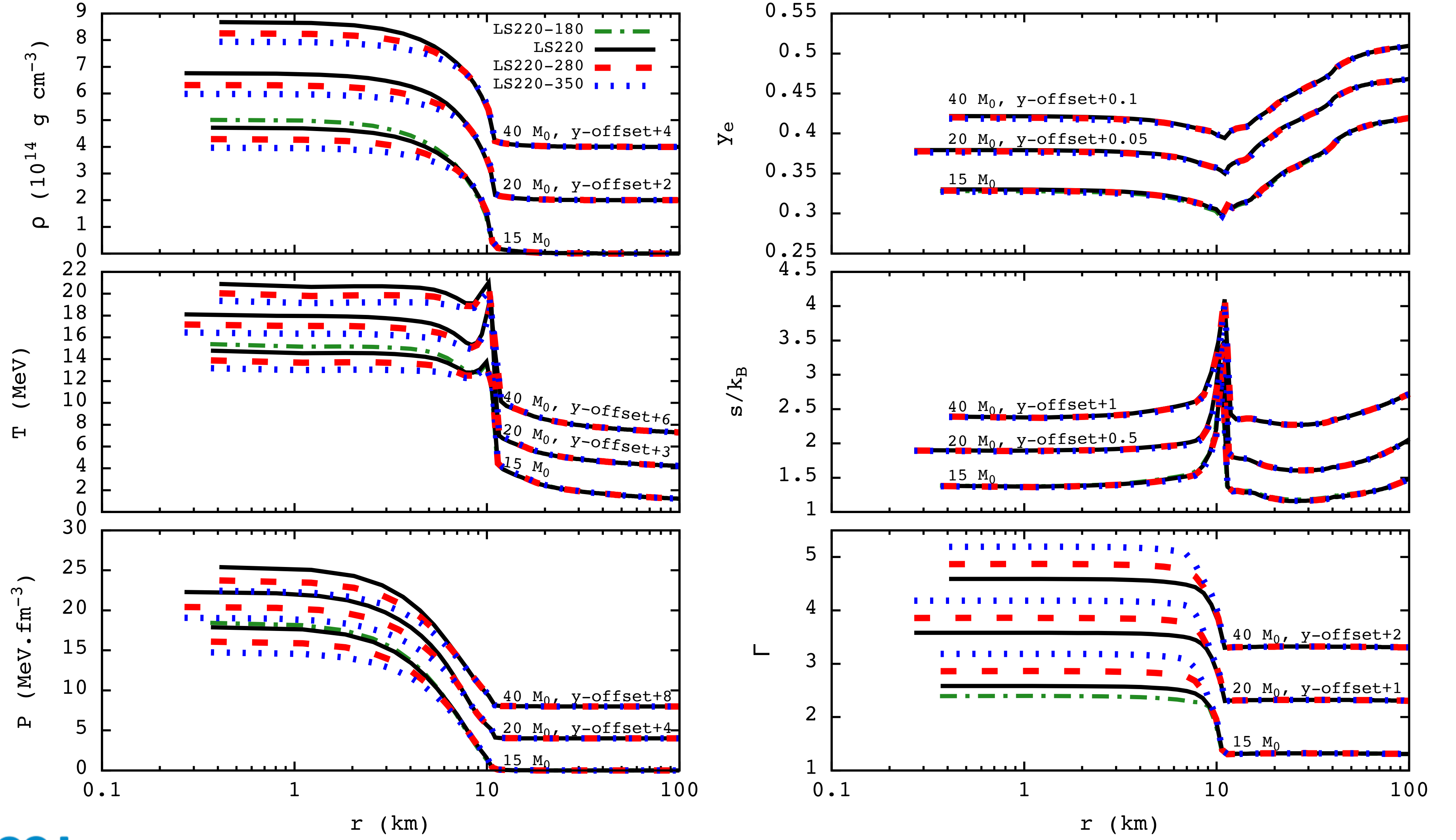


Core-collapse supernovae

Effect of K0



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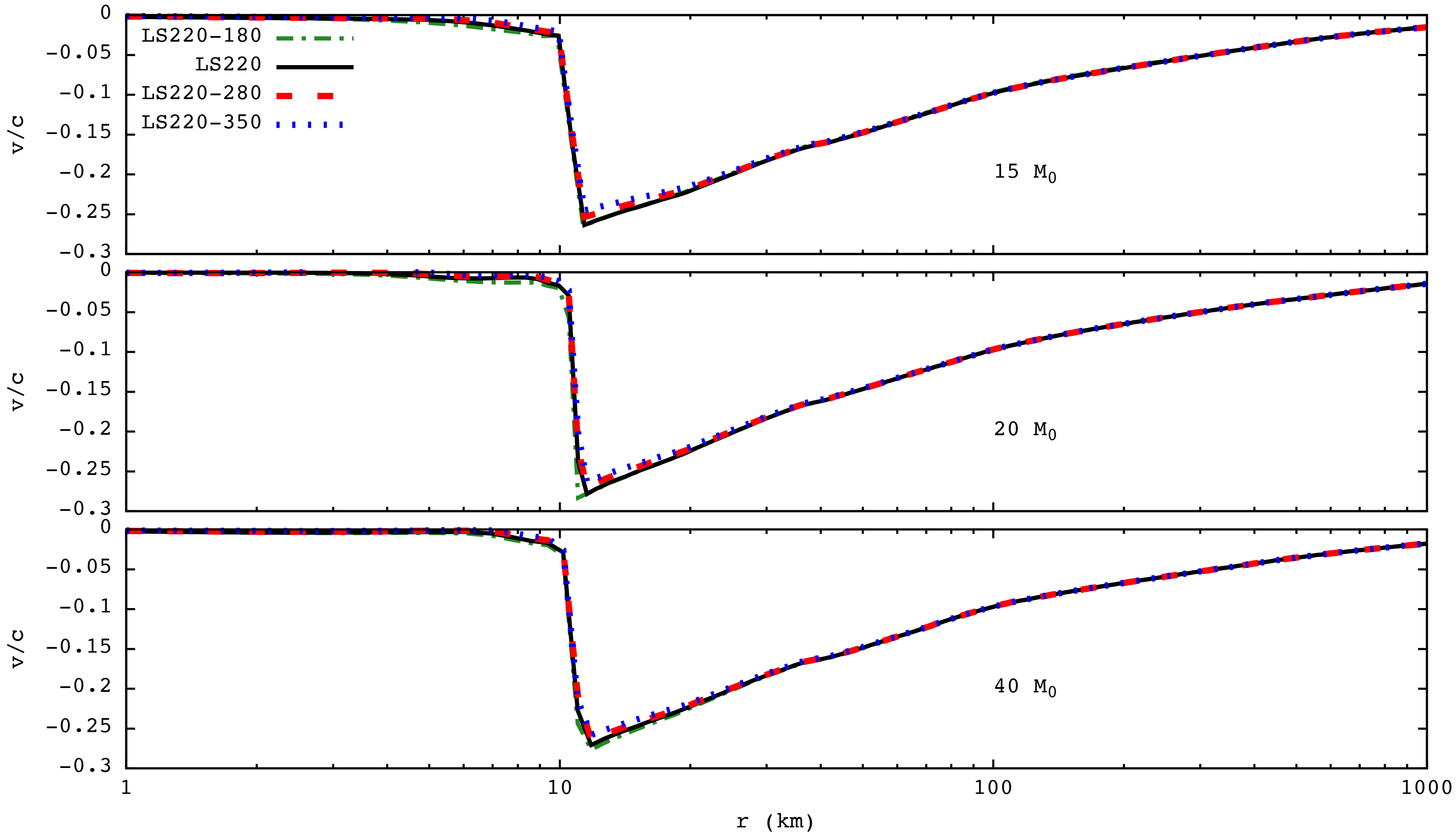

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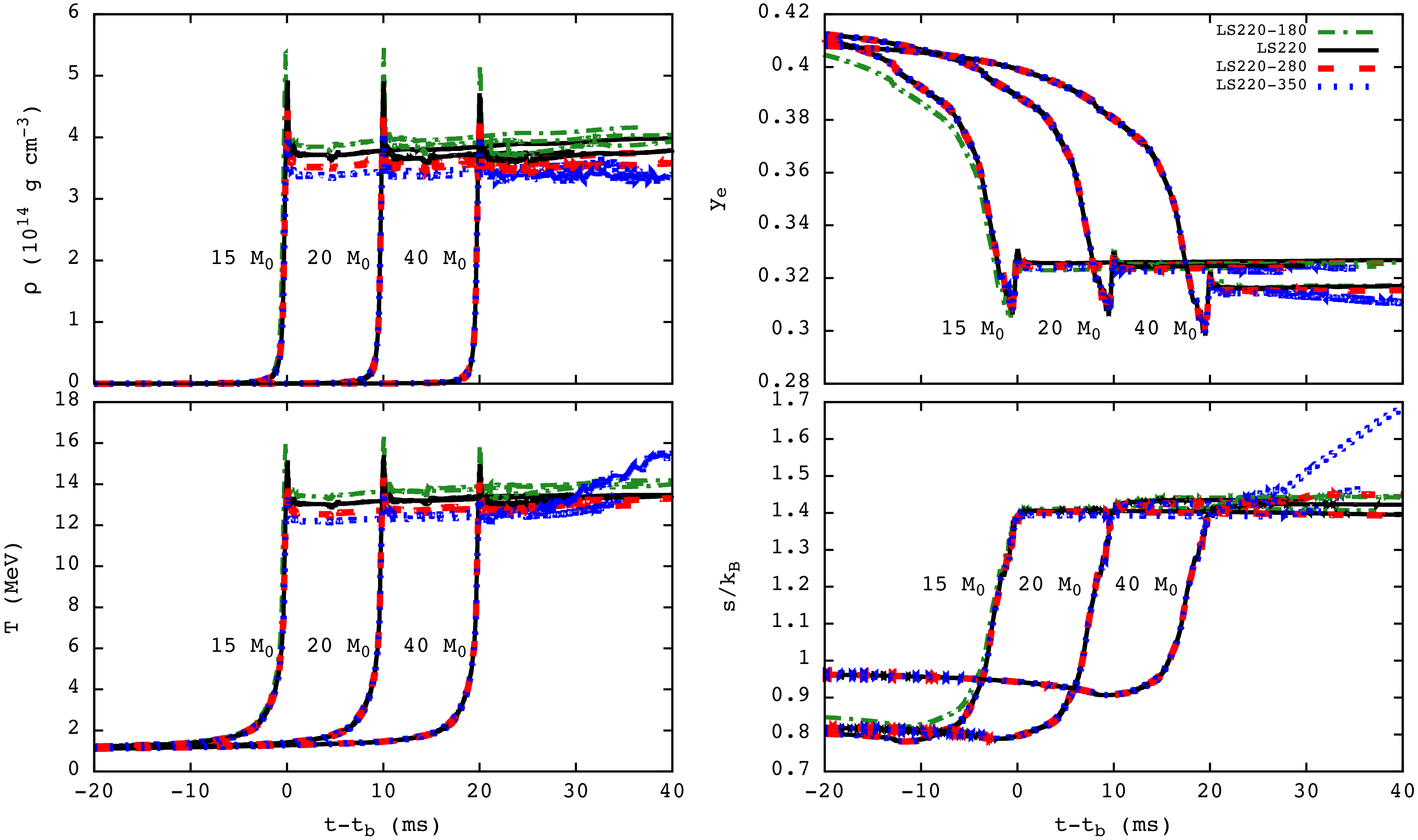
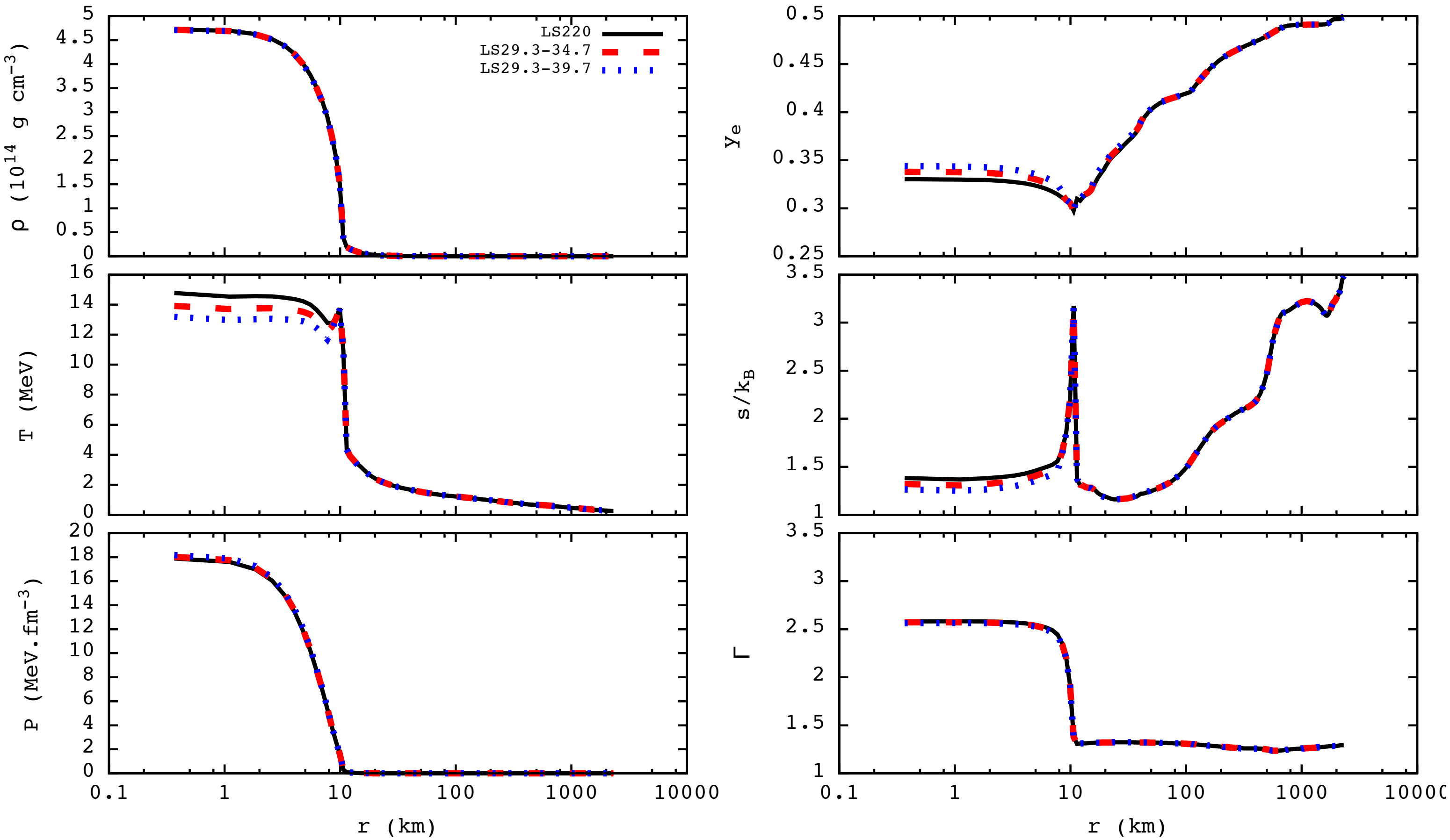


(weak) Impact on ρ and T

Core-collapse supernovae

Effect of Esym



Core-collapse supernovae

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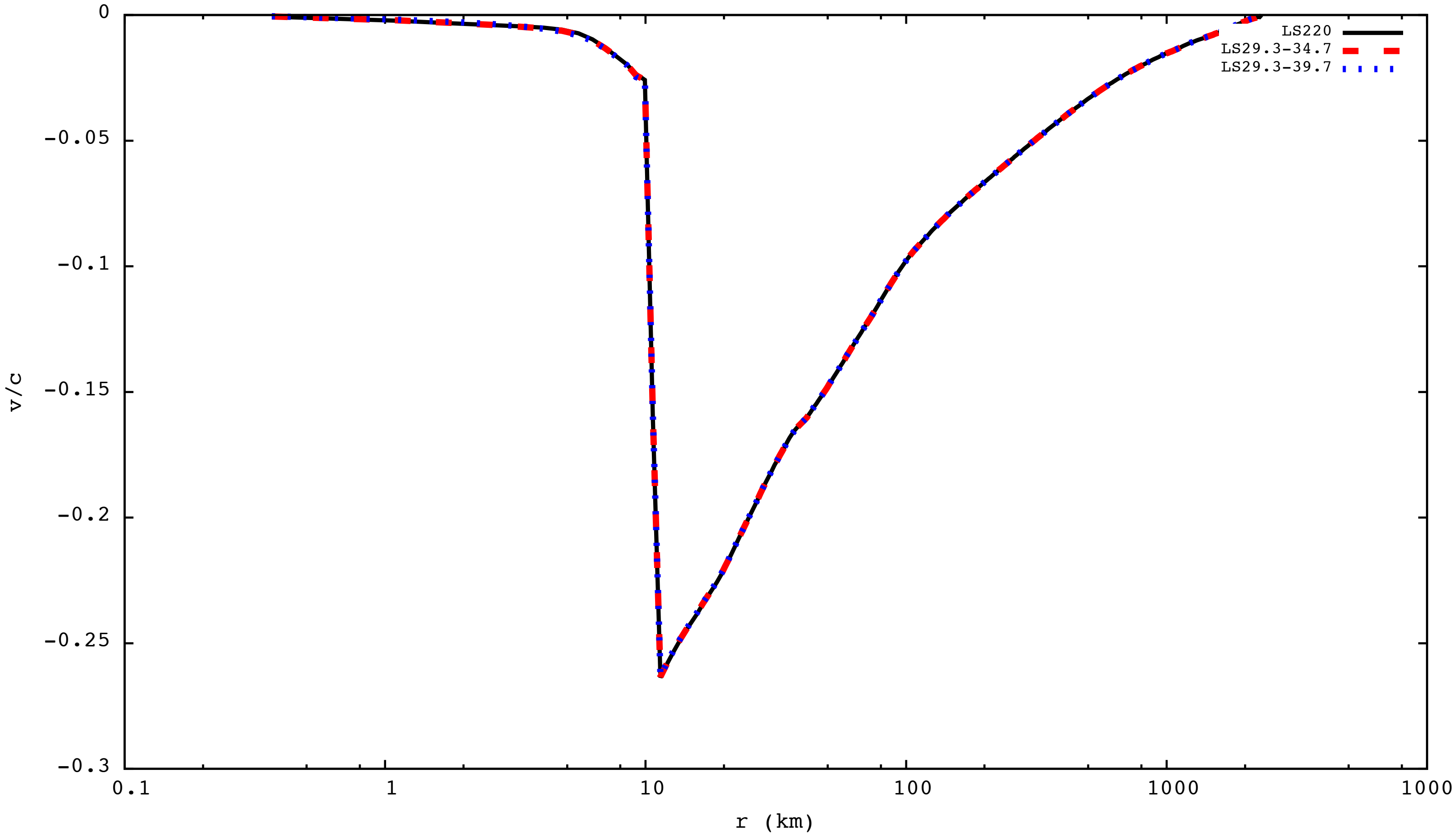

Core-collapse supernovae

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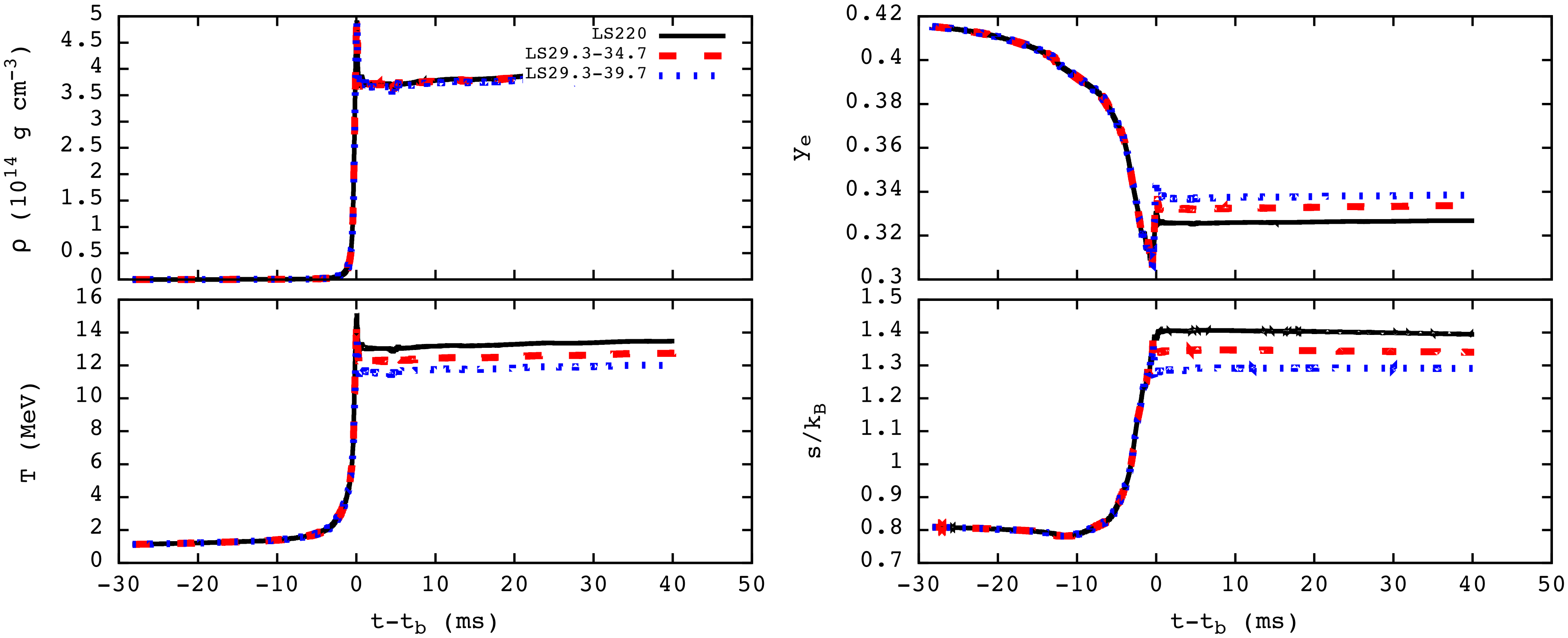

(weak) Impact on Ye, T and S

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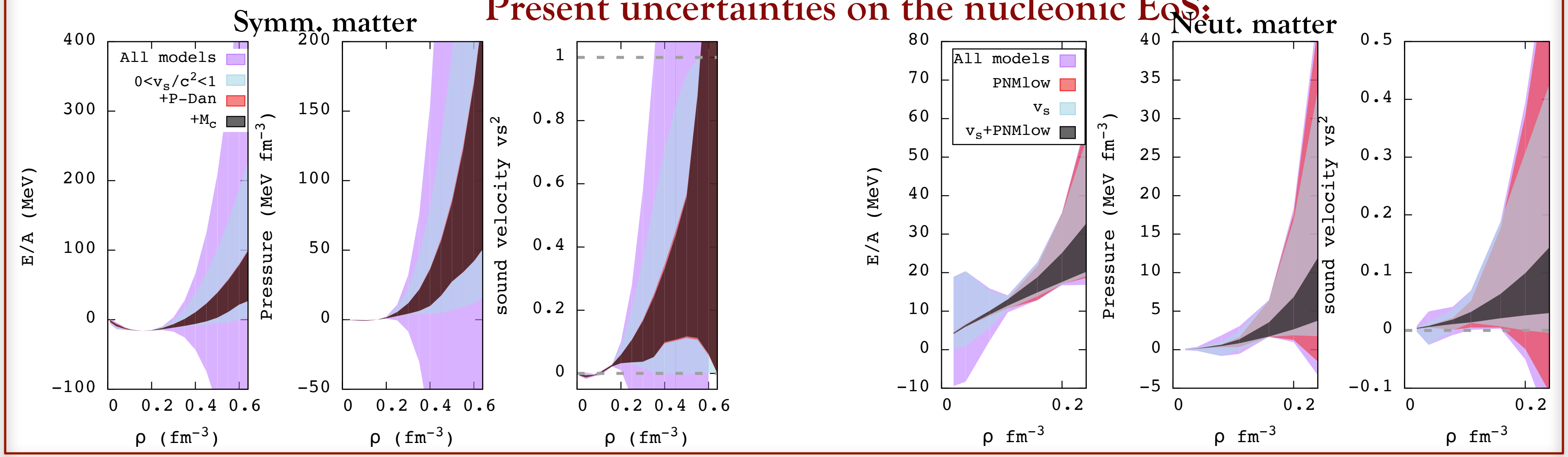
Conclusions

- We propose a flexible form of the EoS which can mimic most of existing nucleonic EoS.
- Information from nuclear physics can be easily encoded, but are not enough.
- There is a natural separation between the parameters related to nuclear experiments, and the ones which requires information from higher densities (HIC, NS).
- **Most important parameters are L_{sym} and K_{sym}**
- We can predict the boundaries in M/R of nucleonic EoS.
- Impact on CC-SN at bounce under study, but quite weak.

Present uncertainties on the nucleonic EoS:



Outlooks

- Interplay with pairing at low density, explore various pairing modeling.
- Relativistic formulation (for high densities).
- Employing this model for structure of finite nuclei, HIC, NS and SN physics.
- For SN physics: Combining with statistical modelling based also on empirical parameters -→ unifying model for sub and supra-nuclear densities.

See talk of D. Chatterjee

Thank you for your attention !!