NEUTRON STAR RADII AND CRUSTS: UNCERTAINTIES AND UNIFIED EQUATIONS OF STATE

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HECOLS
Polish-French collaboration in astrophysics

compstar

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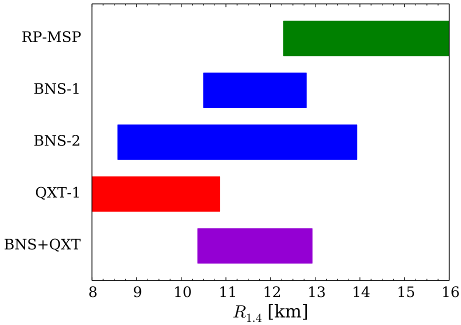
Astrophysical constraints: radius

Fitting the spectrum of

- X-ray emission from radio millisecond pulsars (RP-MSP);
- X-bursts from accreting NSs (BNS);
- the quiescent thermal emission of accreting NSs (QXT).



Summary

Based on most recent publications.
Adapted from Fortin et al. A&A (2015)

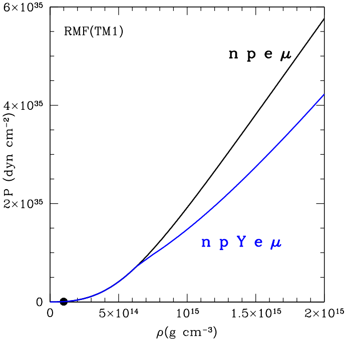
- RP-MSP: Bodganov, ApJ (2013)
- BNS-1: Nättilä et al. arXiv:1509.06561
- BNS-2: Güver & Özel, ApJ (2013)
- QXT-1: Guillot & Rutledge, ApJ (2014)
- BNS+QXT: Steiner et al., ApJ (2013)

Conclusion

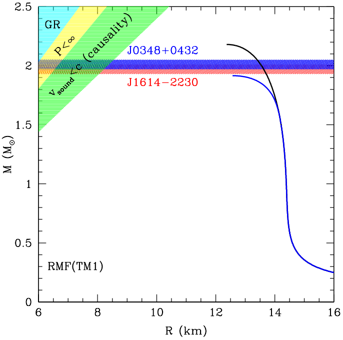
- marginally consistent (see QXT-1 and RP-MSP),
- many remaining uncertainties in the modelling,
- inclusion of rotation: effect $\simeq 10\%$.
- future X-ray telescopes (NICER, Athena, LOFT?): $M - R$ constraints with a precision of $\sim 5\%$

Context: equation of state (EoSs) and hyperons (Y)

Equation of state



$M - R$ plot



Hyperons

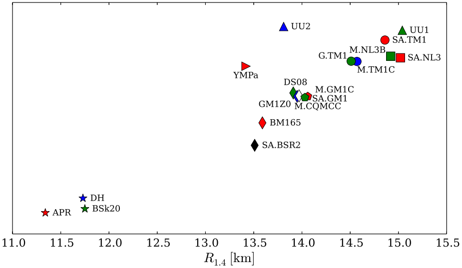
- reduce the pressure in the inner core. ie. softening of the EoS;
- reduce the maximum mass.

Claims that $M_{\max} \geq 2\,M_{\odot}$ rules out hyperonic equations of state ...

Hyperonic equations of state and radii

Fortin, Zdunik, Haensel and, Bejger, A&A (2015)

Radius of a 1.4 $M_\odot$ NS



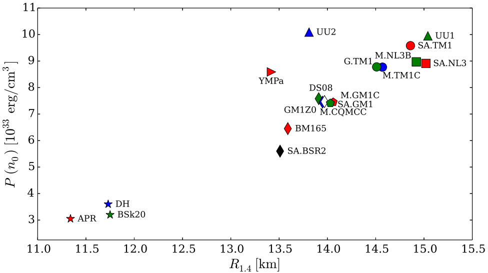
- 14 hyperonic with $M_{\rm max} \geq 2\,M_\odot$, all but one (Yamamoto et al. PRC 2014) are RMF models;
- 3 nucleonic as reference.

Hyperonic EOS:
for $M \in [1.0 - 1.6]\ M_\odot$, $R > 13$ km.

Hyperonic equations of state and radii

Fortin, Zdunik, Haensel and, Bejger, A&A (2015)

Pressure at $n_0 = 0.16\ \mathrm{fm}^{-3}$ near the core-crust transition



- large radius for Y.EoSs correlated with a large pressure at n_0.

$\rightarrow$ $M_{\max} \geq 2\, M_\odot$ possible if the decrease in the pressure at high density due to Y is compensated by a large pressure at low density.

Hyperonic equations of state and radii

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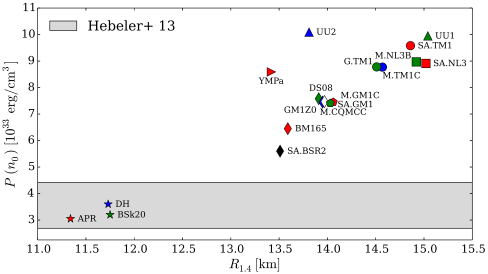


- large radius for Y.EoSs correlated with a large pressure at n_0.

→ $M_{\max} \geq 2\,M_\odot$ possible if the decrease in the pressure at high density due to Y is compensated by a large pressure at low density.

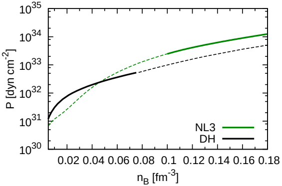
- gray strip: chiral effective field theory calculations up to n_0 (Hebeler et al. ApJ 2013).
- over-pressure at n_0 for hyperonic EOS inconsistent with this constraint.

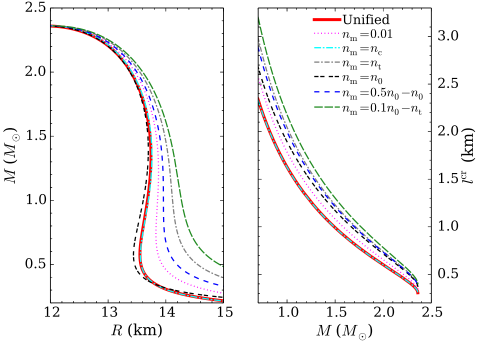
Recent work

Oertel et al. JPG (2015): hyperonic EoS consistent with Hebeler et al. constraint and with $M_{\max} \geq 2\,M_\odot$.

How to glue core and crust: NL3 & DH?

Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, arXiv:1604.01944





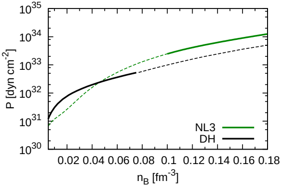
- core glued to BPS+BBP EOS at 0.01 fm^{-3};
- transition at the crossing density between the 2 EoSs;
- transition at the core-crust transition density n_t;
- transition at $n_0 = 0.16$ fm^{-3};
- crust below $0.5n_0$ and core above n_0;
- crust below $0.1n_0$ and core above n_t;
- reference: unified EoS.

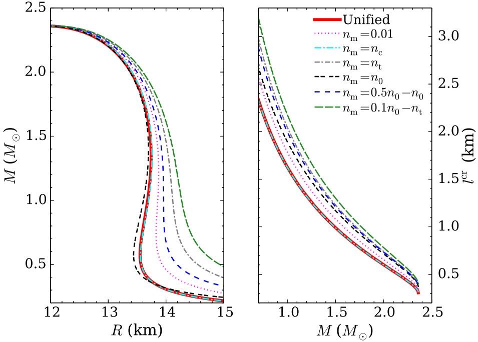
Uncertainty on R

- due to the treatment of the core-crust transition: up $\sim$ 4% (up to $\sim$ 30% on the crust thickness),
- decreases if crust and core EOS with similar saturation properties.

How to glue core and crust: NL3 & DH?

Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, arXiv:1604.01944





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Uncertainty on R

- due to the treatment of the core-crust transition: up $\sim 4\%$
- with NICER, Athena or LOFT(?): expected precision $\sim 5\%$ ....
- how to, if not solve, at least handle this problem?

1. Thermodynamically consistent 'gluing'

Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, arXiv:1604.01944

EOS as a function of n

- crust EOS: $P_{\rm cr}(n),\ \rho_{\rm cr}(n)$
- core EOS: $P_{\rm co}(n),\ \rho_{\rm co}(n)$
- matching between n_1 and $n_2 > n_1$.

Matched EOS: $P(n),\ \rho(n)$

- for $n < n_1$, $P(n) = P_{\rm cr}(n)$
- for $n > n_2$, $P(n) = P_{\rm co}(n)$
- for $n_1 < n < n_2$, assume $P(n)$ with $P(n_1) = P_{\rm cr}(n_1)$ & $P(n_2) = P_{\rm co}(n_2)$.
- for $n < n_1$, $\rho(n) = \rho_{\rm cr}(n)$
- for $n_1 < n < n_2$, $\rho(n) = n\mu(n) - P(n)$ with $\mu(n) = \mu_1 + \int_{n_1}^{n} \frac{{\rm d}P(n)}{n}$
- for $n > n_2$, $\rho(n) = \rho_{\rm co}(n) + n(\mu(n_2) - \mu_{\rm co}(n_2))$
- affect the $M - R$ relation

EOS as a function of ρ

- crust EOS: $P_{\rm cr}(\rho),\ n_{\rm cr}(\rho)$
- core EOS: $P_{\rm co}(\rho),\ n_{\rm co}(\rho)$
- matching between ρ_1 and $\rho_2 > \rho_1$.

Matched EOS: $P(\rho),\ n(\rho)$

- for $\rho < \rho_1$, $P(\rho) = P_{\rm cr}(\rho)$
- for $\rho > \rho_2$, $P(\rho) = P_{\rm co}(\rho)$
- for $\rho_1 < \rho < \rho_2$, assume $P(\rho)$ with $P(\rho_1) = P_{\rm cr}(\rho_1)$ & $P(\rho_2) = P_{\rm co}(\rho_2)$.
- for $\rho < \rho_1$, $n(\rho) = n_{\rm cr}(\rho)$
- for $\rho_1 < \rho < \rho_2$, $n(\rho) = n_1 \exp\left(\int_{\rho_1}^{\rho} \frac{{\rm d}\rho}{P(\rho)+\rho}\right)$.
- for $n > n_2$, $n(\rho) = n_{\rm co}(\rho) n(\rho_2)/n_{\rm co}(\rho_2)$.
- affect the model of dense matter

2. Approximate formula for the radius and crust thickness

Zdunik, Fortin, and Haensel, in prep.

How?

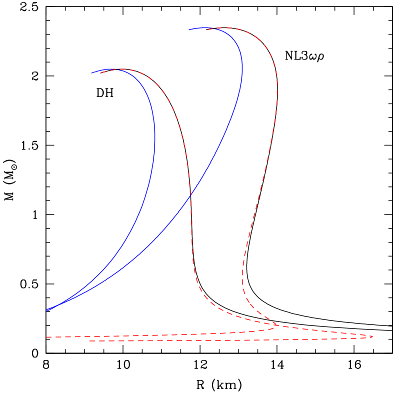
- All you need is . . . : the core EOS down to a chosen density $n_{\rm b}$ with $\mu(n_{\rm b}) = \mu_{\rm b}$.
- Obtain the $M(R_{\rm core})$ relation solving the TOV equations.
- Obtain $M(R)$ with

  $$R = R_{\rm core} \Big/ \left(1 - (\frac{\mu_{\rm b}^2}{\mu_0^2} - 1)(\frac{R_{\rm core}c^2}{2GM} - 1)\right)$$

  where $\mu_0 = \mu(P = 0) = 930.4$ MeV - minimum energy per nucleon of a bcc lattice of ^{56}Fe.

Results

- uncertainty in the radius: $\lesssim 1\%$ for $M > 1\ M_\odot$
- uncertainty in the crust thickness: $\sim 1\%$ for $M > 1\ M_\odot$



Solution of the TOV equation with a unified EoS
TOV solution for the core $M(R_{\rm core})$
Approximate $M(R)$ for $n_{\rm b} = 0.077$ fm^{-3}

2. Approximate formula for the radius and crust thickness

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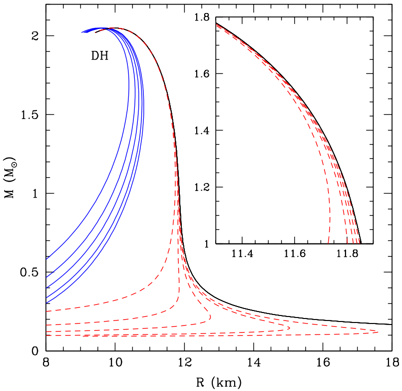


Solution of the TOV equation with a unified EoS
TOV solution for the core $M(R_{\mathrm{core}})$
Approximate $M(R)$ for $n_{\mathrm{b}} = 0.16, 0.13, 0.11, 0.09, 0.077$ fm^{-3} from left to right.

3. Unified equations of state

Very few unified EoSs for NSs exist eg. DH (Douchin & Haensel 2001), BSk (Brussels Uni.)

Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, arXiv:1604.01944

9 RMF models

NL3, NL3$_{\omega\rho}$, DDME2, GM1, TM1, DDHδ, DD2, BSR2, and BSR6 with

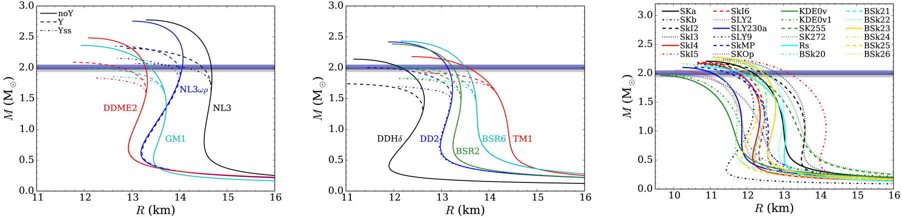
- outer-crust non consistently calculated but hardly affect the $M-R$ relations
- inner-crust with pasta phase from Thomas-Fermi calculations
- noY: a purely nucleonic core
- Y: a transition to hyperonic matter in the core: SU(6) with the ϕ meson; $U_\Lambda^N(n_0) = -28$ MeV, $U_\Sigma^N(n_0) = 30$ MeV, $U_\Xi^N(n_0) = -18$ MeV
- Yss: a transition to hyperonic matter in the core: SU(6) with the ϕ and σ^* mesons; $U_\Lambda^\Lambda(n_0) = -5$ MeV, $U_\Xi^\Xi \simeq 2U_\Lambda^\Lambda$, $g_{\sigma^*\Sigma} = g_{\sigma^*\Lambda}$

24 Skyrme models

SKa, SKb, SkI2, SkI3, SkI4, SkI5, SkI6, Sly2, Sly230a, Sly9, SkMP, SKOp, KDE0V, KDE0V1, SK255, SK272, Rs, BSk20, BSk21, BSk22, BSk23, BSk24, BSk25, and BSk26 with

- purely nucleonic core, causal up to 2 $M_\odot$
- compressible liquid drop model
- no shell effect and curvature terms

3. Unified equations of state



33 nucleonic EoSs and 17 hyperonic EoSs

- tables with n, ρ, P as supplemental material to the paper (for observers mainly)
- available on the open-source CompOSE database: `http://compose.obspm.fr/`

Fits by piecewise polytropes à la Read et al. PRD (2009)

In progress

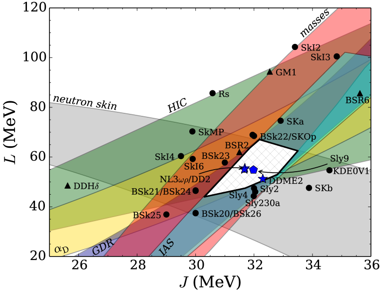
Potential applications to:

- I-Love-Q relations
- modelling of binary neutron star systems

with different crust models and thus consistent models.

Comparison with nuclear constraints

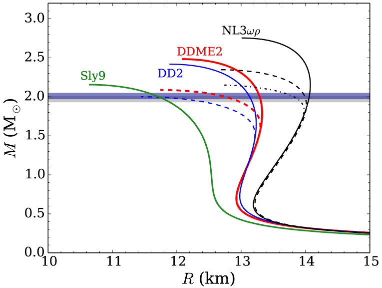
a. Low-density constraints from Hebeler et al. ApJ (2013): chiral effective field theory; Gandolfi et al. PRC (2012): Quantum Monte Carlo technique
b. Incompressibility: $K = 230 \pm 40$ MeV
c. $L - J$ constraints see eg. Lattimer & Steiner, EPJA (2015)



EOS fulfilling:

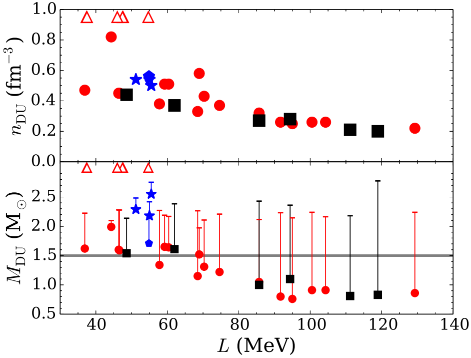
- all constraints: DDME2
- constraint a$\pm$10%+ constraints b+c:
  DD2, NL3$\omega\rho$ and Sly9.

$R_{1.4} = 13.10 \pm 0.65$ km.



Nucleonic DUrca process

- $n \to p + e^- + \bar{\nu}_e$ and $p + e^- \to n + \nu_e$
- momentum conservation $\to$ density $n_{\rm DU}$ and mass $M_{\rm DU}$ threshold



Additional DUrca processes for hyperonic EOS.

- Beznogov & Yakovlev MNRAS (2015): DUrca process needed to explain the thermal emission of isolated and accreting NS.
- For $L \gtrsim 70$ MeV, DUrca process always on for $M > 1.5\,M_\odot$.
- For $L \lesssim 70$ MeV, EOS with DUrca and others without.
- $L - J$ plane: the intersection of all constraints gives $L \lesssim 70$ MeV.
- Popov et al. A&A (2006): population synthesis of isolated NS requires $M_{\rm DU} > 1.5\,M_\odot$.

Conclusions

- Most hyperonic EoSs consistent with 2 $M_\odot$ have a large $R_{1.4}$ and overpressure close to saturation density.
- Treatment of the gluing of non-unified core and crust EoSs introduces an uncertainty on the radius that can be as large as the expected precision from NICER, Athena or LOFT(?).
- Approximate formula for $M(R)$ as a function of $M(R_{\rm core})$.
- Development of unified nucleonic and hyperonic EoSs based on 9 RMF and 24 Skyrme models;
- available on the CompOSE database: `http://compose.obspm.fr` and as supplemental material to the paper;
- confrontation with nuclear constraints and selection of 4 EoS.

Perspectives

- calculation of fits by piecewise polytropes for various applications,
- study of rotating NS (Keplerian frequency, minimum mass, ...) with `LORENE` and of the surface gravitational redshift (spectral lines),
- development of more EOS consistent with Hebeler et al. constraint and 2 $M_\odot$.