Probing the universality of I-Love-Q relations

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Based on collaboration with
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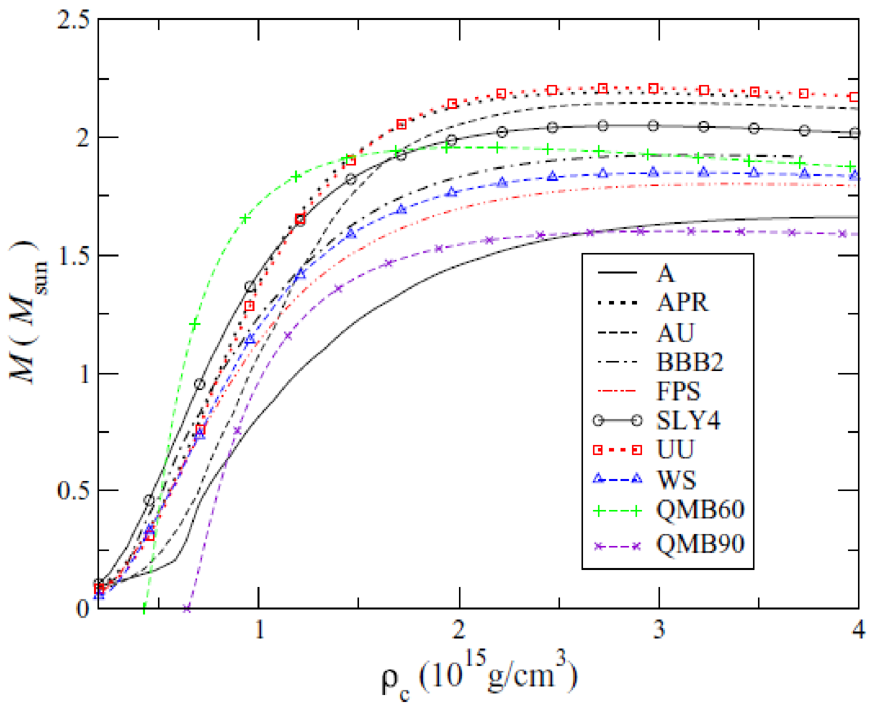
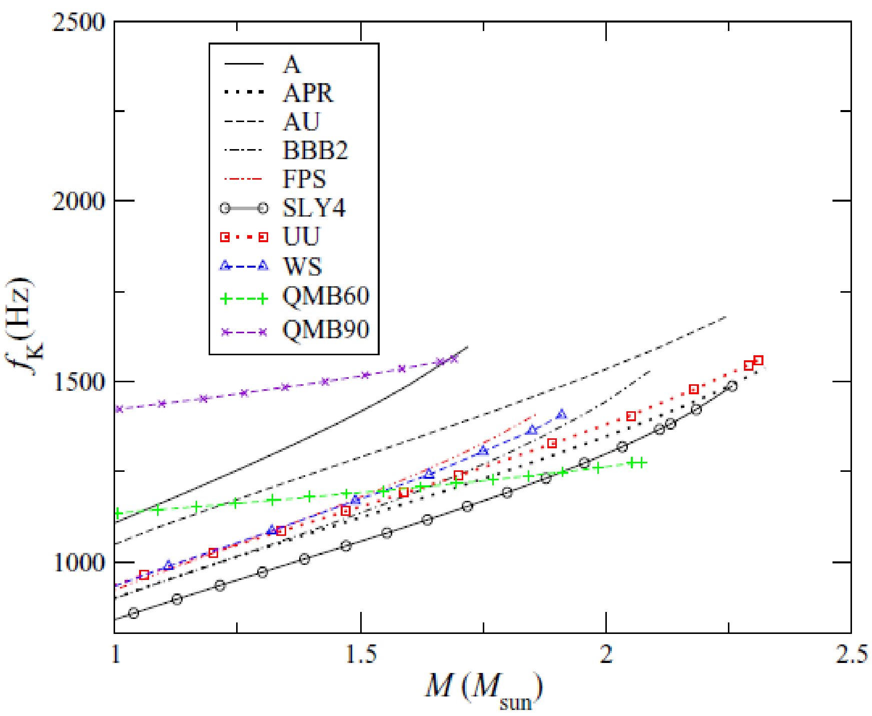
EOS & neutron stars

- It is well known that many physical quantities of neutron stars depend strongly on the EOS....(which is good news to nuclear physicists)

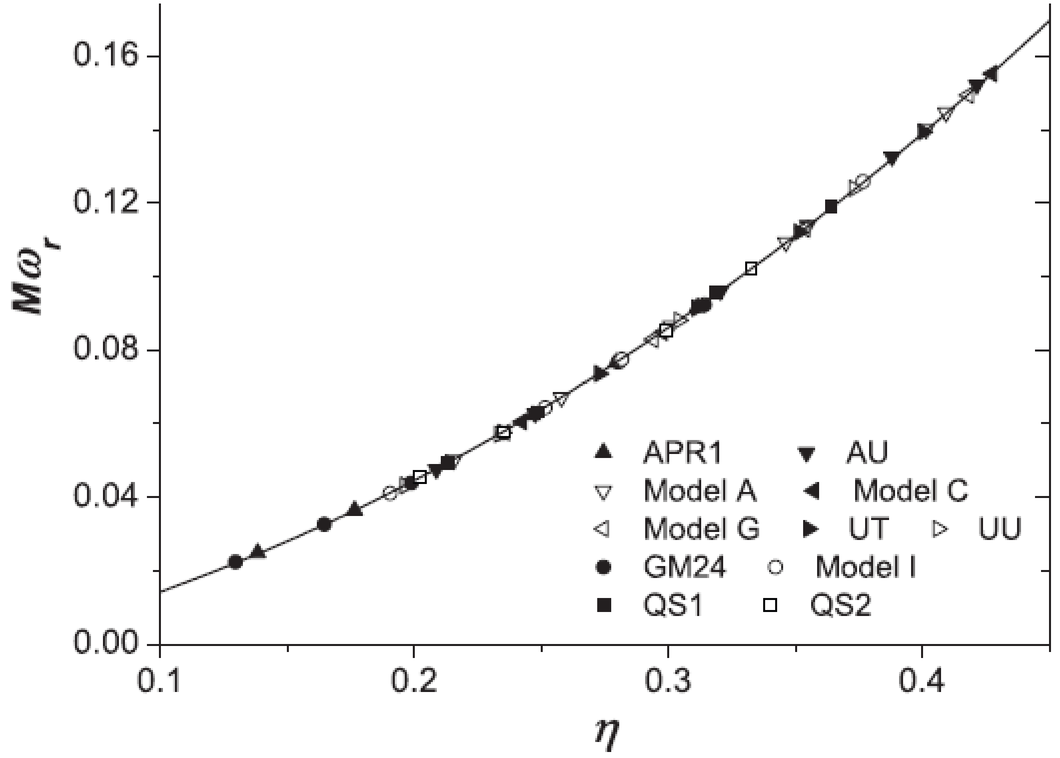




Keplerian frequency: f_K

- EOS insensitive relations do exist!

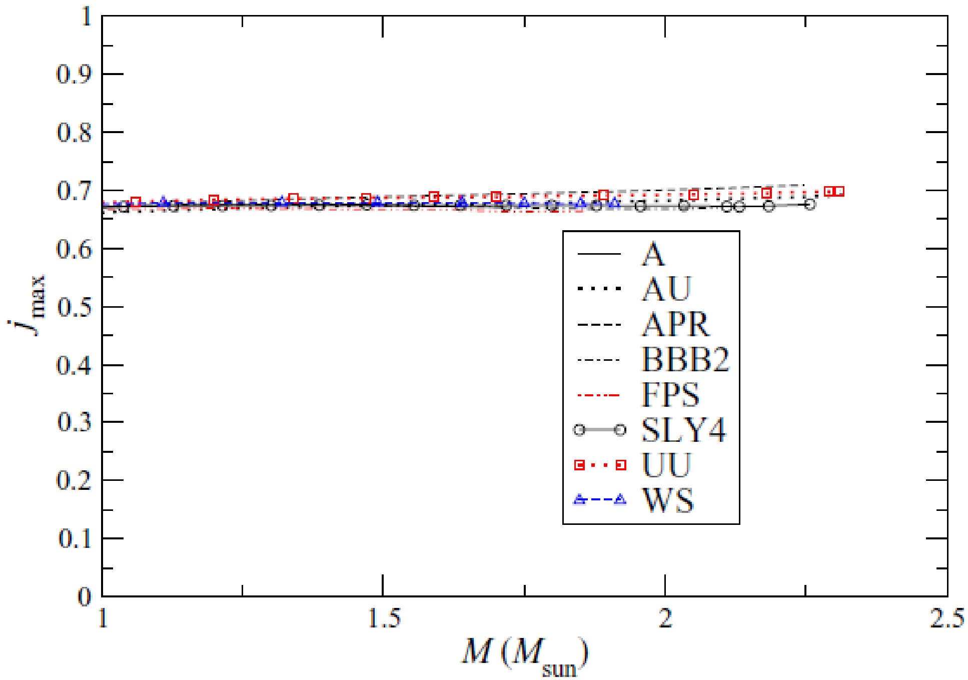
Scaled f-mode oscillation frequency

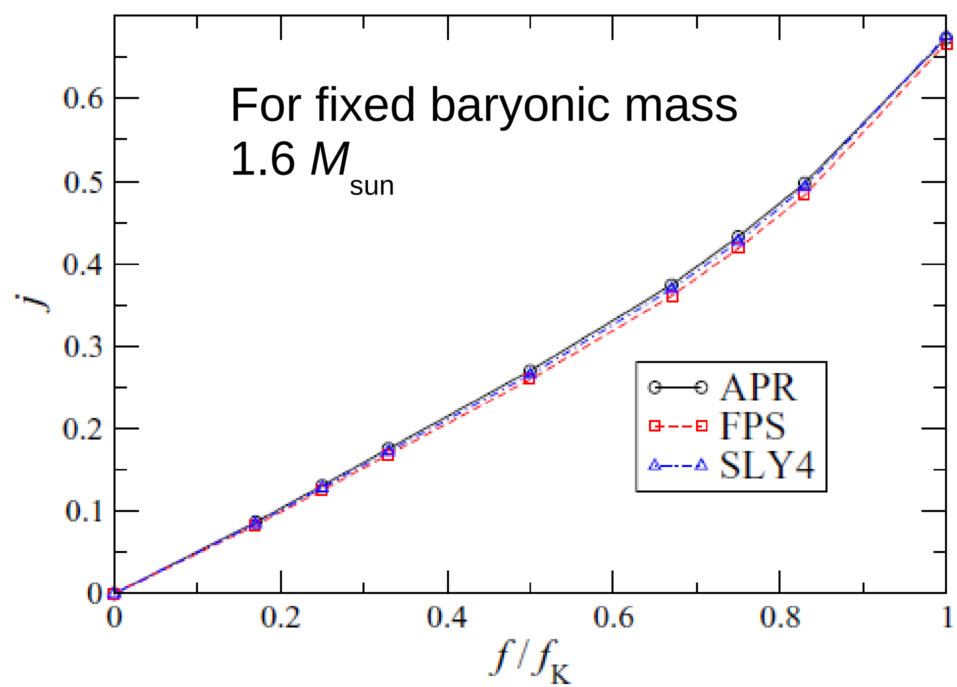


Effective compactness: $\eta \equiv \sqrt{\frac{M^3}{I}}$ ← Moment of inertia

Lau, Leung, & Lin
ApJ, 714, 1234 (2009)

- EOS insensitive quantities for rotating neutron stars





(Scaled rotation frequency)

Dimensionless spin parameter

$$j \equiv \frac{J}{M^2}$$ (G=c=1)

Lo & Lin
ApJ, 728, 12 (2011)

I-Love-Q relations

- In 2013, Yagi & Yunes discovered the universal I-Love-Q relations which connect the following dimensionless quantities:

$$\bar{I} \equiv \frac{I}{M^3} \quad , \quad \bar{Q} \equiv -\frac{Q}{M^3 j^2} \quad , \quad \bar{\lambda} \equiv \frac{\lambda}{M^5}$$

- Q = rotation-induced quadrupole moment
- λ = tidal deformability

$$k_2 = \frac{3}{2} \lambda R^{-5}$$

(Love number)

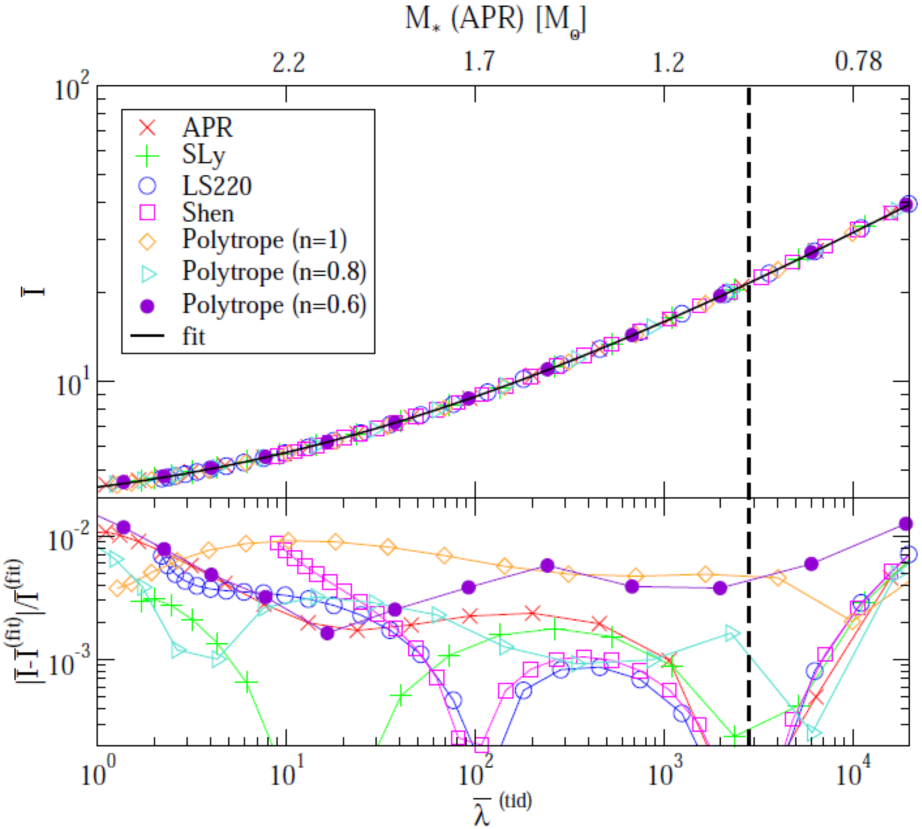
$$Q_{ij} = -\lambda \epsilon_{ij}$$

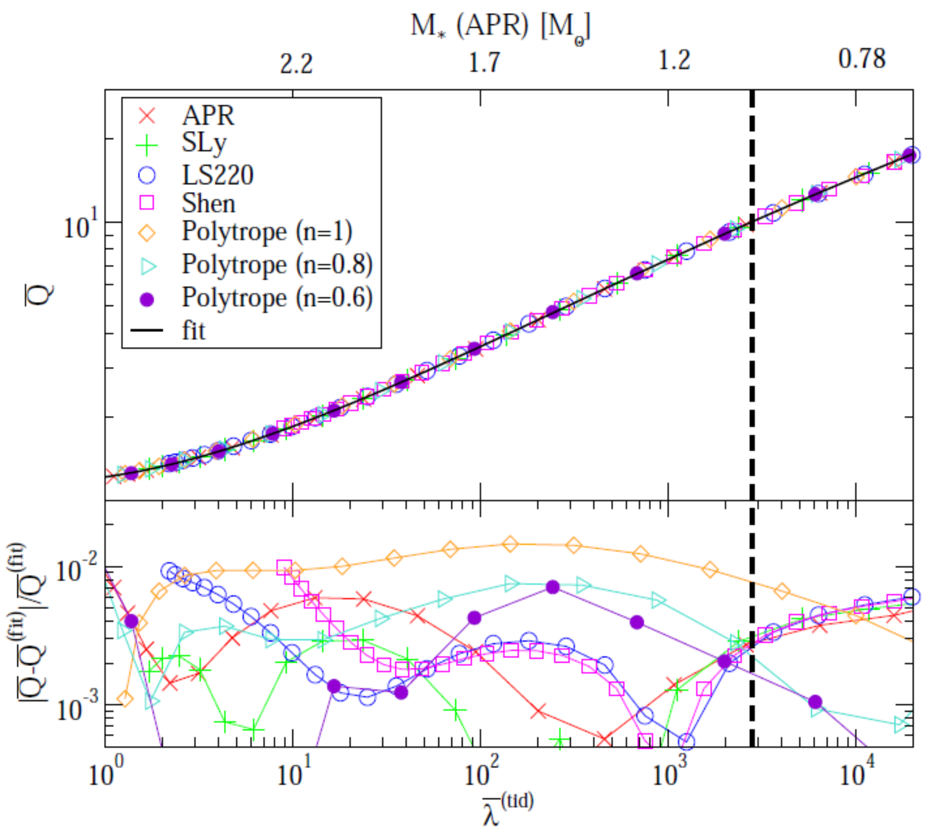
Tidal-induced quadrupole moment

External quadrupolar tidal field

Yagi & Yunes
* Science, 341, 365 (2013)
* PRD, 88, 023009 (2013)

• Original results from Yagi & Yunes:

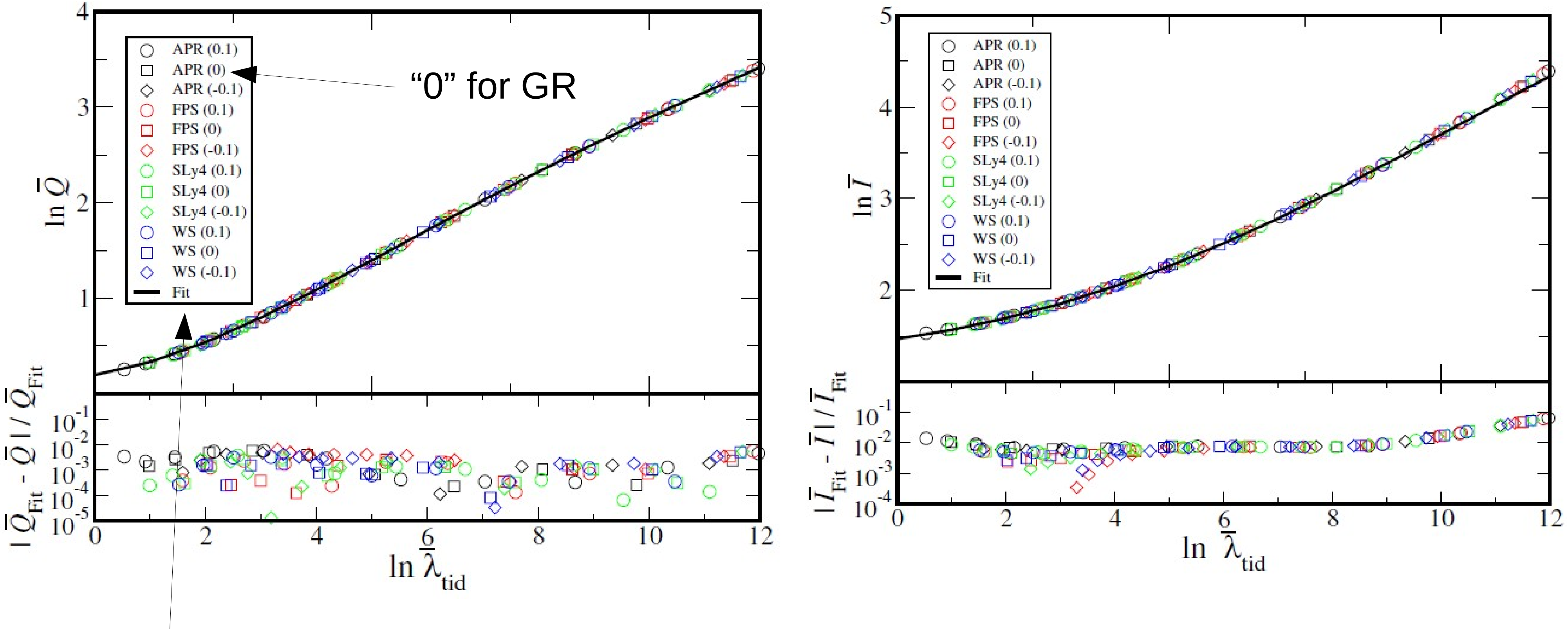




Yagi & Yunes
* Science, 341, 365 (2013)
* PRD, 88, 023009 (2013)

• I-Love-Q relations for GR and EiBI gravity

EiBI = Eddington-inspired Born-Infeld



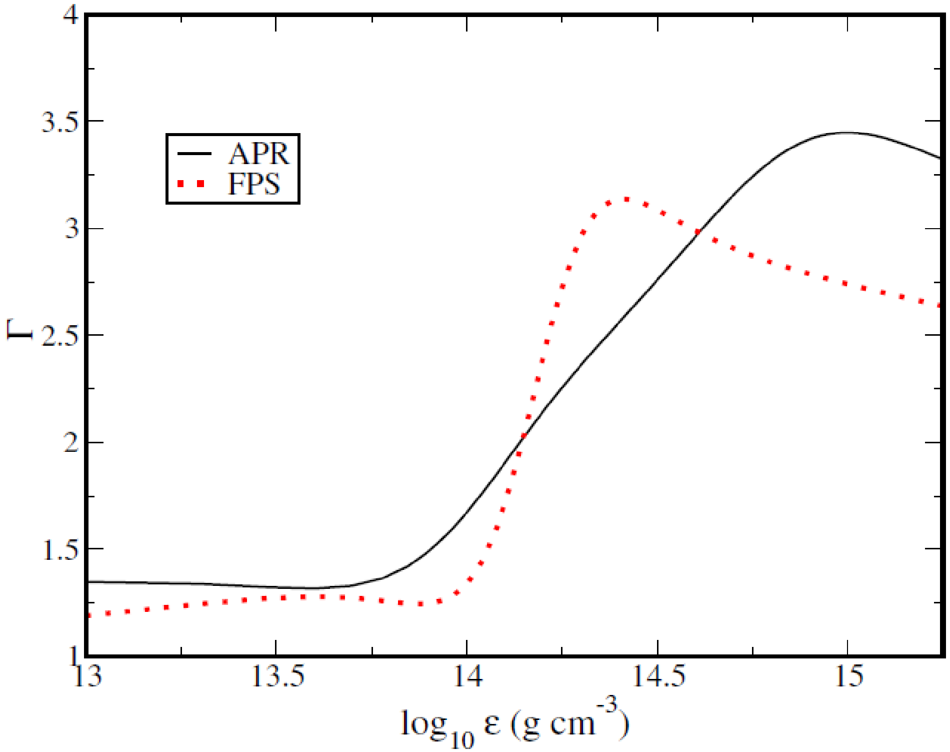
Fit = fitting curve obtained by Yagi & Yunes

Sham, Lin, & Leung
ApJ, 781, 66 (2014)

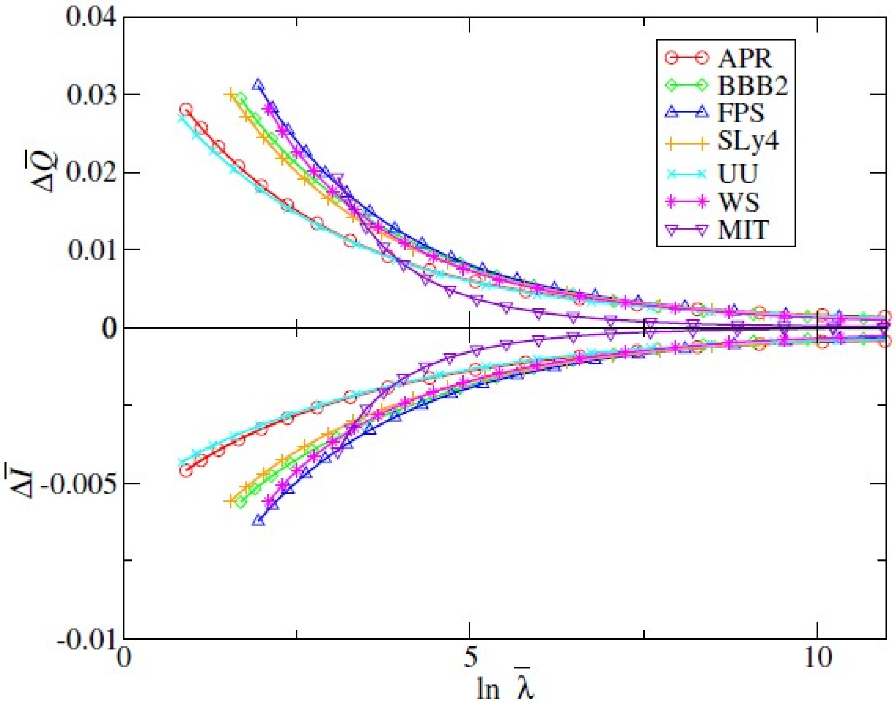
Why I-Love-Q?
EOSs are stiff enough!

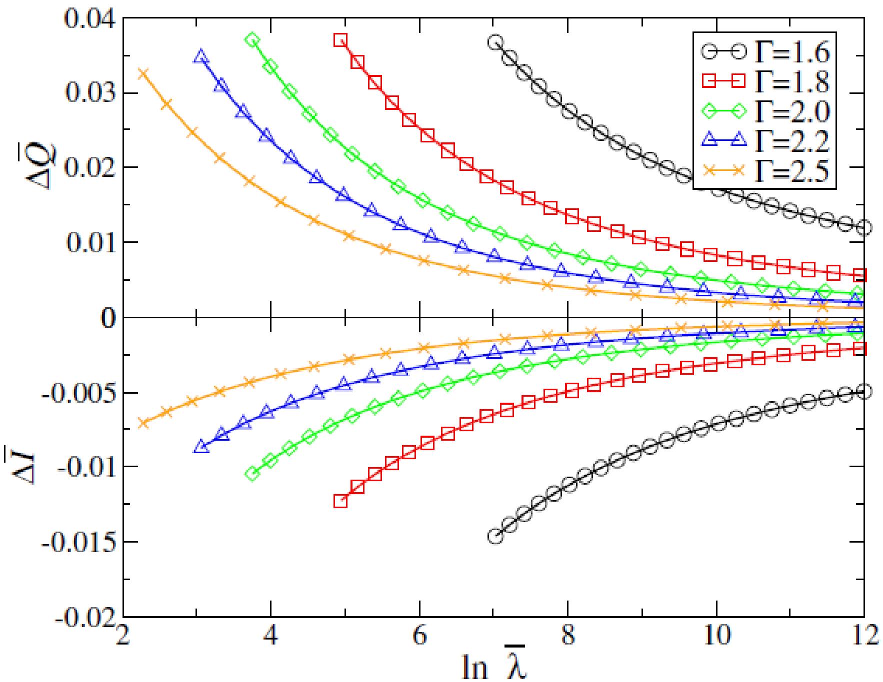
- We propose that the stiffness of modern EOS is the key!
- Realistic EOS models typically have effective adiabatic index $\Gamma \geq 2$

(above nuclear density)



- The EOS models are so stiff that the I-Love-Q relations are well modeled by the incompressible limit





$$\Delta \bar{I} \equiv \frac{\bar{I} - \bar{I}_{incom}}{\bar{I}_{incom}}$$

For incompressible model

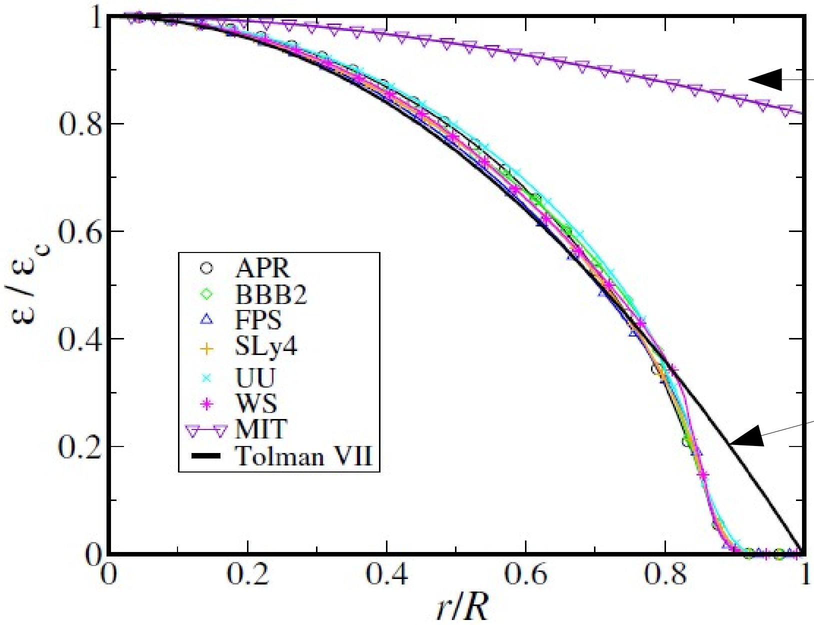
similarly for $\Delta \bar{Q}$ and $\Delta \bar{\lambda}$

Sham, Chan, Lin, & Leung
ApJ, 798, 121 (2015)

Analytical study

- Here we study the I-Love relation analytically in Newtonian gravity and show that the incompressible limit is a key point to the universal relations
- Neutron stars are modeled well by the density profile:

$$x \equiv \frac{r}{R} \qquad \rho = \rho_0 \left(1 - \delta x^2\right) \qquad \delta = \text{free parameter}$$



Quark star (MIT bag model) $\delta \approx 0$

$\delta = 1$

- Scaled moment of inertia

$$\bar{I} = \frac{\int_0^R \rho r^4 dr}{24\pi^2 \left(\int_0^R \rho r^2 dr \right)^3}$$

With the density profile $\rho = \rho_0 \left(1 - \delta x^2 \right)$

$$\bar{I} = \frac{2(7-5\delta)}{7(5-3\delta)} C^{-2}$$

$$C \equiv \frac{M}{R}$$

- Tidal deformability λ (in Newtonian gravity)

$$\frac{d^2h}{dr^2}+\frac{2}{r}\frac{dh}{dr}-\left(\frac{6}{r^2}-4\pi\rho\frac{d\rho}{dP}\right)h=0$$

$$\bar{\lambda}=\frac{\lambda}{M^5}$$

$$\bar{\lambda}=\frac{2-y(R)}{3[3+y(R)]}C^{-5}$$

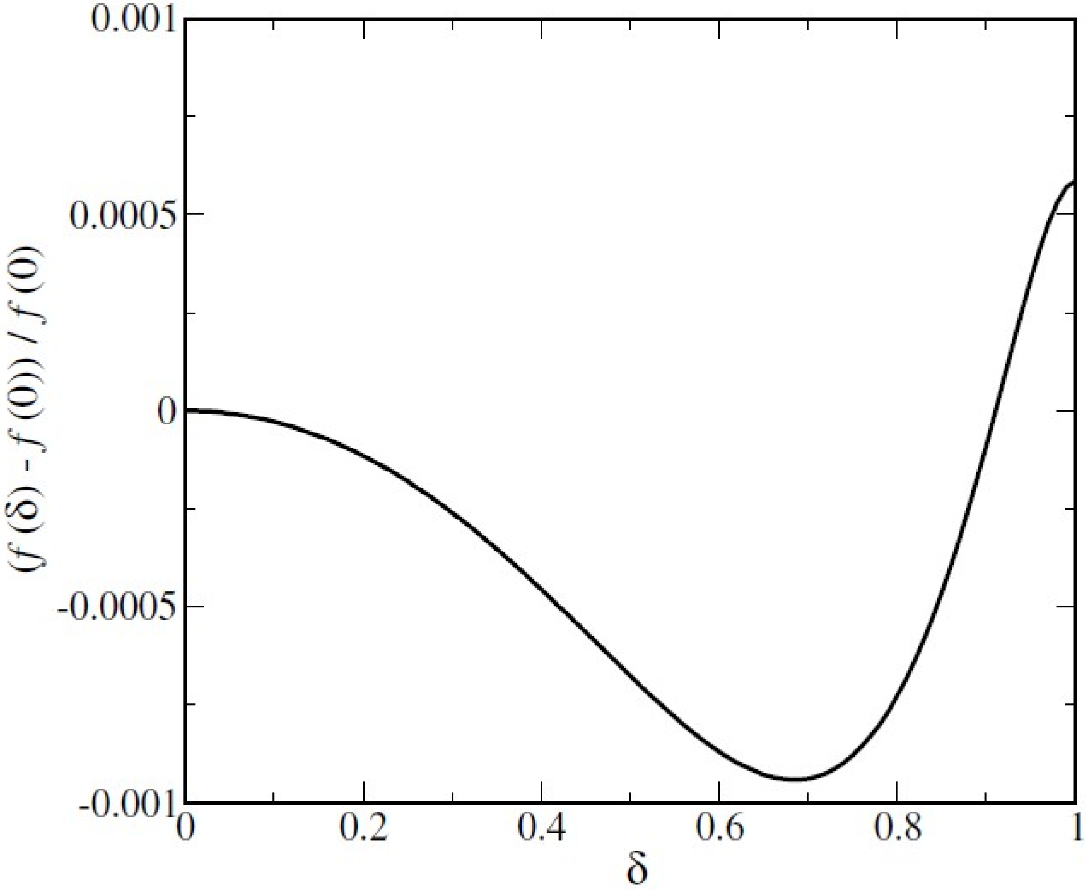
$$y(R)\equiv\frac{R\,h'(R)}{h(R)}$$

- By eliminating the compactness C, we obtain the I-Love relation

$$f(\delta)\equiv\bar{\lambda}\,\bar{I}^{-5/2}=\frac{2-y(R)}{3(3+y(R))}\left[\frac{2(7-5\delta)}{7(5-3\delta)}\right]^{-5/2}$$

- The I-Love relation *f*(δ) depends weakly on δ

$$f(\delta) \equiv \bar{\lambda}\,\bar{I}^{-5/2} = \frac{2-y(R)}{3(3+y(R))}\left[\frac{2(7-5\delta)}{7(5-3\delta)}\right]^{-5/2}$$



$$\rho = \rho_0 (1 - \delta x^2)$$

$$x \equiv \frac{r}{R}$$

$\delta = 0$ incompressible limit

- We can expand the I-Love relation about the incompressible limit ($\delta=0$)

$$\bar{\lambda}\,\bar{I}^{-5/2}=5\sqrt{\frac{5}{2}}\left(\frac{5}{8}-\frac{1}{588}\delta^2+\ldots.\right)$$

2nd order

- The incompressible stellar model is a stationary point for the I-Love relation

Remark: $y(R)$ has a formal solution given by the hypergeometric function

$$f(\delta)\equiv\bar{\lambda}\bar{I}^{-5/2}=\frac{2-y(R)}{3(3+y(R))}\left[\frac{2(7-5\delta)}{7(5-3\delta)}\right]^{-5/2}$$

$$y(R)=2-\frac{6\delta}{7}\frac{{}_2F_1\left(\frac{9-\sqrt{65}}{4},\frac{9+\sqrt{65}}{4},\frac{9}{2},\frac{3\delta}{5}\right)}{{}_2F_1\left(\frac{5-\sqrt{65}}{5},\frac{5+\sqrt{65}}{4},\frac{7}{2},\frac{3\delta}{5}\right)}-\frac{15(1-\delta)}{5-3\delta}.$$

- Brief summary

$$\rho = \rho_0 \left(1 - \delta x^2\right)$$

δ is used to model the stellar structure and EOS

$$\bar{I} = \frac{2(7-5\delta)}{7(5-3\delta)} C^{-2}$$

$$\bar{\lambda} = \frac{2-y(R)}{3[3+y(R)]} C^{-5}$$

They depend non-trivially on δ

- However, the I-Love relation depends weakly on δ (and hence EOS)

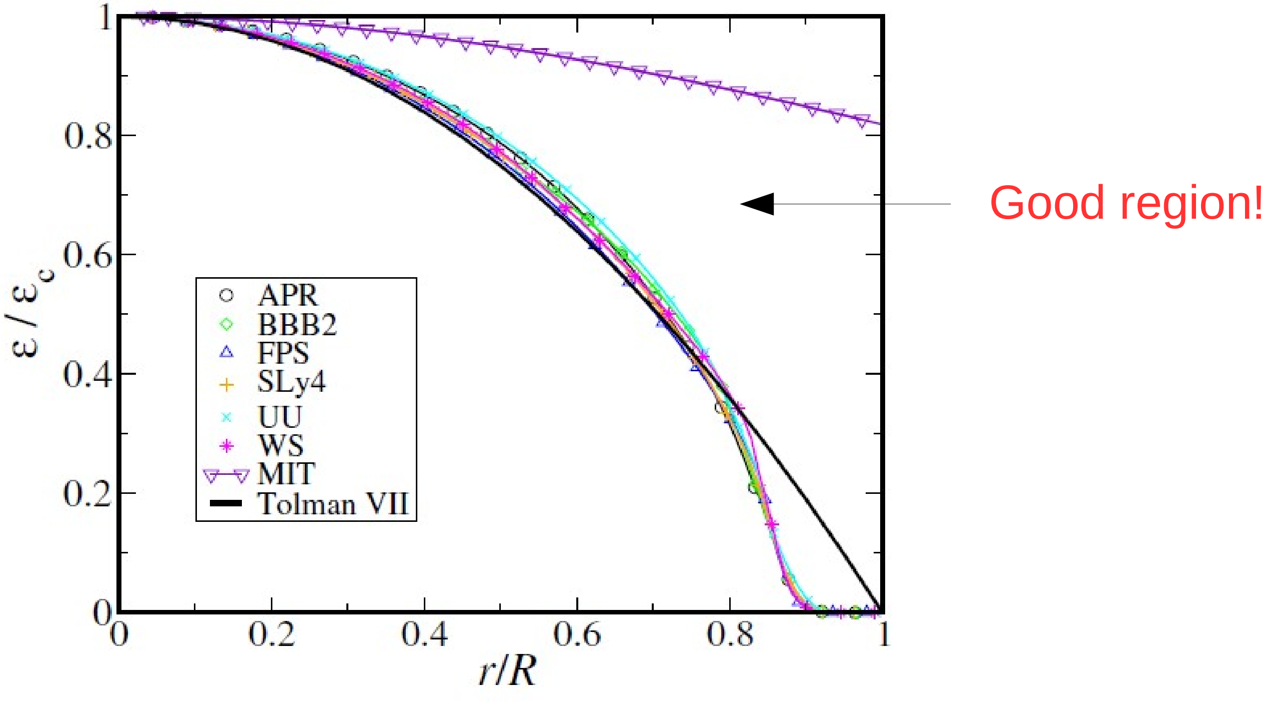
$$\bar{\lambda}\,\bar{I}^{-5/2} = \frac{2-y(R)}{3(3+y(R))} \left[\frac{2(7-5\delta)}{7(5-3\delta)}\right]^{-5/2}$$

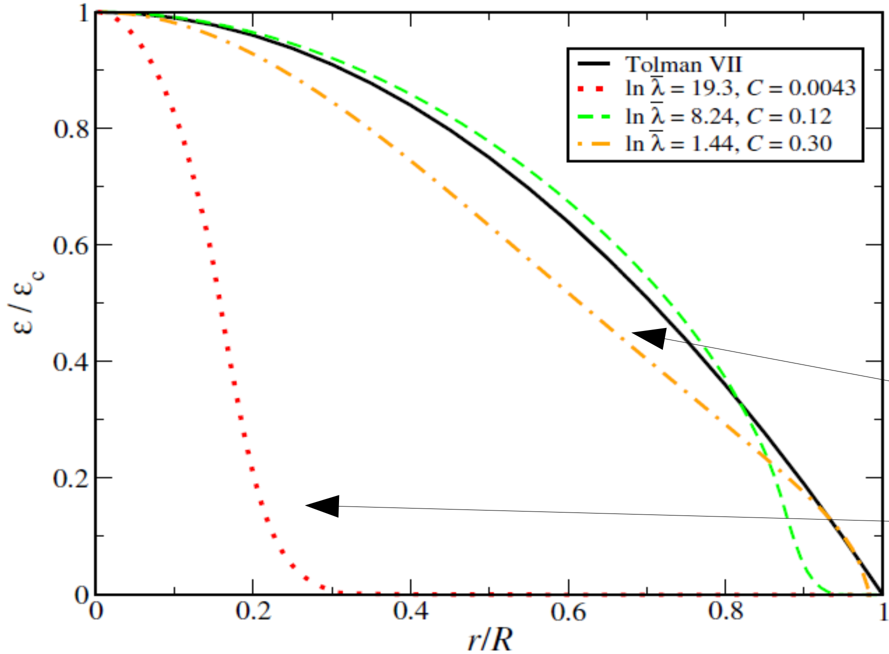
$$= a + b\,\delta^2 + \ldots.$$

Breakdown of I-Love-Q

- We expect that the universality is good when the stellar density profile can be approximated by

$$\rho = \rho_0 (1 - \delta x^2) \qquad \delta \in [0, 1]$$

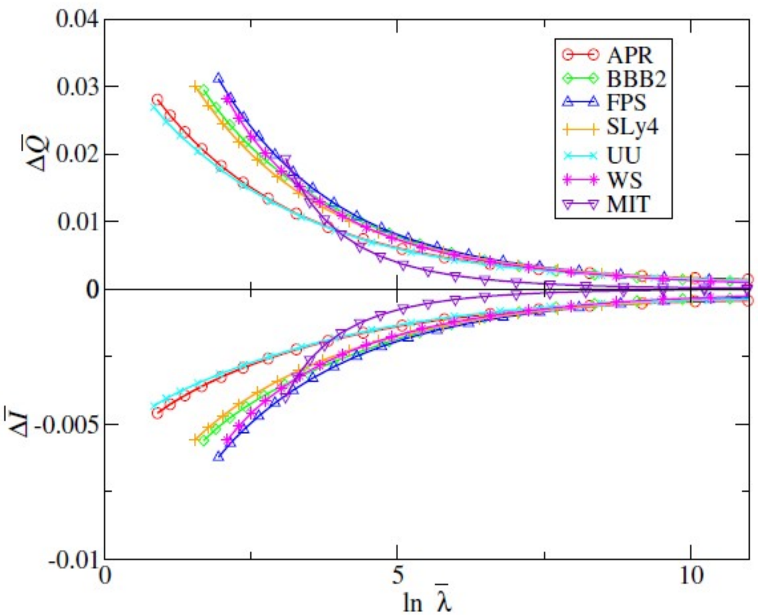


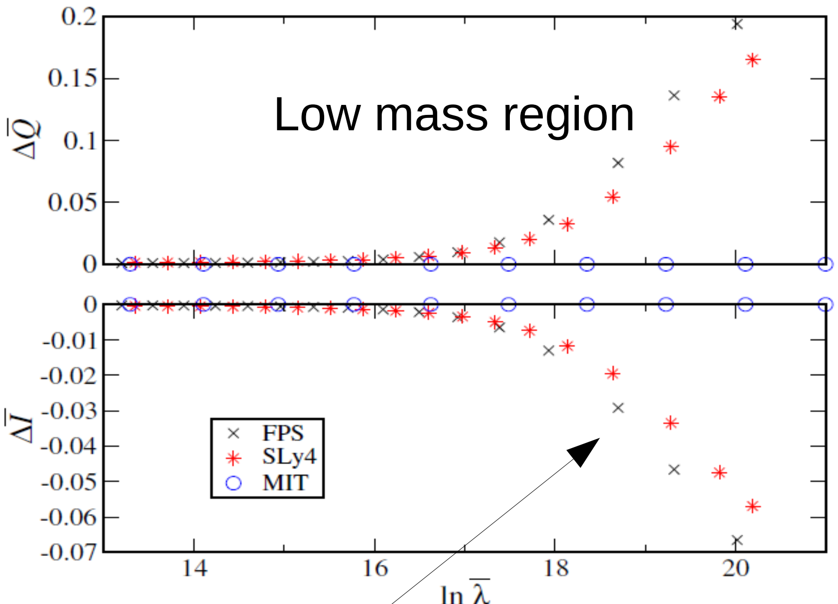


For a fixed EOS model

Maximum mass model

Minimum mass model
(composed mainly of "softer" matter)





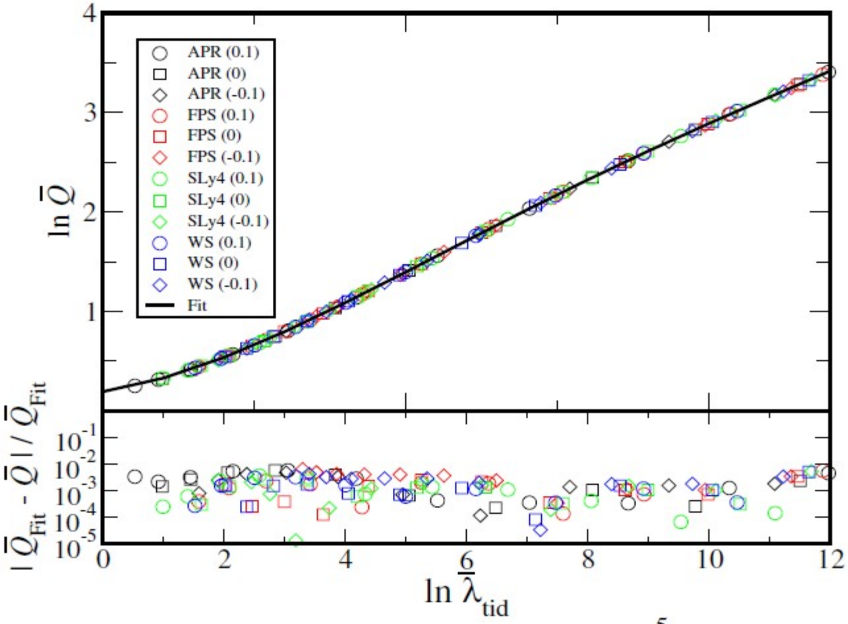
The relations depend more sensitively on EOS

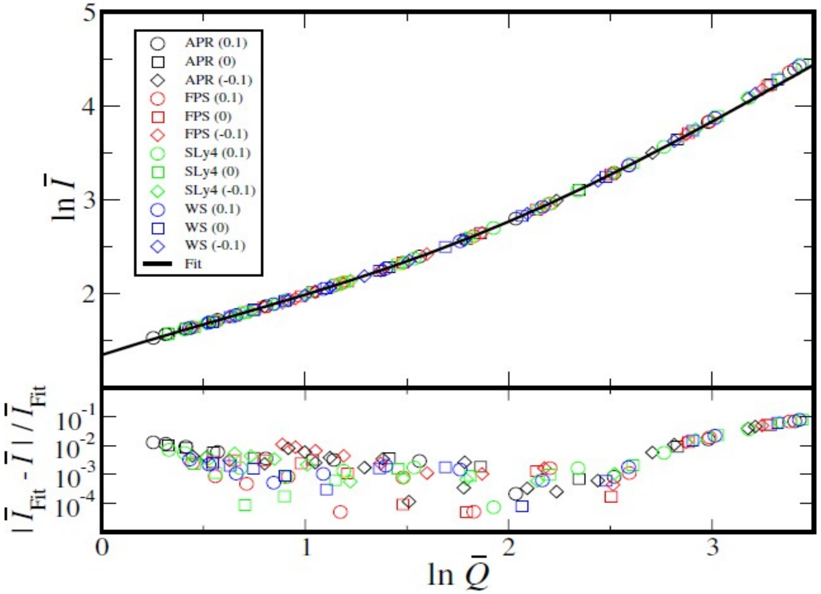
Conclusion

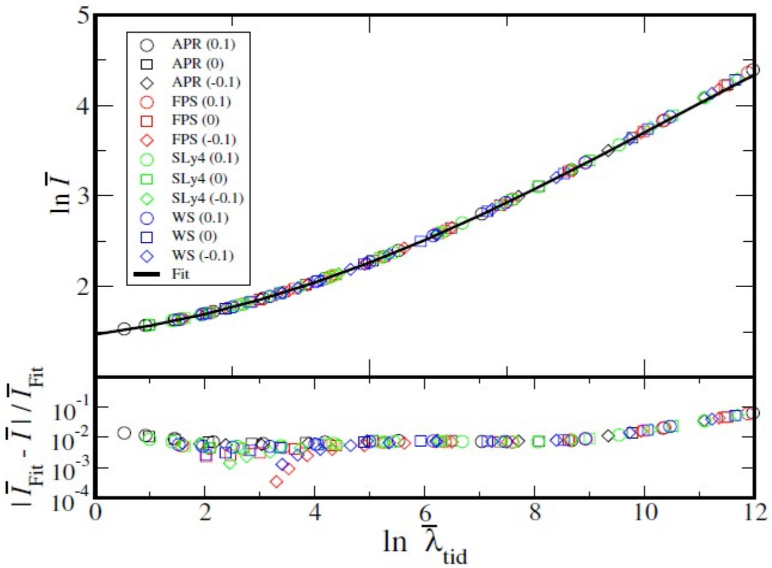
- We have demonstrated numerically that the universality of I-Love-Q relations can be attributed to the incompressible limit
- Typical nuclear EOSs are already stiff enough to establish the universality
- We have also provided an analytical study to show the weak EOS-dependence of the I-Love relation in Newtonian gravity

Appendix

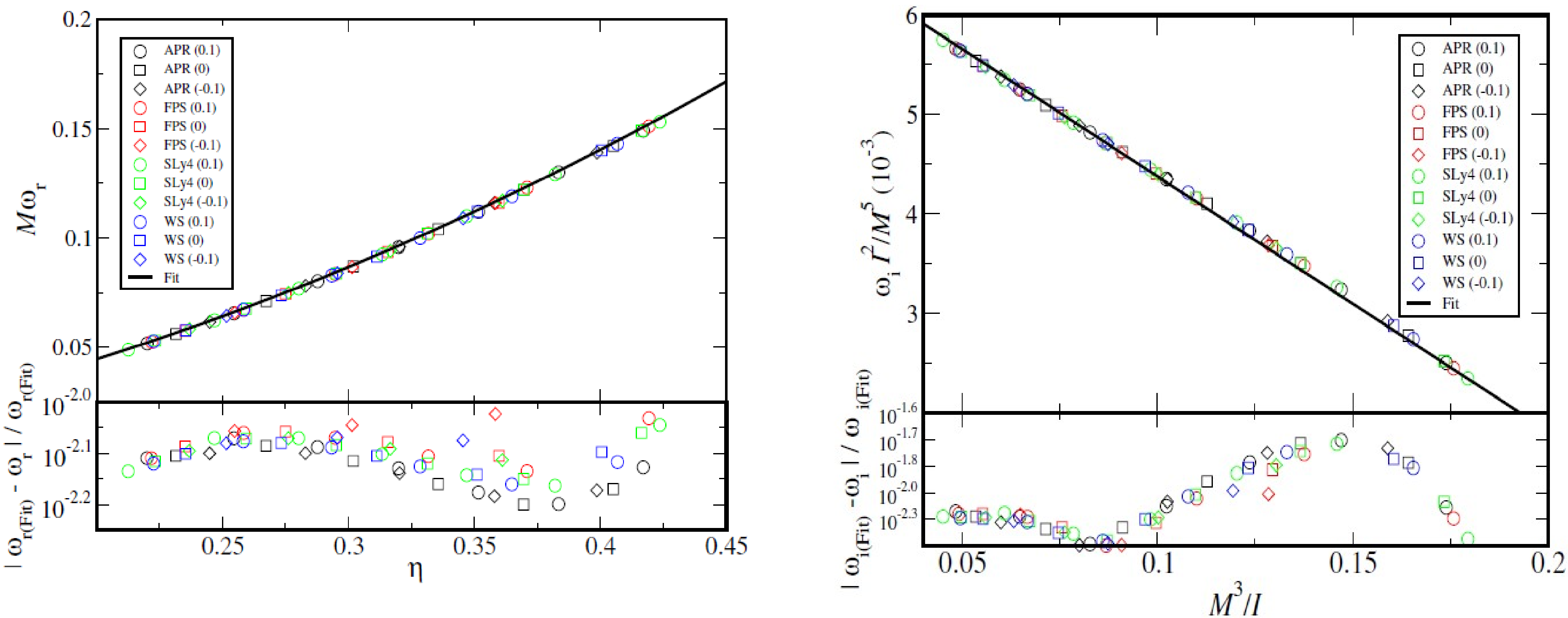
I-Love-Q relations







F-mode universality



The universal relations found by Lau et al. (2010) are given by[2]

$$M\omega_{\rm r} = -0.0047 + 0.133\eta + 0.575\eta^2, \qquad (14)$$

$$I^2\omega_{\rm i}/M^5 = 0.00694 - 0.0256\eta^2, \qquad (15)$$

I-Love-Q: Extensions

- I-Q relations for rapidly rotating stars
  (Doneva et al 2014; Pappas & Aposolatos 2014; Chakrabarti et al. 2014; Yagi et al. 2014)

- I-Love relation during inspiral; Proto-neutron stars
  (Maselli et al. 2013; Martinon et al. 2014)

- Strong magnetic field (Haskell et al. 2014)

- Non-GR gravity theories
  (Yagi et al 2013; Sham et al. 2014, Pani & Berti 2014; Doneva et al. 2014; Kleihaus et al. 2014)

- Anisotropic stars, Gravastars (Yagi & Yunes 2015; Pani 2015)

- Possible origins (Yagi et al. 2014; Sham et al. 2015; Chan et al. 2015)

The list is most likely not complete!

EiBI gravity

The EiBI theory is based on a Palatini formulation of the action (Bañados & Ferreira 2010)

$$S = \frac{1}{16\pi}\frac{2}{\kappa}\int d^4x\left(\sqrt{|g_{\mu\nu} + \kappa R_{\mu\nu}|} - \lambda\sqrt{-g}\right) + S_M[g, \Psi_M], \quad (1)$$

Varying the action (Equation (1)) with respect to the metric $g_{\mu\nu}$ and $\Gamma^{\alpha}_{\beta\gamma}$ separately yields

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}, \quad (2)$$

$$\sqrt{-q}q^{\mu\nu} = \sqrt{-g}g^{\mu\nu} - 8\pi\kappa\sqrt{-g}T^{\mu\nu}, \quad (3)$$

where $q_{\mu\nu}$ is an auxiliary metric compatible with the connection:

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2}q^{\alpha\sigma}(\partial_\gamma q_{\sigma\beta} + \partial_\beta q_{\sigma\gamma} - \partial_\sigma q_{\beta\gamma}). \quad (4)$$