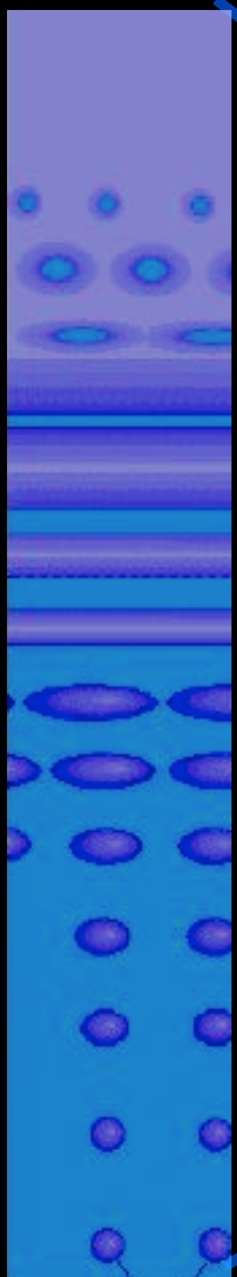
An Empirical Equation of State and its applications in nuclear physics and astrophysics



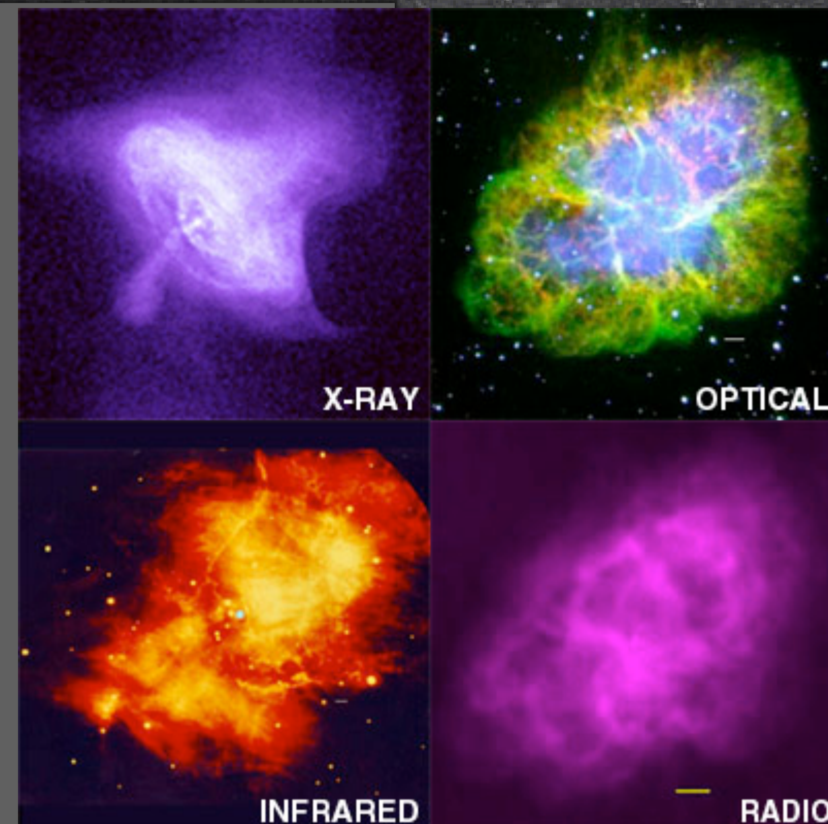
Debarati Chatterjee
LPC/ENSICAEN,
Caen, Normandy, France

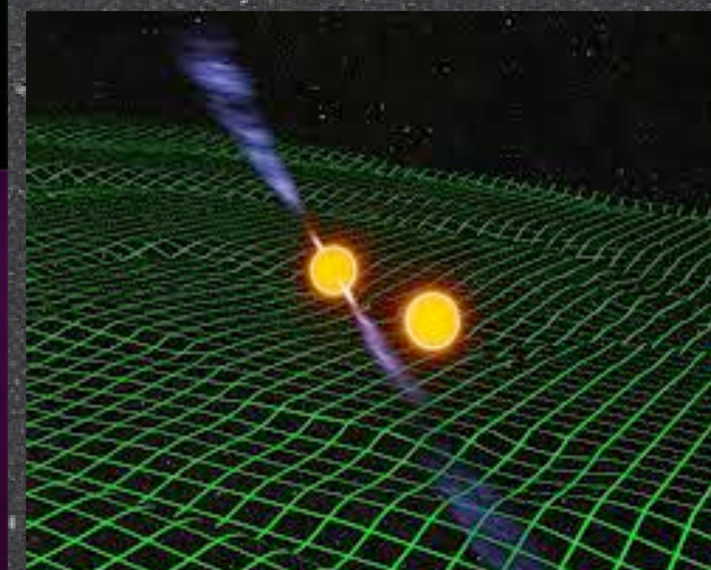
Collaborators:

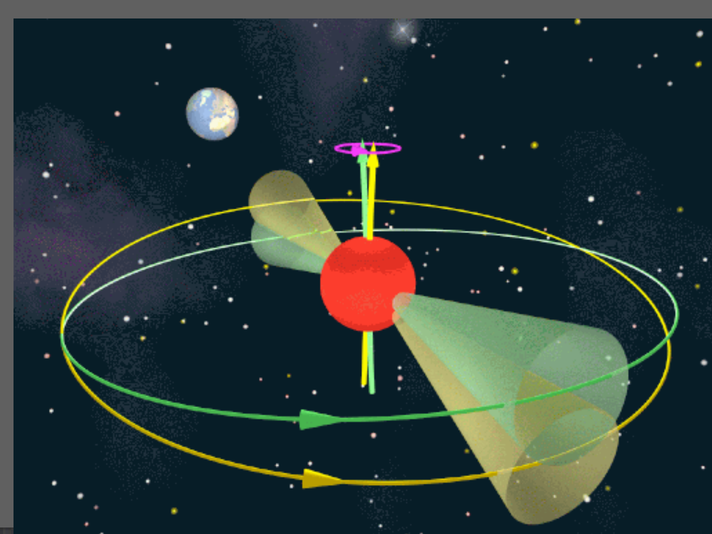
Francesca Gulminelli
Jerome Margueron

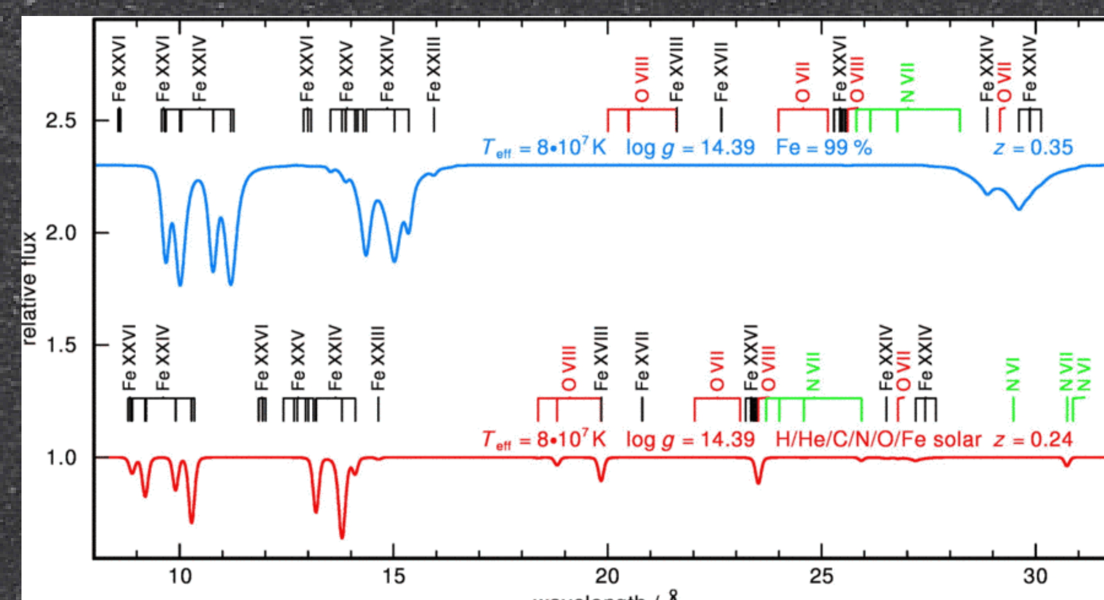
NS: Theory vs Observations

- *Mass M*
- *Radius R*
- *Moment of inertia I~MR²*
- *Gravitational redshift z~M/R*
- *Period, Period derivative*

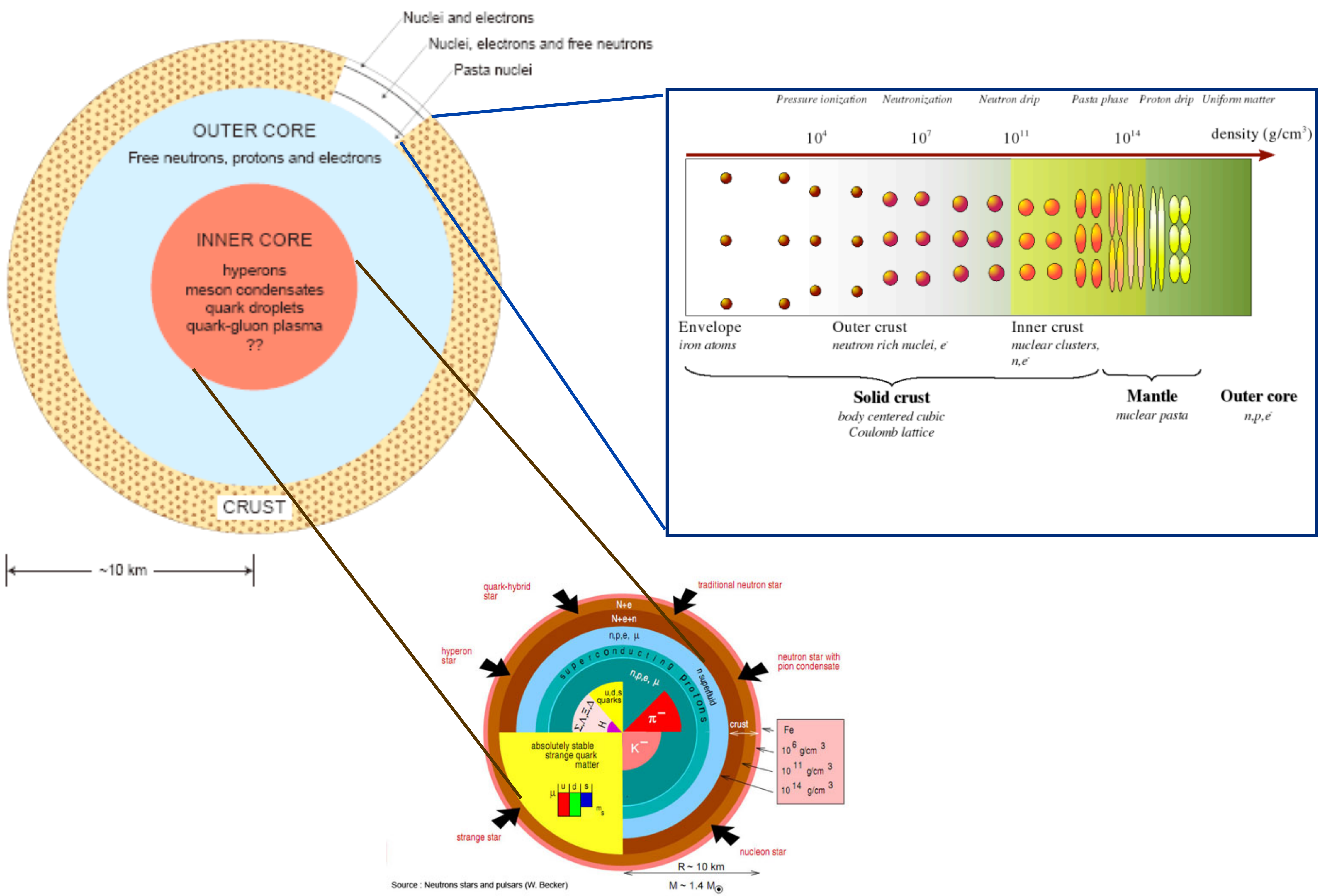




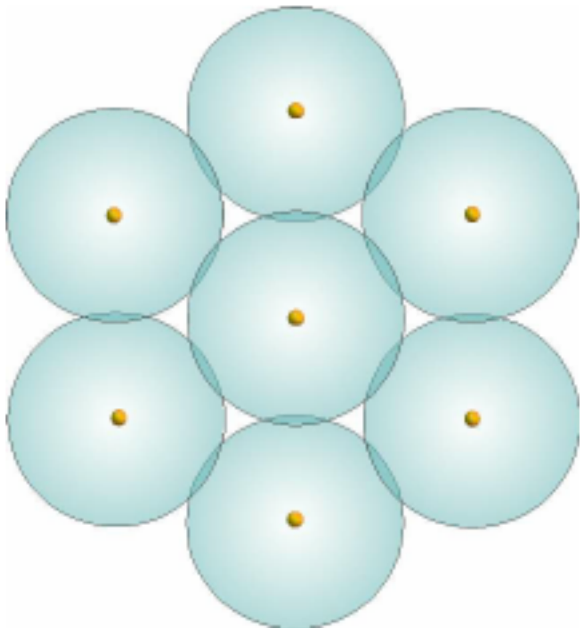




MODELING NS EOS: INTERNAL STRUCTURE

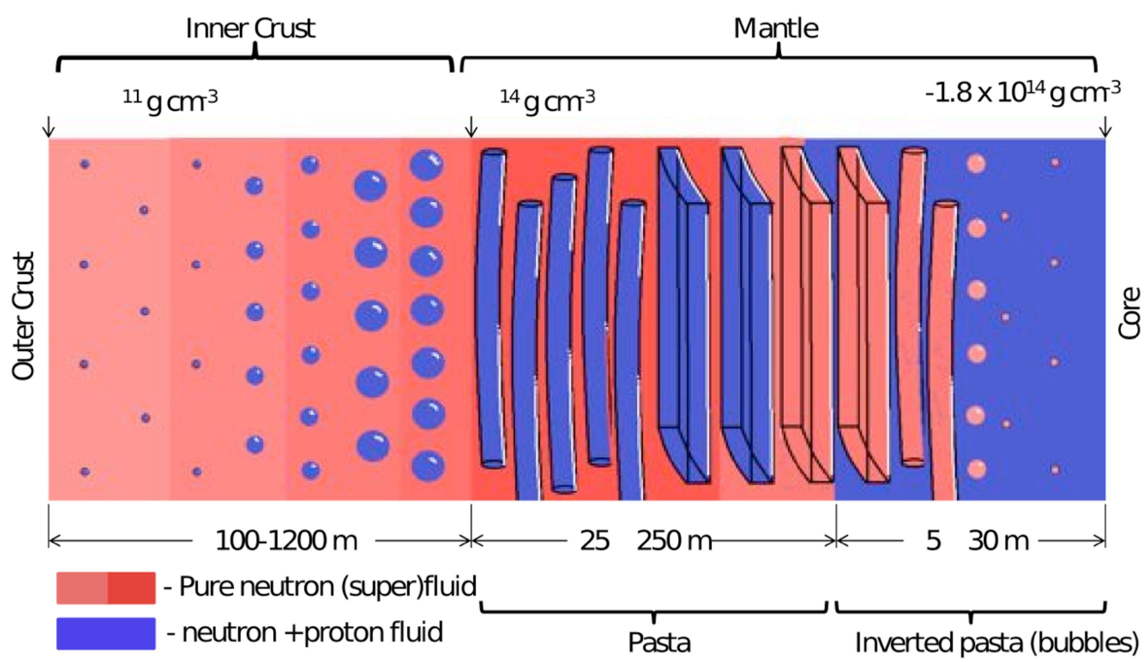


MODELING NS CRUST & CORE



Outer crust

- *nuclei in lattice + e gas*
- *each lattice volume represented by a Wigner-Seitz cell*
- *Each cell assumed to be charge neutral, Coulomb interaction between cells neglected*
- *electrons uniformly distributed*
- *system in chemical equilibrium*
- *Composition largely sensitive to experimental determination of nuclear masses [Audi tables]*



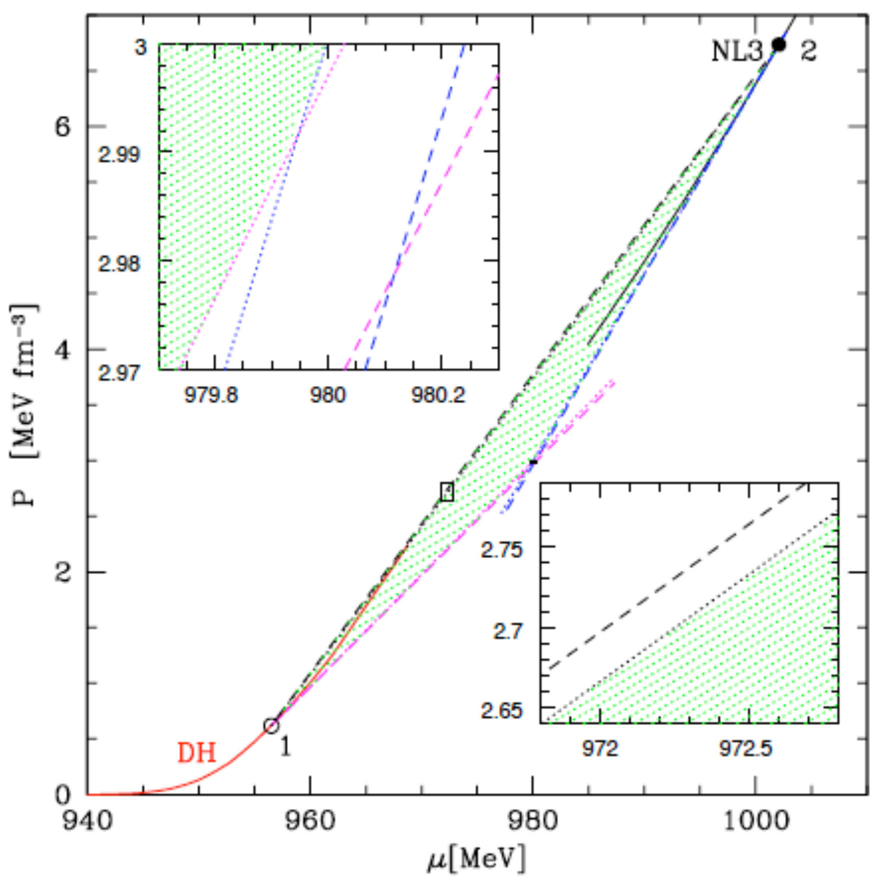
Inner crust

- *as density increases, nuclei become n rich*
- *Beyond density $4x10^{11}$ g/cm^3 (n drip density), n drip out of nuclei*
- *nuclear clusters + e gas + n gas*
- *At T=0, ground state energy: E minimization*
- *Composition dominated by clusters beyond current experimental data*

Core

- *Homogeneous liquid composed of n,p,e (exotic matter) in chemical equilibrium*

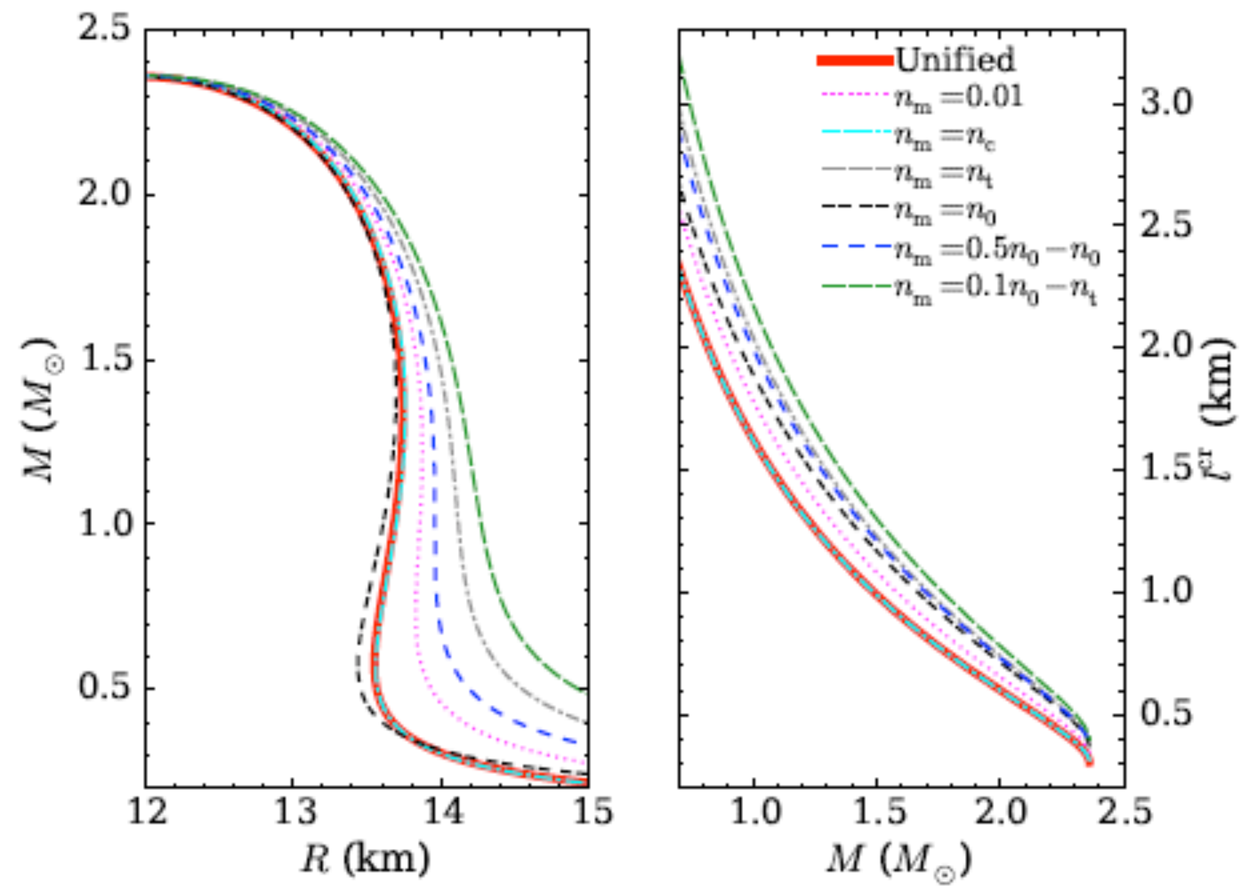
WHY WE NEED A UNIFIED EOS



Fortin et al., ArXiv: 2016

Crust-Core Matching for non-unified EoSs

- Matching is done so that pressure is an increasing function of energy density
- different models leads to arbitrary results
- uncertainty in crust thickness upto 30% and for radius 4%

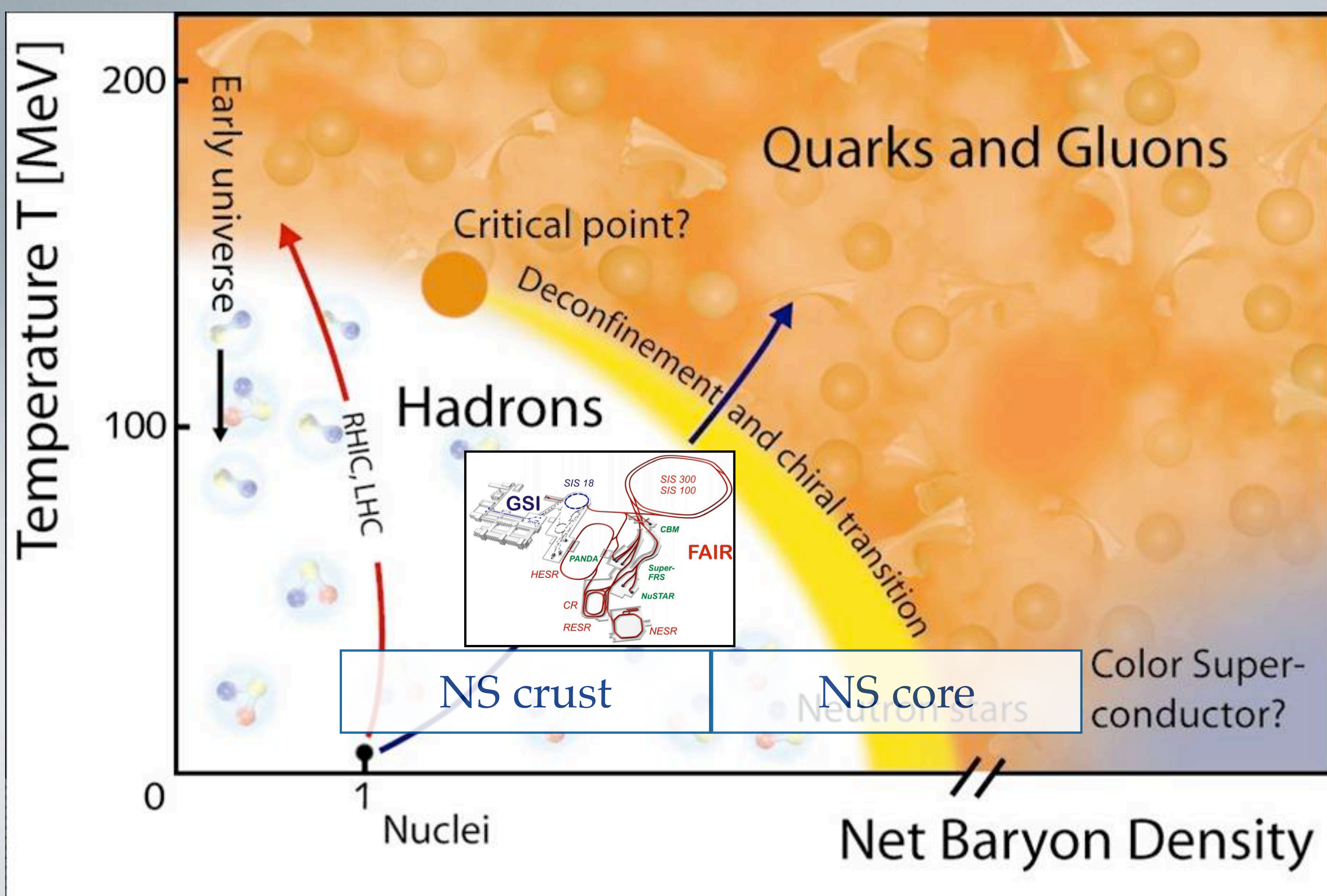


thermonuclear explosion
Photo Credit: David A. Hardy & PPARC

MOTIVATION

- *study of the properties of NS crust-core transition is crucial for understanding astrophysical phenomena such as:*
  - Cooling of neutron stars
  - Pulsar glitches
  - X-ray Bursts and Superbursts in accreting neutron stars
  - Moment of inertia
  - R-modes

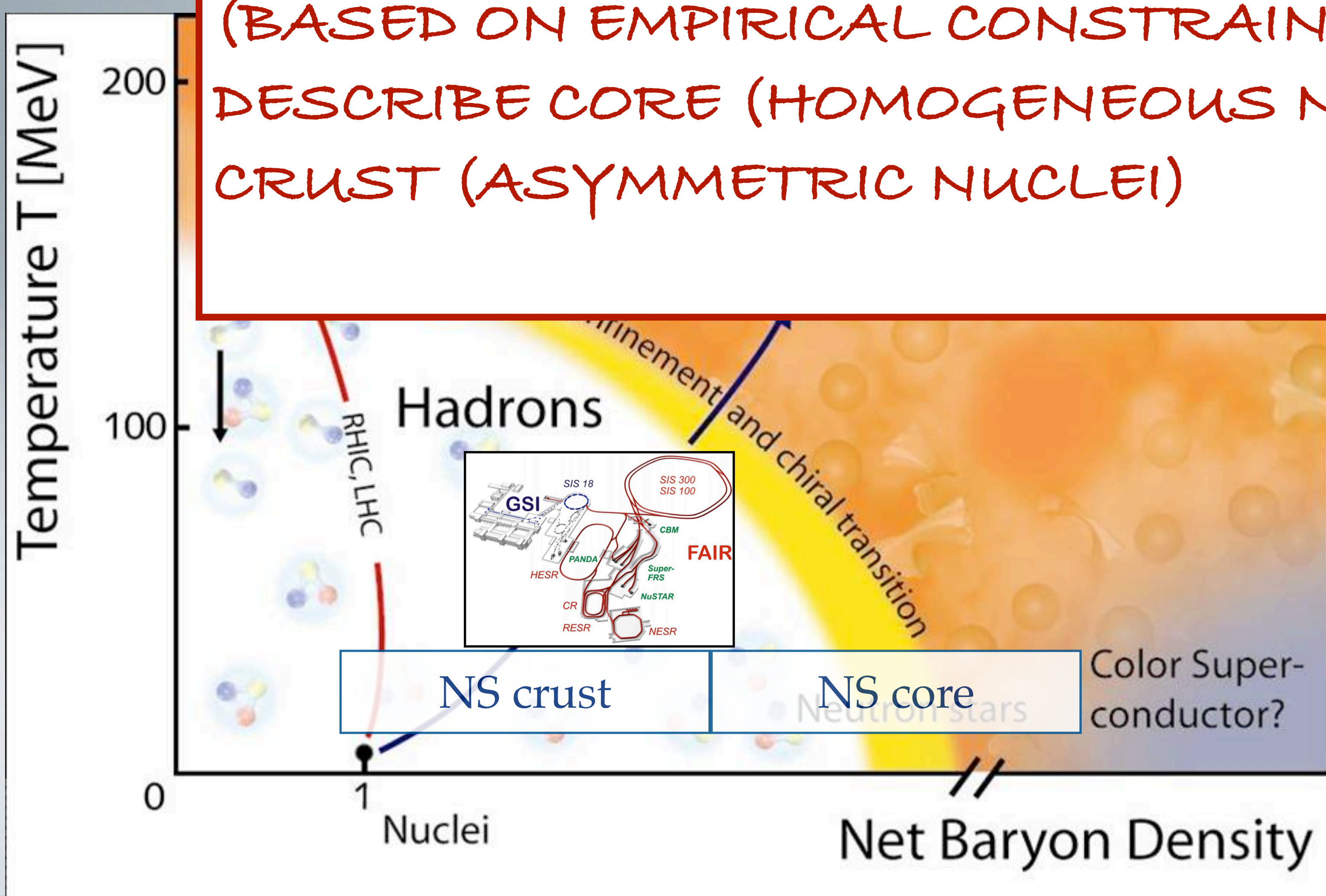
- *outer layers of neutron stars with subnuclear density* $n < 0.16\ fm^{-3}$
  => empirical constraints from terrestrial nuclear physics



MOTIVATION

- *study of the properties of NS crust-core transition is crucial for understanding astrophysical phenomena such as:*

AIM OF THIS WORK: TO CONSTRUCT A MODEL INDEPENDENT (BASED ON EMPIRICAL CONSTRAINTS) "UNIFIED" EOS TO DESCRIBE CORE (HOMOGENEOUS NUCLEAR MATTER) + CRUST (ASYMMETRIC NUCLEI)



- *outer layers of neutron stars with subnuclear density $n < 0.16\ fm^{-3}$ => empirical constraints from terrestrial nuclear physics*

EMPIRICAL EOS COEFFICIENTS

Nuclear matter model (ab-initio, pheno, empirical)

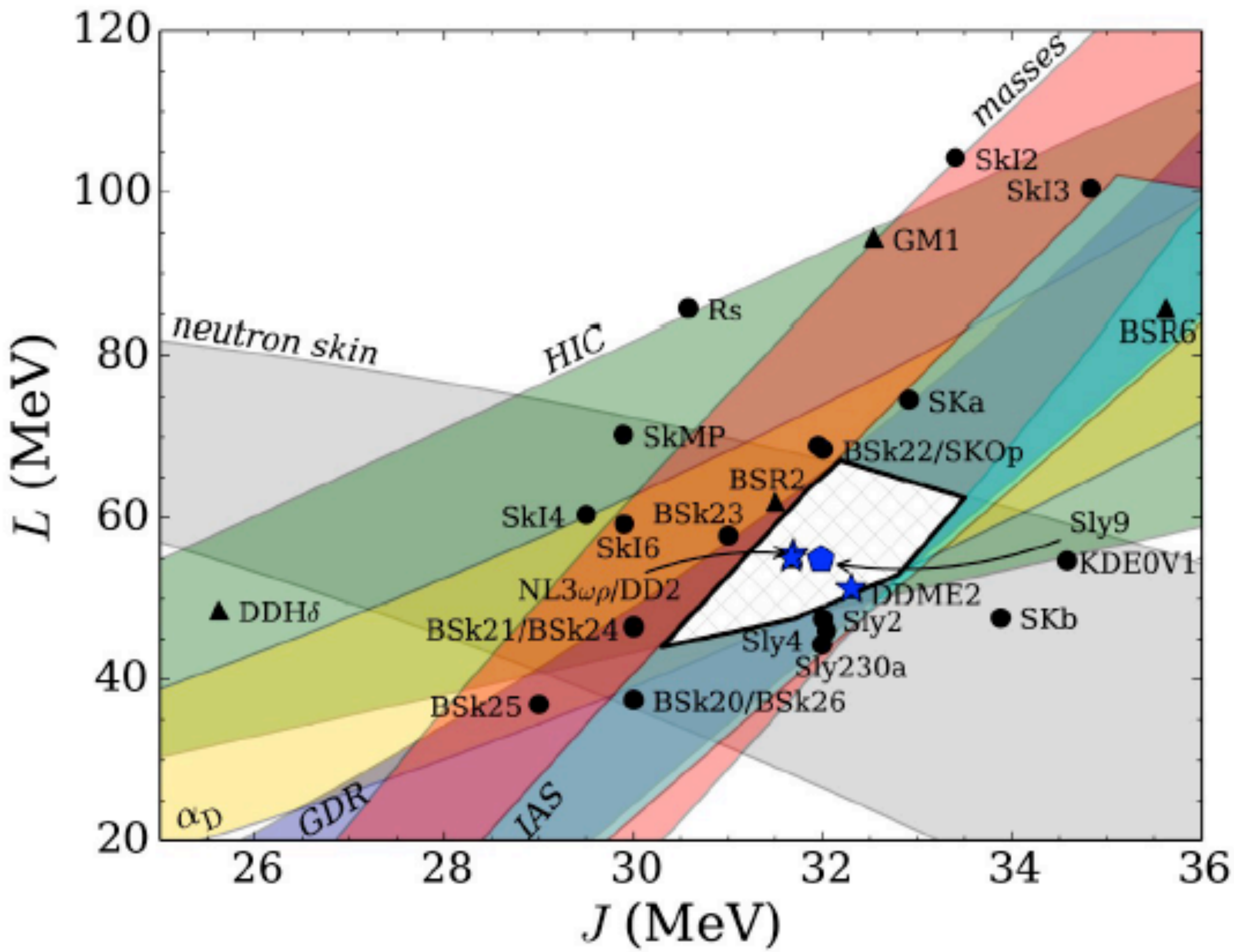
$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

$$x = \frac{\rho - \rho_0}{\rho_0}$$

$$\begin{aligned} e(\rho_n, \rho_p) &= e(\rho, \delta) \\ &= e_{IS}(\rho) + e_{IV}(\rho)\delta^2 + O(\delta^3) \\ &= \left(\boldsymbol{E_0} + \frac{1}{18}\boldsymbol{K_0}x^2 + O(x^3)\right) + \left(\boldsymbol{J_0} + \frac{1}{3}\boldsymbol{L}x + \frac{1}{18}\boldsymbol{K_{sym}}x^2 + O(x^3)\right)\delta^2 \end{aligned}$$

- Around saturation: If $(E_0, \rho_0, K_0, J_0, L, K_{sym})$ are known, the EoS is known
- Above saturation: it is the same, but you need more coefficients

EXPERIMENTAL CONSTRAINTS

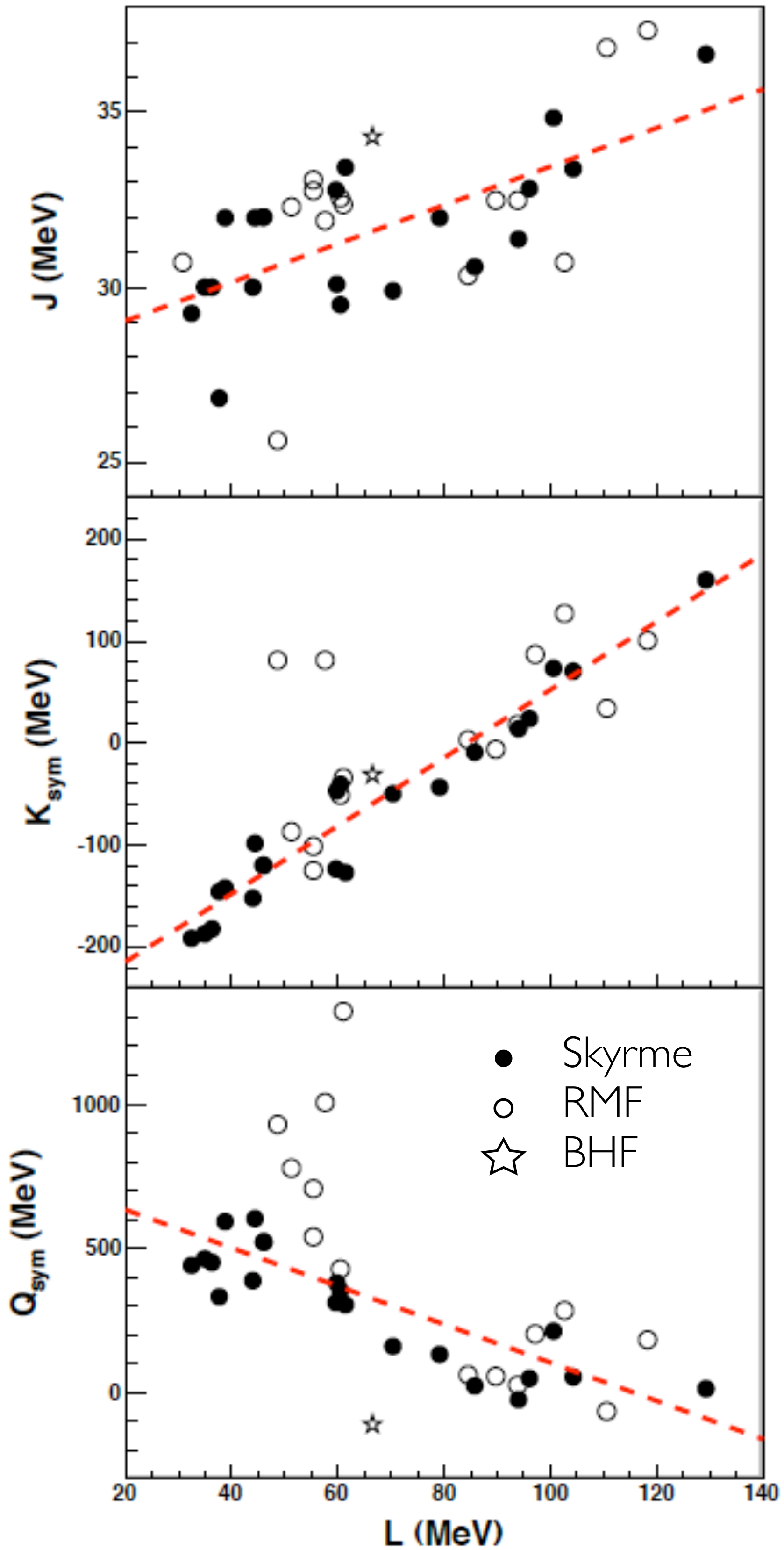


Constraints in J-L plane

- From n skin thickness of ^{208}Pb
- From HIC
- From electric dipole polarizability α_D
- From giant dipole resonance (GDR) of ^{208}Pb
- From measured nuclear masses
- From isobaric analog states (IAS)

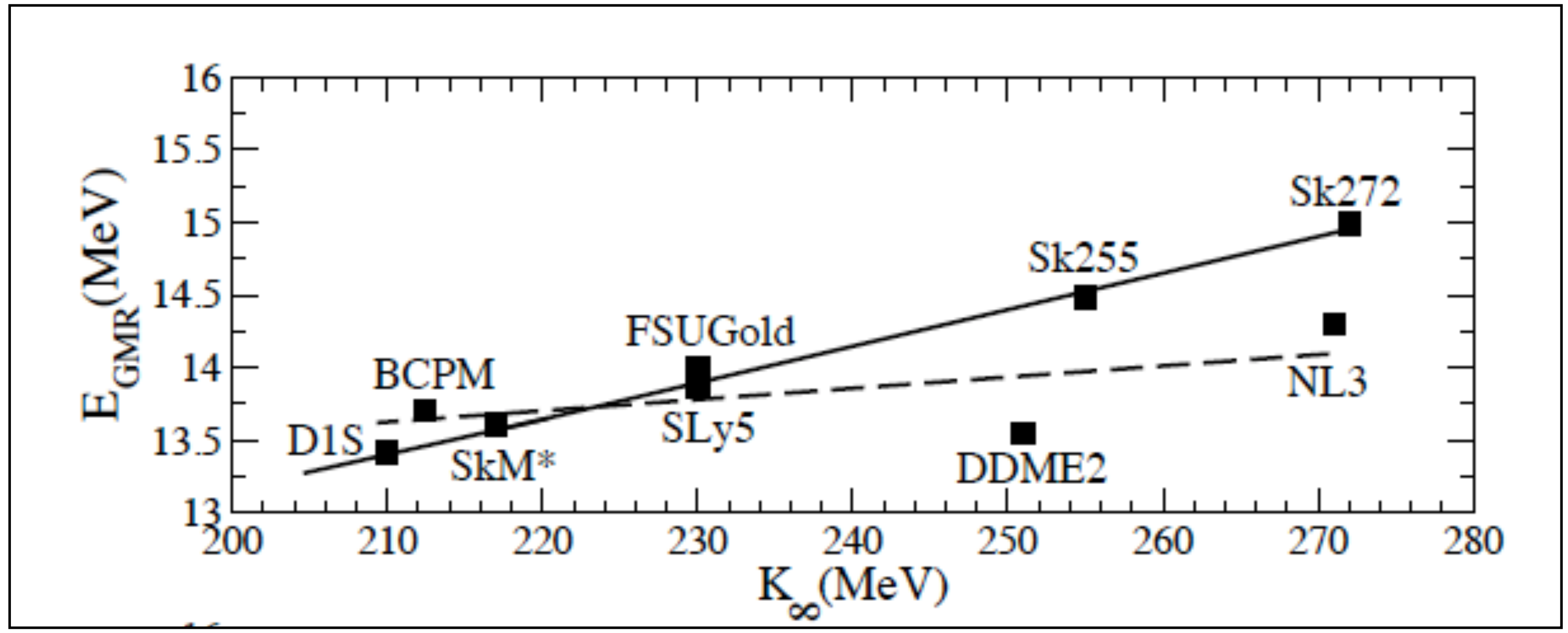
Fortin et al., ArXiv: 2016

WHY WE NEED A MODEL INDEPENDENT EOS



Ducoin et al. PRC 2011

Spurious correlations



Khan Margueron PRC 2013

e.g.: Skyrme interactions

$$e_{IV}(\rho) = e_{IV}^{(kin)}(\rho) + e_{IV}^{(eff)}(\rho) + D_0\rho + D_3\rho^{1+\sigma}$$

$$\mathrm{L} = 3\rho_0 \frac{de_{IV}}{d\rho}\bigg]_{\rho_0} = L_{kin} + L_{eff} + 3D_0\rho_0 + 3(1+\sigma)D_3\rho_0^{1+\sigma}$$

$$K_{sym} = 9\rho_0^2 \frac{d^2 e_{IV}}{d\rho^2}\bigg]_{\rho_0} = K_{kin} + K_{eff} + 9\sigma(1+\sigma)D_3\rho_0^{1+\sigma}$$

A Unified Model

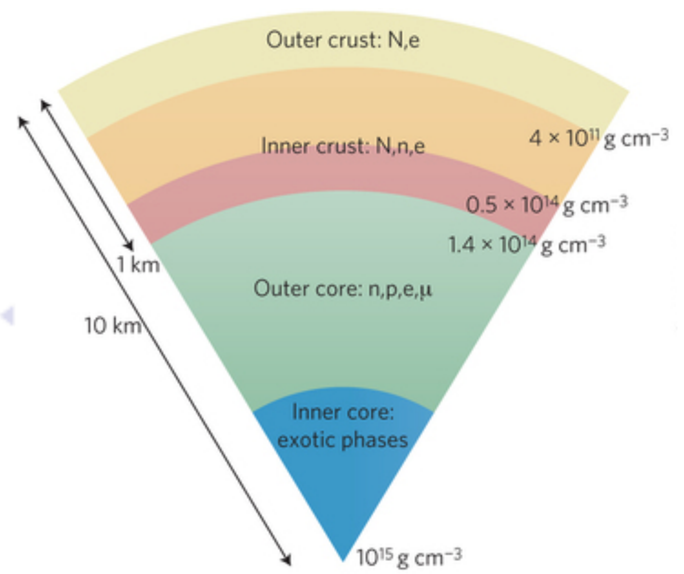
- In density functional theory, the energy density functional

$$\mathcal{H} = \mathcal{H}\left[\rho_q(\vec{r}), \nabla^k_{q'}\rho_q(\vec{r})\right]$$

  is a function of the densities ρ_q and their gradients.
- In general, there are an infinite number of gradients
- If $k = 0 \implies$ Thomas-Fermi approximation (density terms only): applicable only to homogeneous infinite nuclear matter
- For finite nuclei (Inhomogeneous matter), non-zero k (say 2) $\implies$ Extended-Thomas-Fermi approximation of order k
- Separated into isoscalar and isovector parts

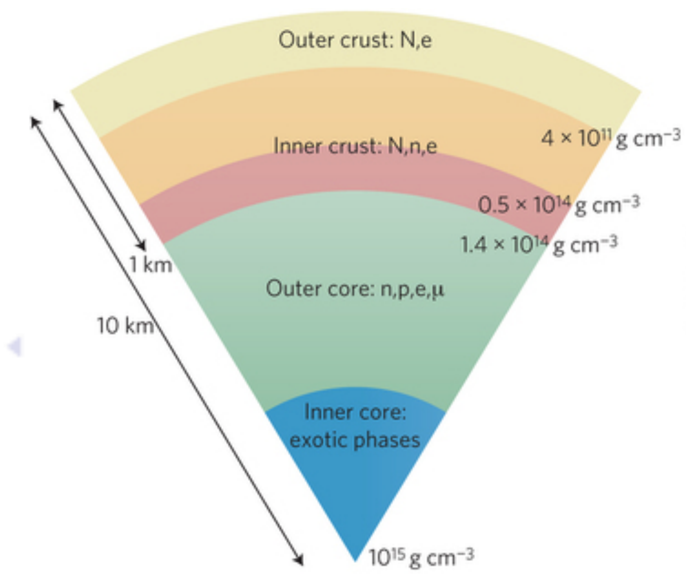
$$\mathcal{H}_{ETF} = \mathcal{H}_{IS} + \mathcal{H}_{IV}\delta^2$$

  where the isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$



A Unified Model

- Each of these contributions consists of kinetic and potential terms
$$\mathcal{H}_{IS,IV} = \mathcal{K} + \mathcal{V}$$
where the potential part can be further separated into contributions from the density-dependent term, the finite range term and the spin-orbit term respectively
$$\mathcal{V} = \mathcal{H}_{\rho} + \mathcal{H}_{fin} + \mathcal{H}_{so}$$
- $\mathcal{K}$ and $\mathcal{H}_{\rho}$ can be determined using empirical constraints
- $\mathcal{H}_{fin}$ and $\mathcal{H}_{so}$ require additional experimental inputs C_{fin} and C_{so}



A Unified Model

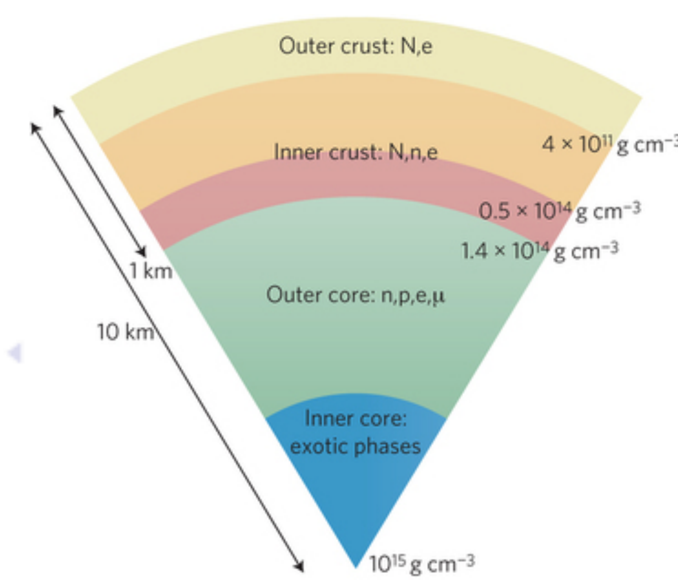
- Each of these contributions consists of kinetic and potential terms

$$\mathcal{H}_{IS,IV} = \mathcal{K} + \mathcal{V}$$

where the potential part can be further separated into contributions from the density-dependent term, the finite range term and the spin-orbit term respectively

$$\mathcal{V} = \mathcal{H}_{\rho} + \mathcal{H}_{fin} + \mathcal{H}_{so}$$

- $\mathcal{K}$ and $\mathcal{H}_{\rho}$ can be determined using empirical constraints
- $\mathcal{H}_{fin}$ and $\mathcal{H}_{so}$ require additional experimental inputs C_{fin} and C_{so}



Homogeneous Nuclear Matter (HNM)

- For infinite homogeneous matter, the finite size effects and spin-orbit terms can be neglected
- Energy per particle $e = \mathcal{H}/\rho$ for HNM : expansion around saturation density ρ_0 in terms of $x = \frac{\rho - \rho_0}{3\rho_0}$

$$e(x,\delta) = e_{kin}(x,\delta) + e_{pot}(x,\delta)$$

- The kinetic part in terms of Fermi gas kinetic energy t_0^{FG}

$$e_{kin}(x,\delta) = \frac{1}{2} t_0^{FG} (1+3x)^{2/3} \left[(1+\delta)^{5/3} \frac{m}{m_n^*} + (1-\delta)^{5/3} \frac{m}{m_p^*} \right]$$

- while the potential part can be written as an expansion

$$e_{pot}(x,\delta) = \sum_{\alpha=0}^{2} (a_{\alpha 0} + a_{\alpha 2}\delta^2) \frac{x^\alpha}{\alpha!}$$

Homogeneous Nuclear Matter (HNM)

- For infinite homogeneous matter, the finite size effects and spin-orbit terms can be neglected
- Energy per particle $e = \mathcal{H}/\rho$ for HNM : expansion around saturation density ρ_0 in terms of $x = \frac{\rho-\rho_0}{3\rho_0}$

$$e(x,\delta) = e_{kin}(x,\delta) + e_{pot}(x,\delta)$$

- The kinetic part in terms of Fermi gas kinetic energy t_0^{FG}

$$e_{kin}(x,\delta) = \frac{1}{2}t_0^{FG}(1+3x)^{2/3}\left[(1+\delta)^{5/3}\frac{m}{m_n^*} + (1-\delta)^{5/3}\frac{m}{m_p^*}\right]$$

- while the potential part can be written as an expansion

$$e_{pot}(x,\delta) = \sum_{\alpha=0}^{2}(a_{\alpha 0} + a_{\alpha 2}\delta^2)\frac{x^\alpha}{\alpha!}$$

An Empirical EoS

- The kinetic and potential terms can be regrouped in terms of isospin asymmetry

$$\frac{E}{A} = e = e_{IS} + e_{IV}\delta^2 + \mathcal{O}(\delta^4) \tag{1}$$

- The density dependence of SNM can be expanded around ρ_0, in terms of $x = \frac{\rho-\rho_0}{3\rho_0}$

$$e_{IS} = E_0 + \frac{K_0}{2}x^2 + \mathcal{O}(x^3) \tag{2}$$

where $K_0 = \left.\frac{\partial^2 e}{\partial x^2}\right|_{x=0,\delta=0}$

An Empirical EoS

- Similarly, symmetry energy around saturation can be written as

$$\boxed{e_{IV}} = J_{sym} + L_{sym}x + \frac{K_{sym}}{2}x^2 + \mathcal{O}(x^3) \qquad (3)$$

where

symmetry energy : $J_{sym} = e_{IV}(x=0) = \frac{1}{2}\frac{\partial^2 e}{\partial \delta^2}\Big|_{\delta=0}$

slope of the symmetry energy : $L_{sym} = \frac{\partial J_{sym}}{\partial x}\Big|_{x=0}$

curvature of symmetry energy : $K_{sym} = \frac{\partial^2 J_{sym}}{\partial x^2}\Big|_{x=0}$

- In terms of empirical parameters, the energy per particle of ANM is

$$e = E_0 + J_{sym}\delta^2 + L_{sym}\delta^2 x + \frac{1}{2}(K_0 + K_{sym}\delta^2)x^2 \qquad (4)$$

Homogeneous Nuclear Matter (HNM)

- One can determine the coefficients of this expansion in terms of empirical parameters $(\rho_0, E_0, K_0, J_{sym}, L_{sym}, K_{sym})$

$$
\begin{aligned}
t_0^{FG} &= \frac{3}{10m}\left(\frac{3\pi^2\rho_0}{2}\right)^{2/3} \\
a_{00} &= E_0 - t_0^{FG}(1+\bar{m}) \\
a_{10} &= -t_0^{FG}(2+5\bar{m}) \\
a_{20} &= K_0 - 2t_0^{FG}(5\bar{m}-1) \\
a_{02} &= J_{sym} - \frac{5}{9}t_0^{FG}\left[1+(\bar{m}+3\bar{\Delta})\right] \\
a_{12} &= L_{sym} - \frac{5}{9}t_0^{FG}\left[2+5(\bar{m}+3\bar{\Delta})\right] \\
a_{22} &= K_{sym} - \frac{10}{9}t_0^{FG}\left[-1+5(\bar{m}+3\bar{\Delta})\right]
\end{aligned}
$$

- $\bar{m}$ and $\bar{\Delta}$ are related to in-medium effective mass and isospin splitting of nucleon masses

PRESENT UNCERTAINTY IN EMPIRICAL PARAMETERS

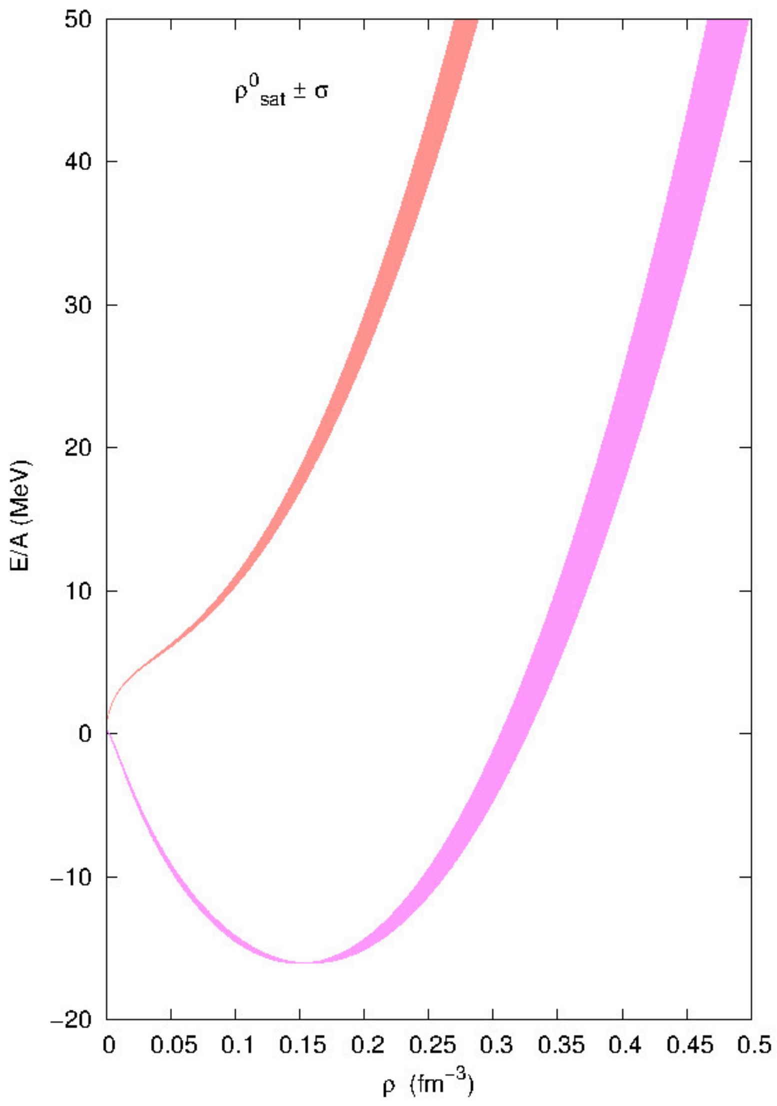
Margueron, Casali, Gulminelli (in preparation)

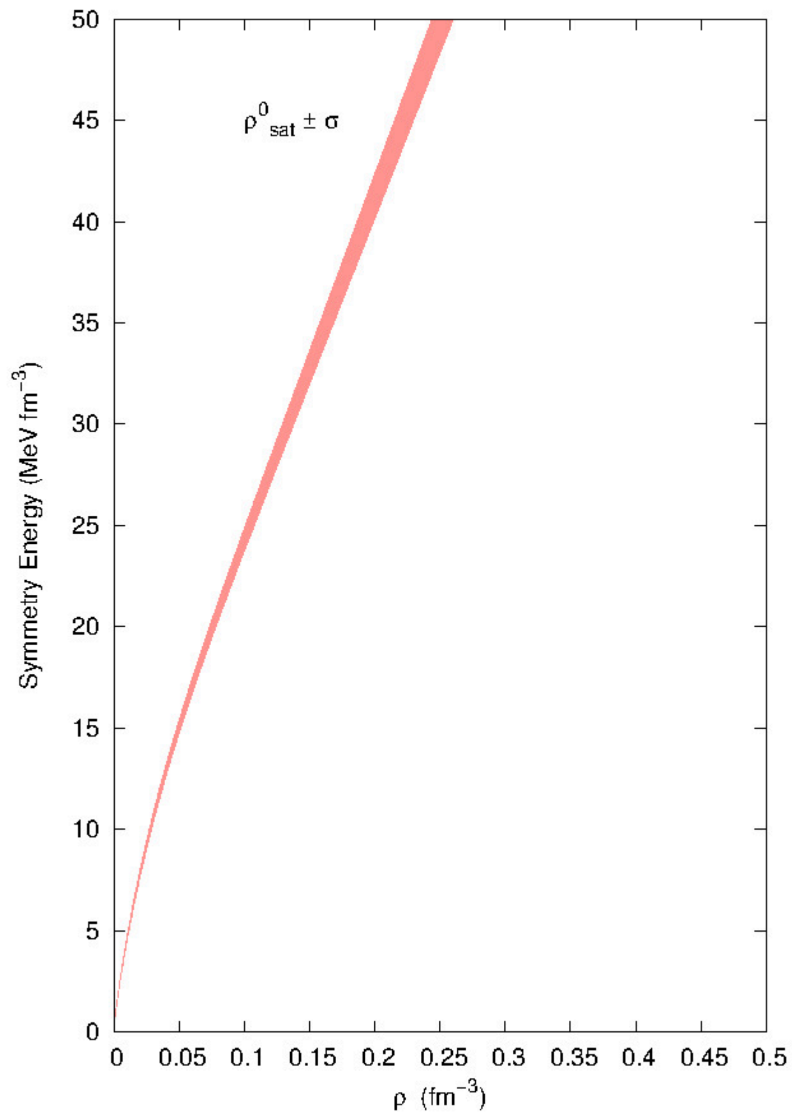
fixed   Explore inside small interval   Consider large interval

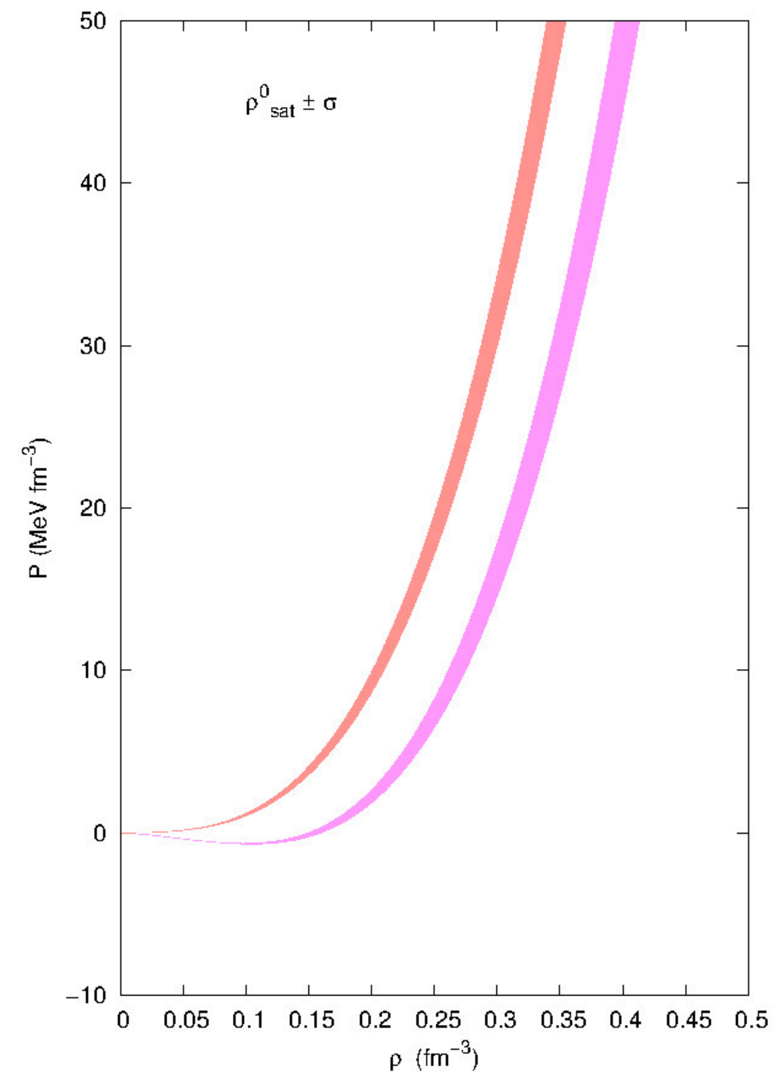
Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
		fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV
Skyrme	Average	0.1586	-15.91	251.68	-300.20	1178.35	31.22	53.52	-130.15	316.68	-1890.99
	σ	0.0040	0.21	45.42	157.81	848.47	2.03	31.06	132.03	218.23	1191.23
RMF	Average	0.1494	-16.24	267.99	-1.94	5058.30	35.11	90.20	-4.58	271.07	-3671.83
	σ	0.0025	0.06	33.52	392.51	2294.07	2.63	29.56	87.66	357.13	1582.34
RHF	Average	0.1540	-15.97	248.06	389.17	5269.07	33.97	90.03	128.16	523.29	-9955.49
	σ	0.0035	0.08	11.63	350.44	838.41	1.37	11.06	51.11	236.80	4155.74
Average		0.1540	-16.04	255.91	29.01	3835.24	33.43	77.92	-2.19	370.34	-5172.77
σ		0.0051	0.20	34.39	424.59	2401.14	2.64	30.84	142.71	298.54	4362.35

$$\frac{E}{A} = (E_0 + E_{sym}\delta^2) + L_{sym}x\delta^2 + \frac{1}{2}(K_0 + K_{sym}\delta^2)x^2 + \ldots$$

RESULTS : HNM

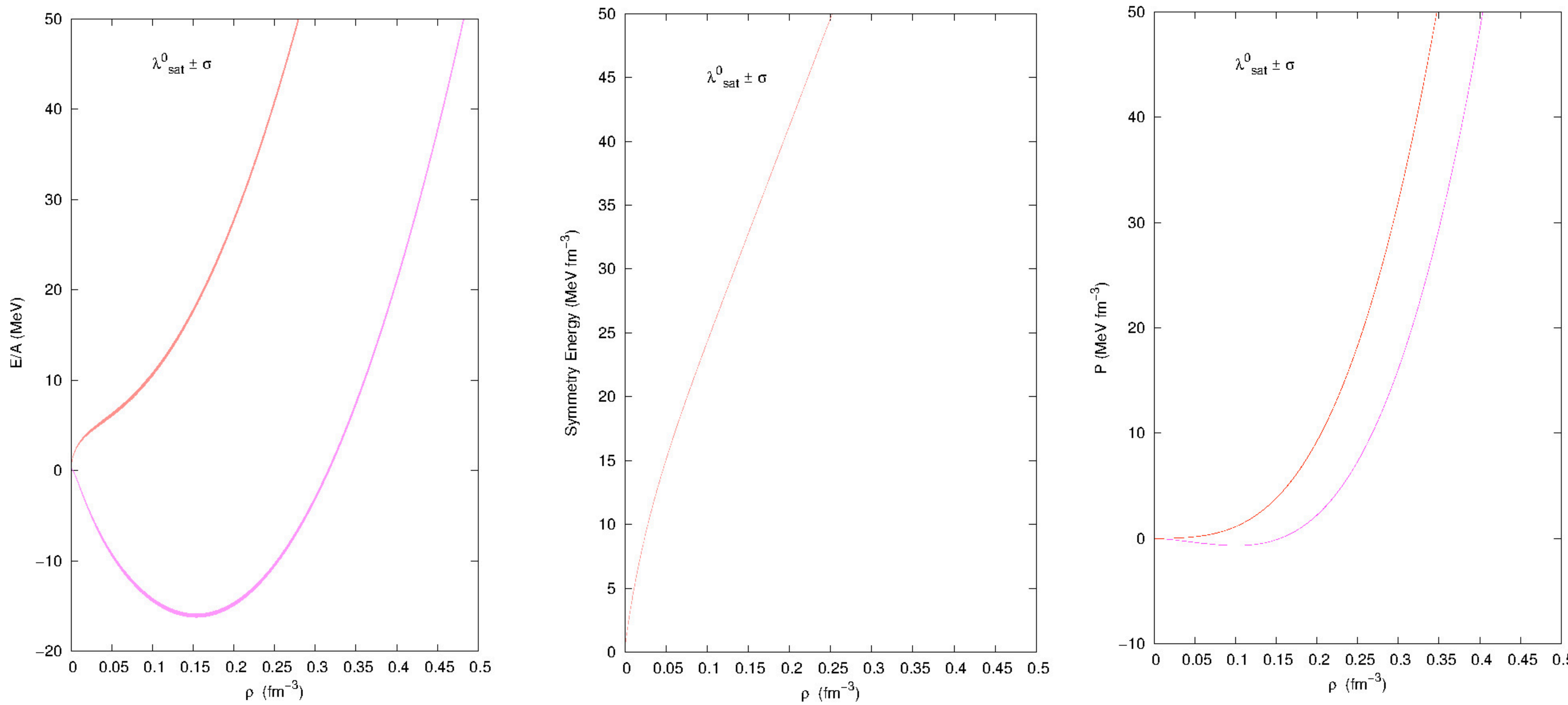






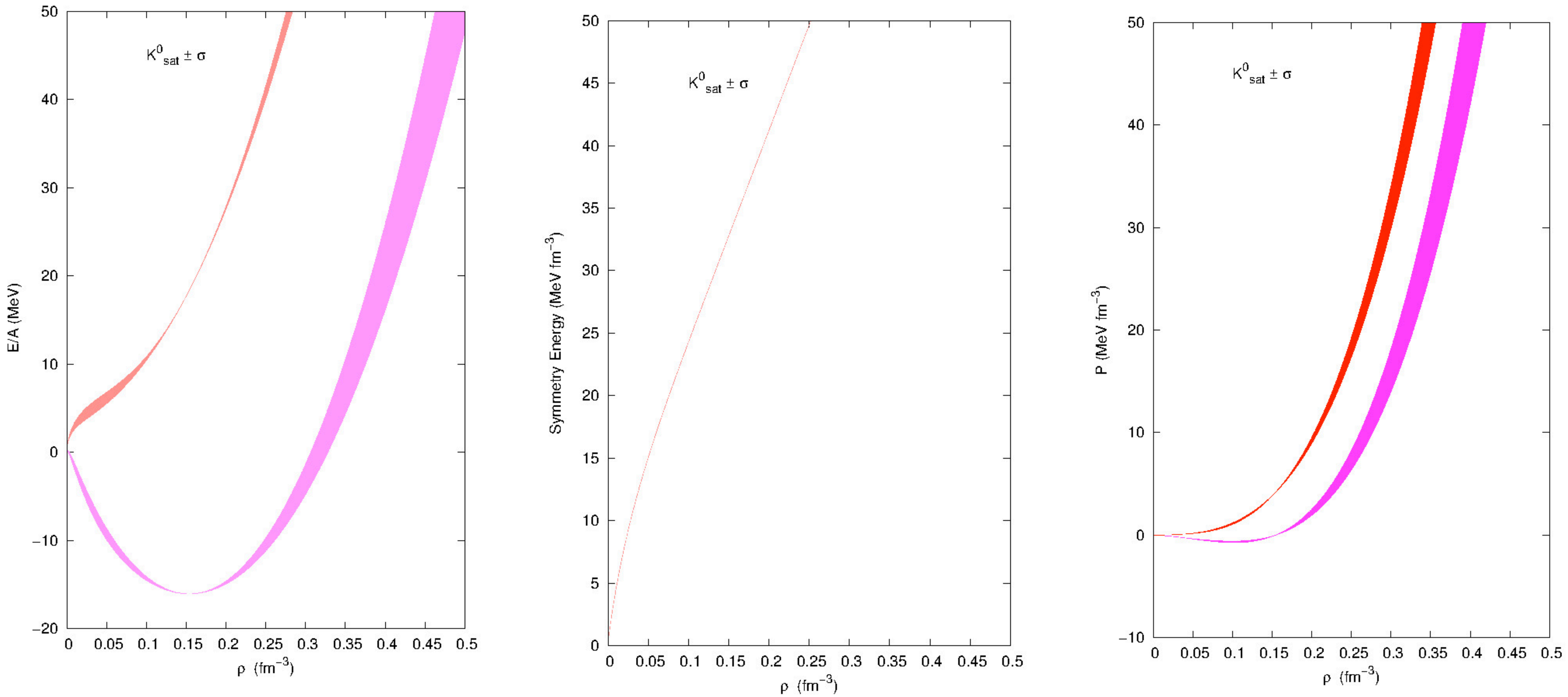
Effect of uncertainty in saturation density
on binding energy, symmetry energy and pressure

RESULTS : HNM



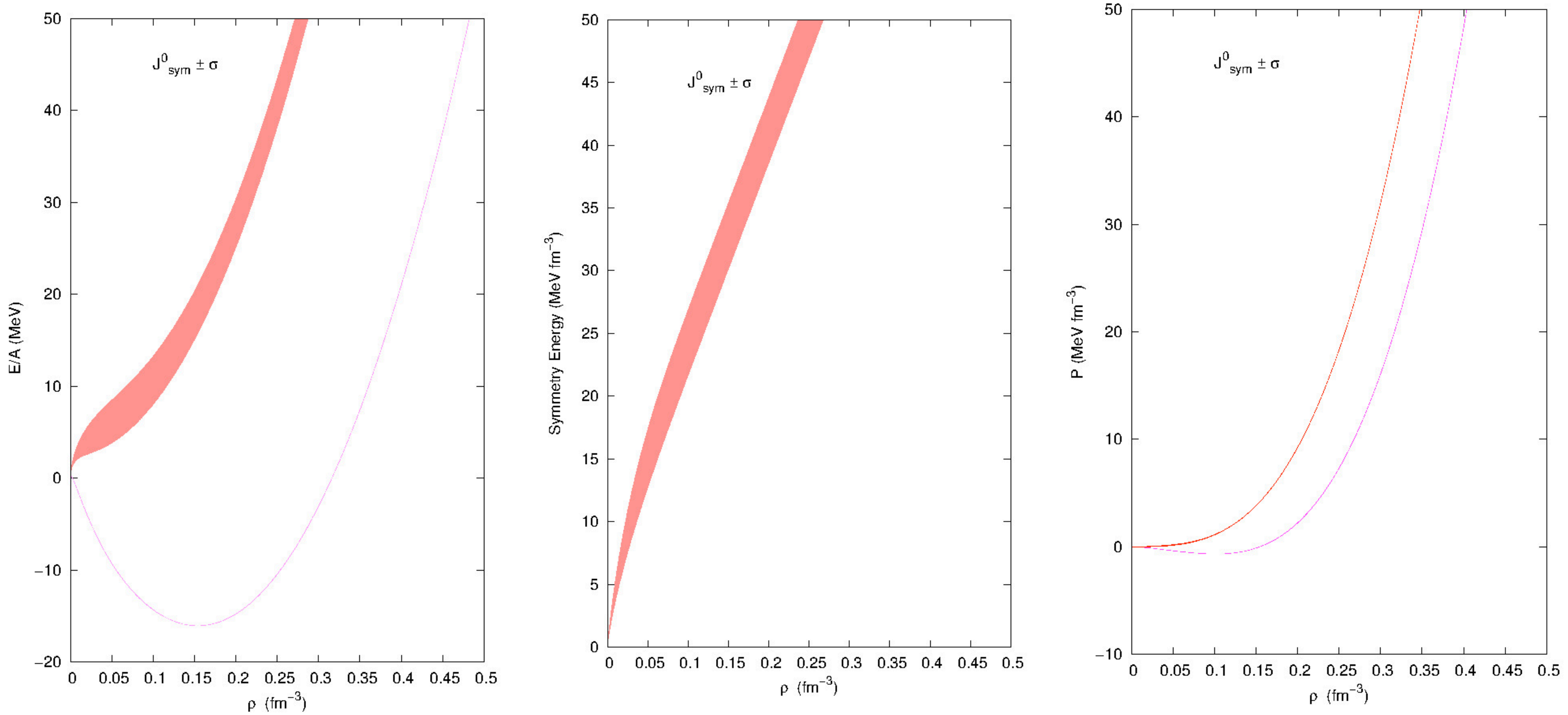
Effect of uncertainty in λ_{sat} on binding energy, symmetry energy and pressure

RESULTS : HNM



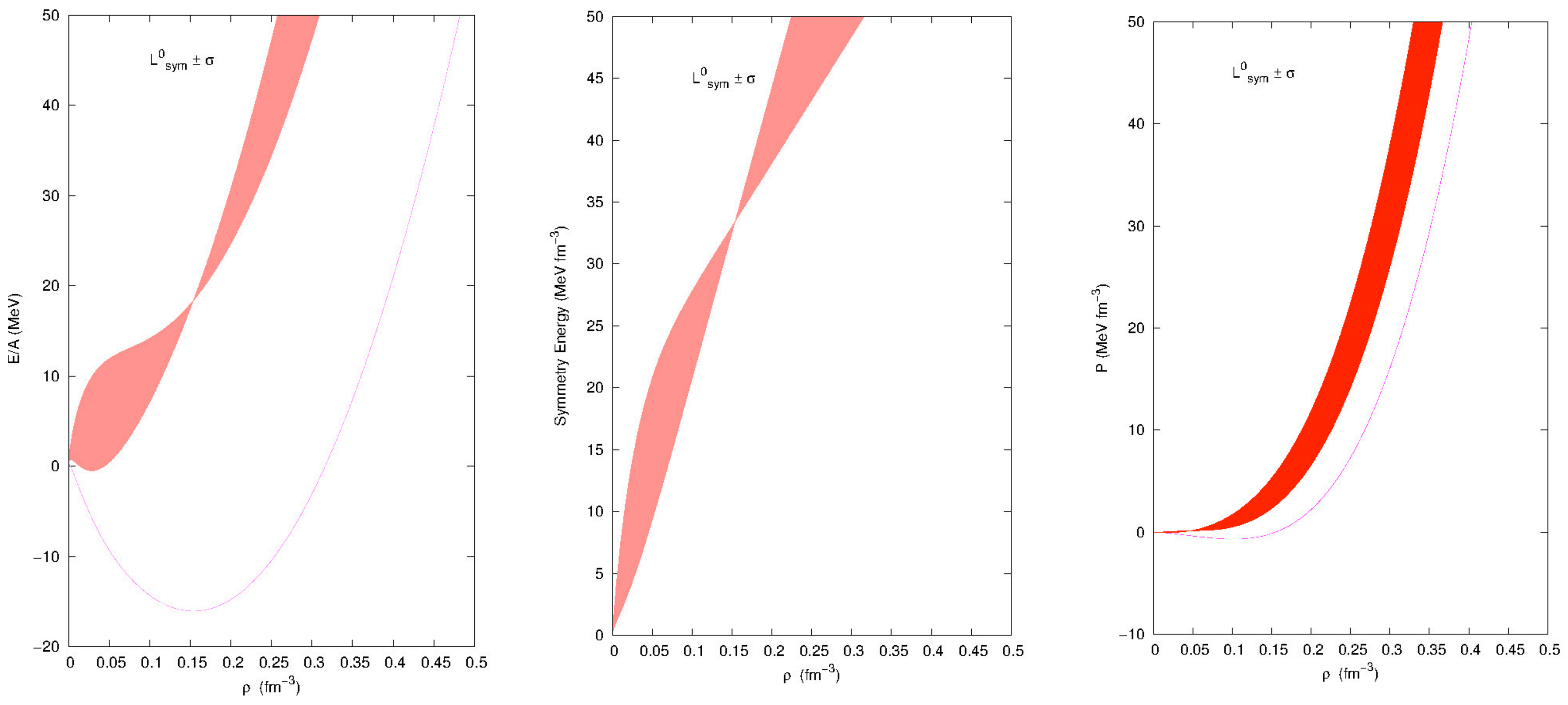
Effect of uncertainty in K_{sat} on binding energy, symmetry energy and pressure

RESULTS : HNM



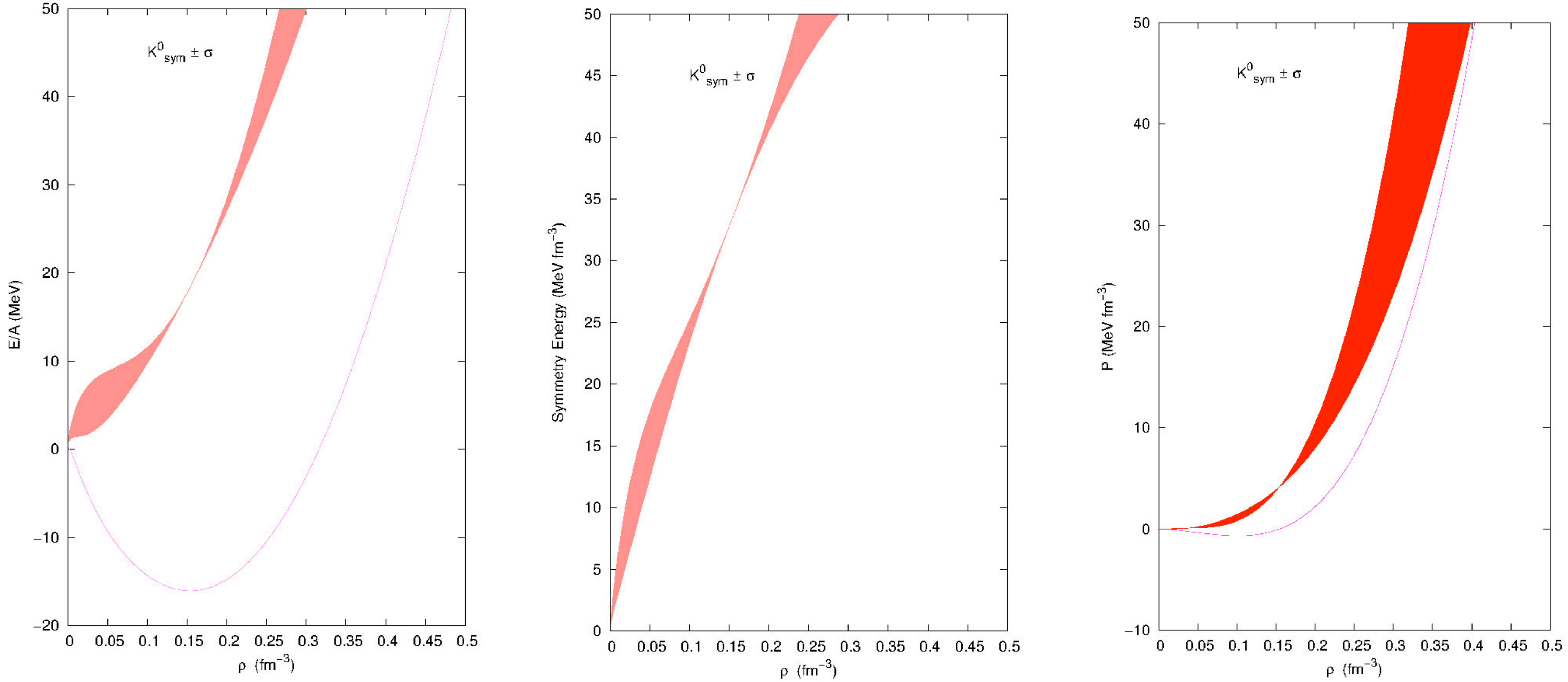
Effect of uncertainty in J_{sym} on binding energy, symmetry energy and pressure

RESULTS : HNM



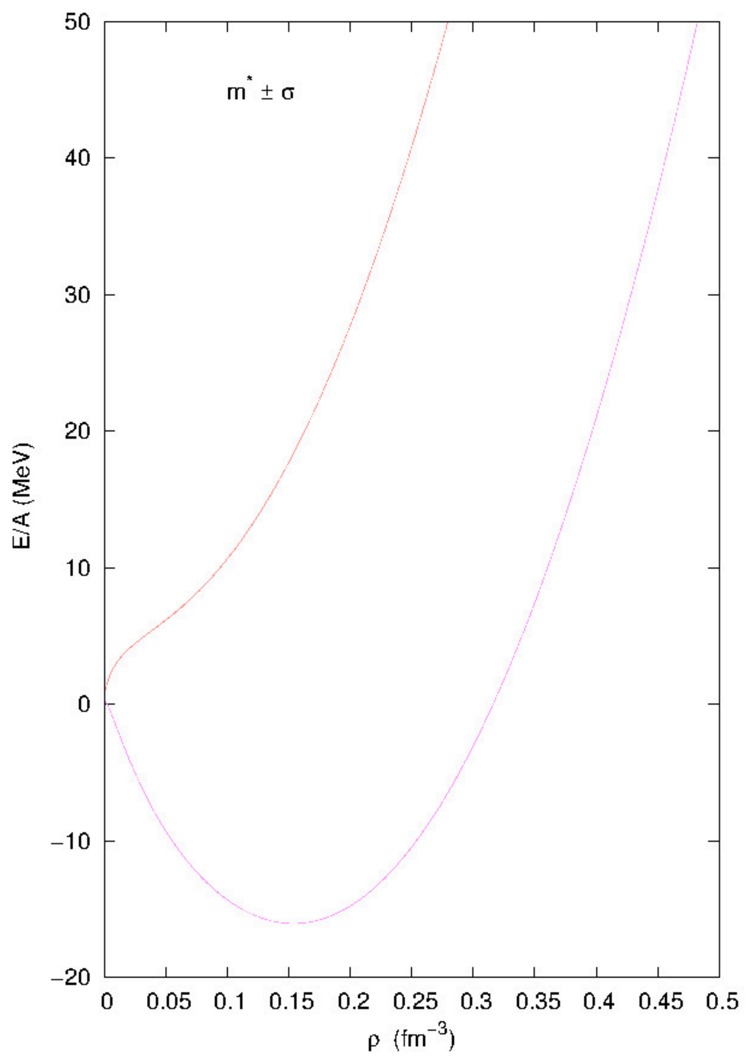
Effect of uncertainty in L_{sym} on binding energy, symmetry energy and pressure

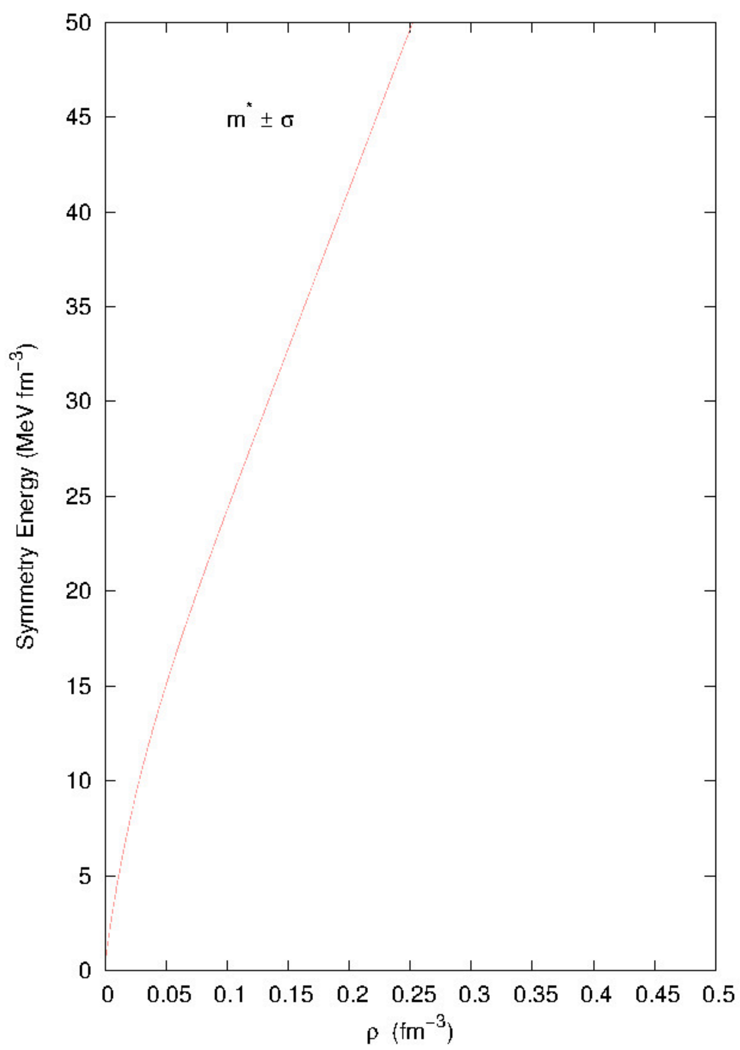
RESULTS : HNM

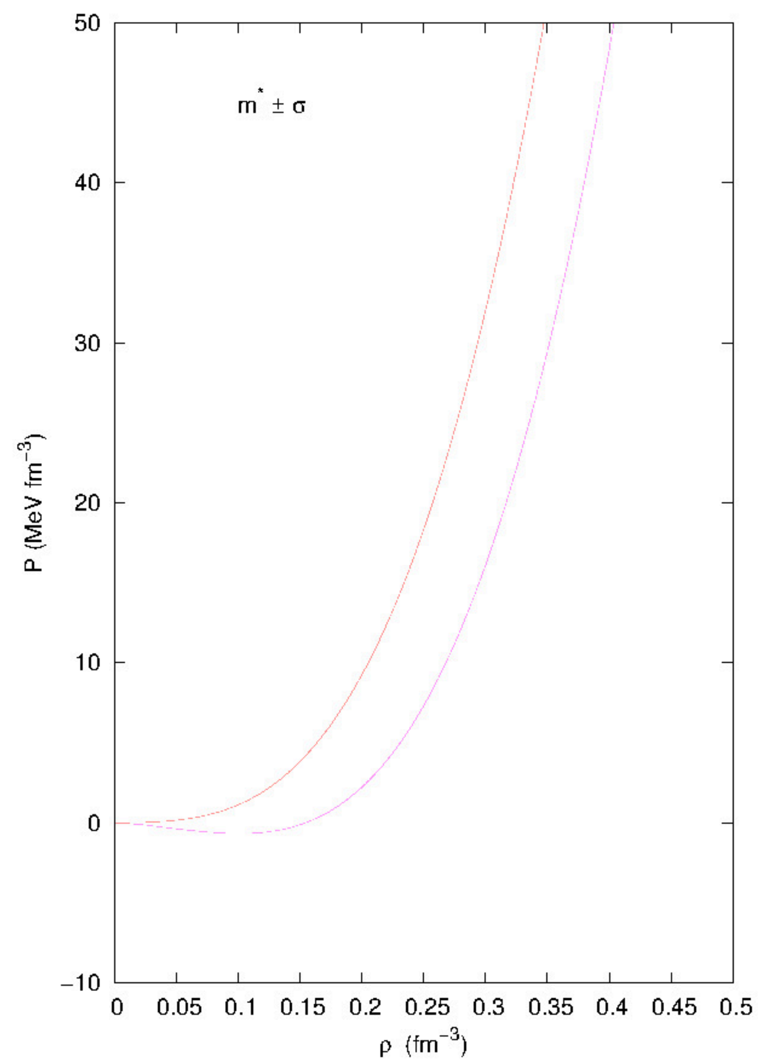


Effect of uncertainty in K_{sym} on binding energy, symmetry energy and pressure

RESULTS : HNM







Effect of uncertainty in effective mass
on binding energy, symmetry energy and pressure

Inhomogeneous matter: Asymmetric nuclei

- Given a parametrized density profile $\rho_q(r)$, the energy of a nucleus

$$E = \int dr \mathcal{H}_{ETF}\left(\rho_q(r)\right)$$

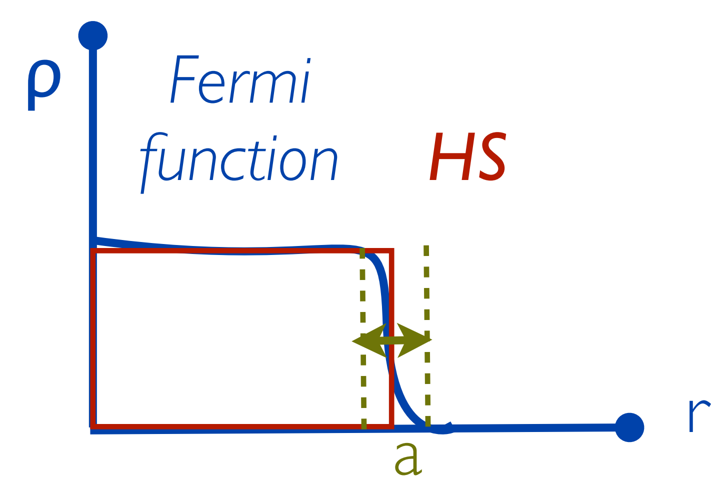
- Fermi function is a reasonable choice for the density profile

$$\rho_q(r) = \rho_0(\delta)F(r), F(r) = (1 + e^{(r-R)/a})^{-1} \tag{5}$$

where a is the diffuseness of the density profile

- saturation density of asymmetric matter

$$\rho_0(\delta) = \rho_0(\delta = 0)\left(1 - \frac{3L_{sym}}{K_{sat} + K_{sym}\delta^2}\right) \tag{6}$$



Inhomogeneous matter: Asymmetric nuclei

- Separating into the bulk and surface contributions,

$$E = E_b + E_s$$

- The bulk part $E_b = \lambda_{sat} A$, where the energy per particle

$$\begin{aligned} \lambda_{sat} &= \left.\frac{\mathcal{H}}{\rho}\right|_{\rho_0(\delta)} \\ &= e_{HNM}(x, \delta) \end{aligned}$$

where $x = \frac{\rho - \rho_0(\delta)}{3\rho_0(\delta)}$

Inhomogeneous matter: Asymmetric nuclei

- Separating into the bulk and surface contributions,

$$E = E_b + E_s$$

- The bulk part $E_b = \lambda_{sat} A$, where the energy per particle

$$\begin{aligned} \lambda_{sat} &= \left.\frac{\mathcal{H}}{\rho}\right|_{\rho_0(\delta)} \\ &= e_{HNM}(x, \delta) \end{aligned}$$

where $x = \frac{\rho - \rho_0(\delta)}{3\rho_0(\delta)}$

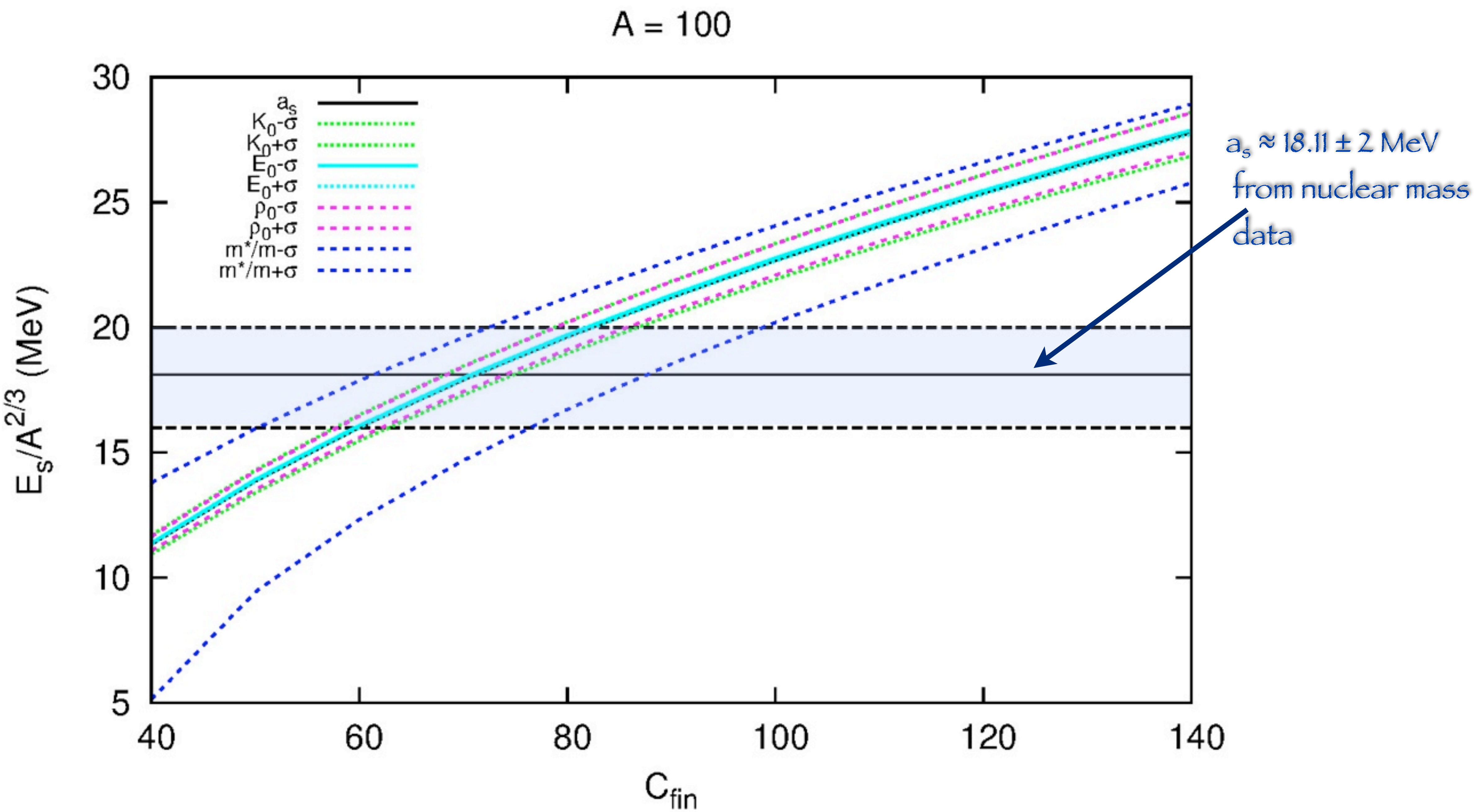
François Aymard, Ph.D. Thesis

Inhomogeneous matter: Asymmetric nuclei

- The surface energy can be separated into local and non-local terms
$$E_s = E_s^L + E_s^{NL}$$
$E_L = E_L$ [Empirical parameters],
$E_{NL} = E_{NL}$ [Empirical parameters $+ C_{fin} + C_{so}$]
- In the limit of purely local energy functional, the optimal configuration is a homogeneous hard sphere $a = 0$.
- The presence of NL terms in the functional results in finite diffuseness for atomic nuclei
- The diffuseness can be determined by $\frac{\partial E}{\partial a} = 0$

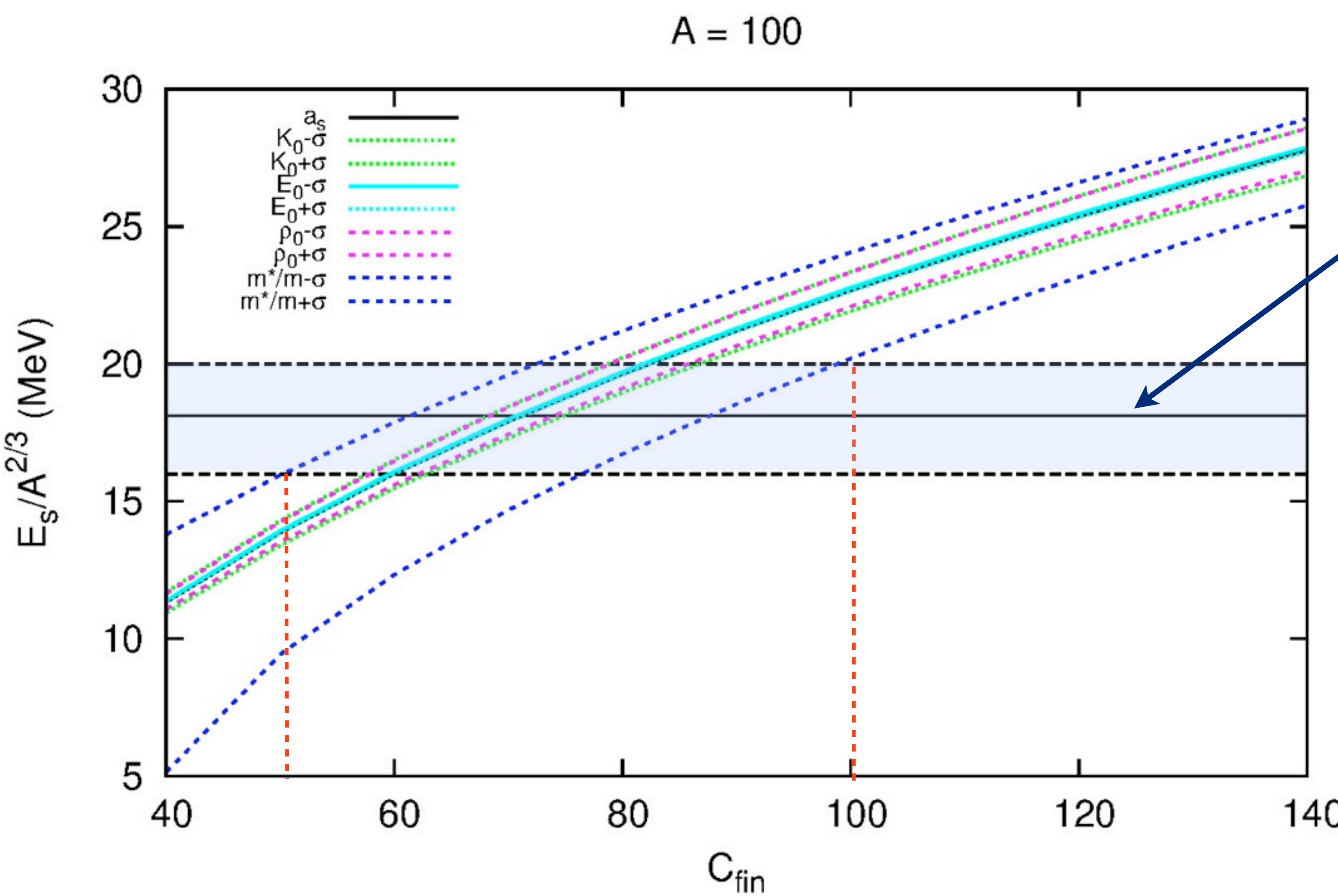
François Aymard, Ph.D.Thesis

RESULTS : SYMMETRIC NUCLEI



Effect of uncertainty in ρ_{sat}, λ_{sat}, K_{sat}, m^*/m on empirical value of C_{fin} using experimentally determined values of a_s

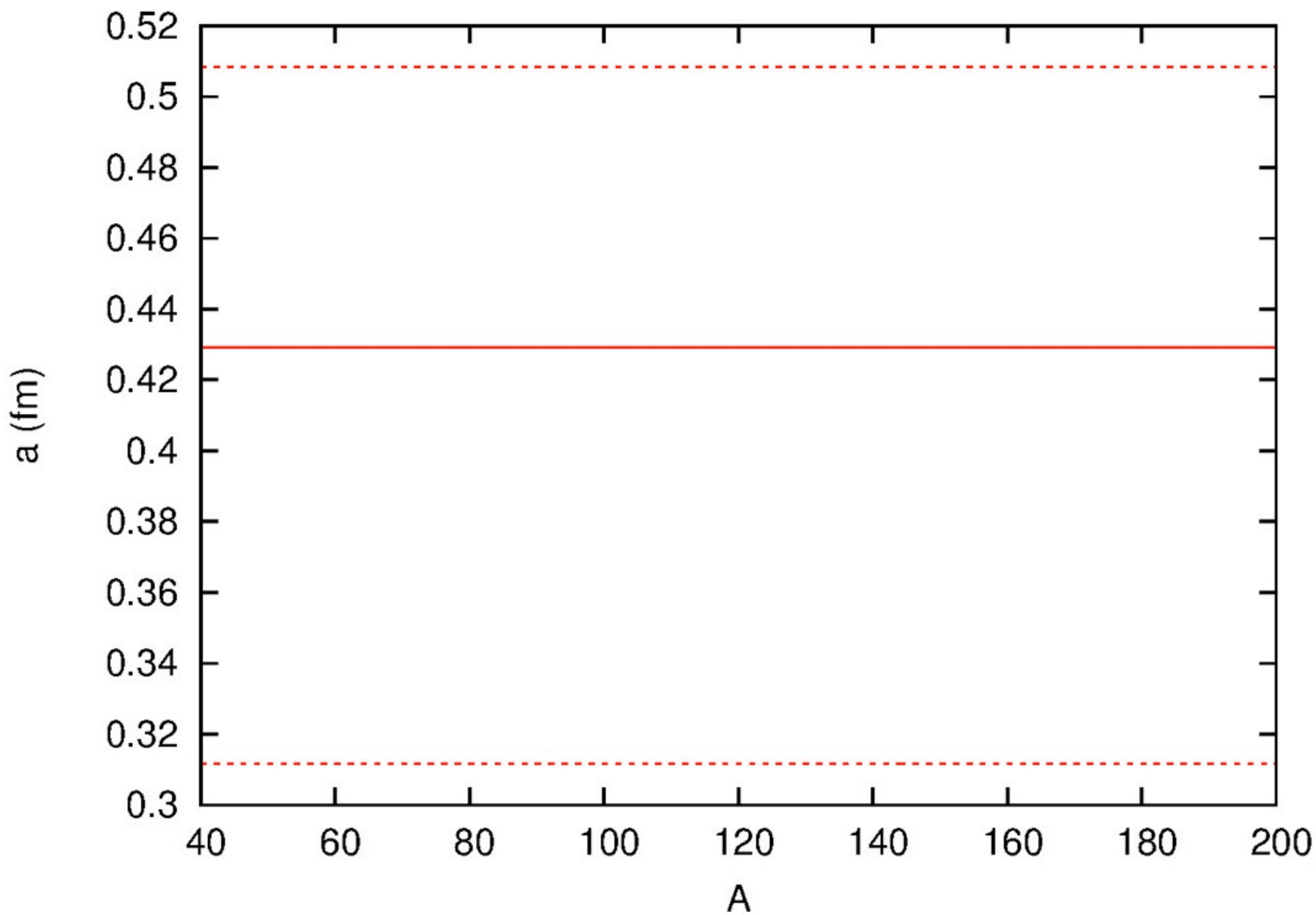
RESULTS : SYMMETRIC NUCLEI



$a_s \approx 18.11 \pm 2$ MeV from nuclear mass data

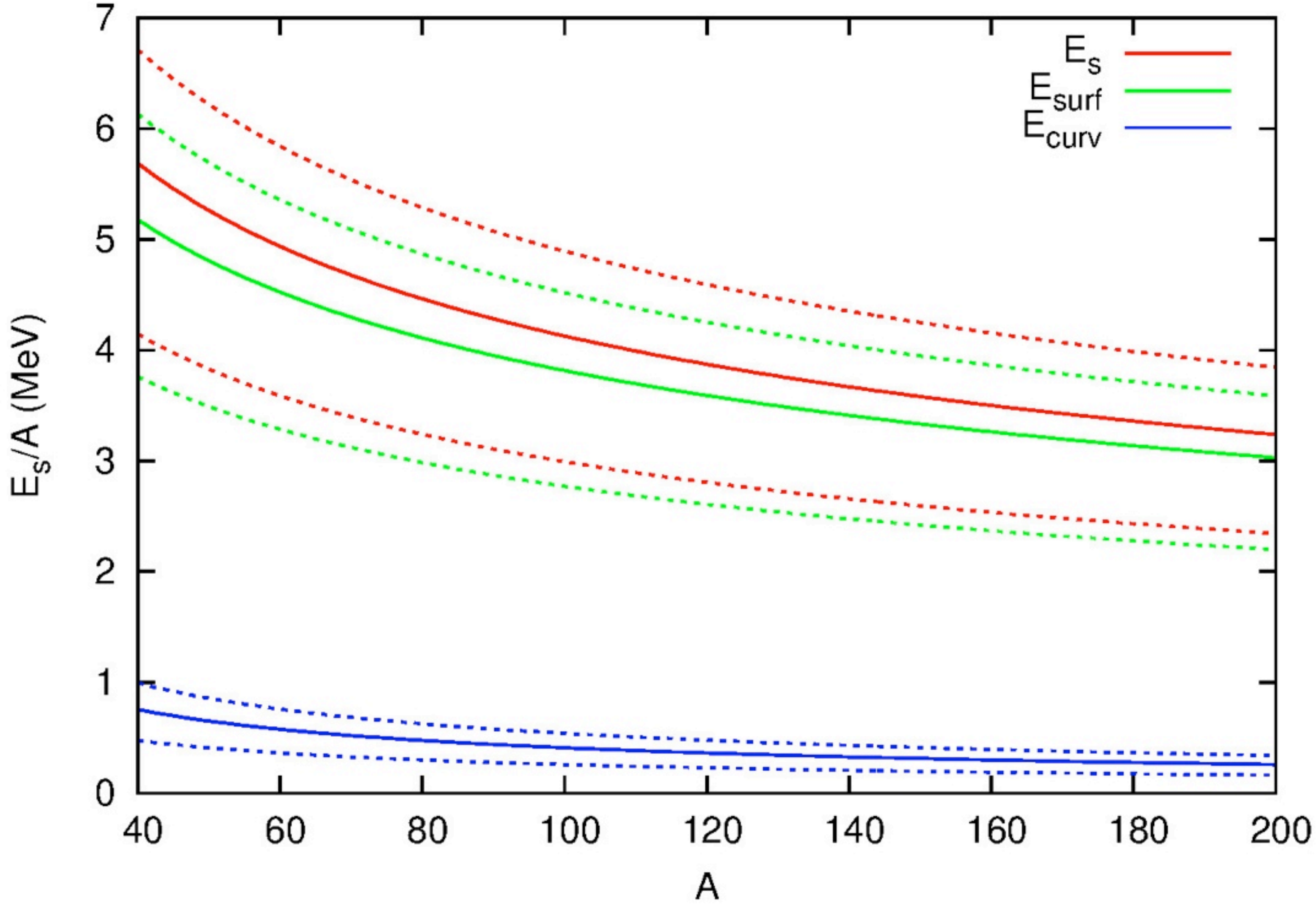
Effect of uncertainty in ρ_{sat}, λ_{sat}, K_{sat}, m*/m on empirical value of C_{fin} using experimentally determined values of a_s

RESULTS : SYMMETRIC NUCLEI



Effect of uncertainty in C_{fin} on the diffuseness parameter a

RESULTS : SYMMETRIC NUCLEI



Effect of uncertainty in C_{fin} on the surface energy and its components

SUMMARY & PROSPECTS

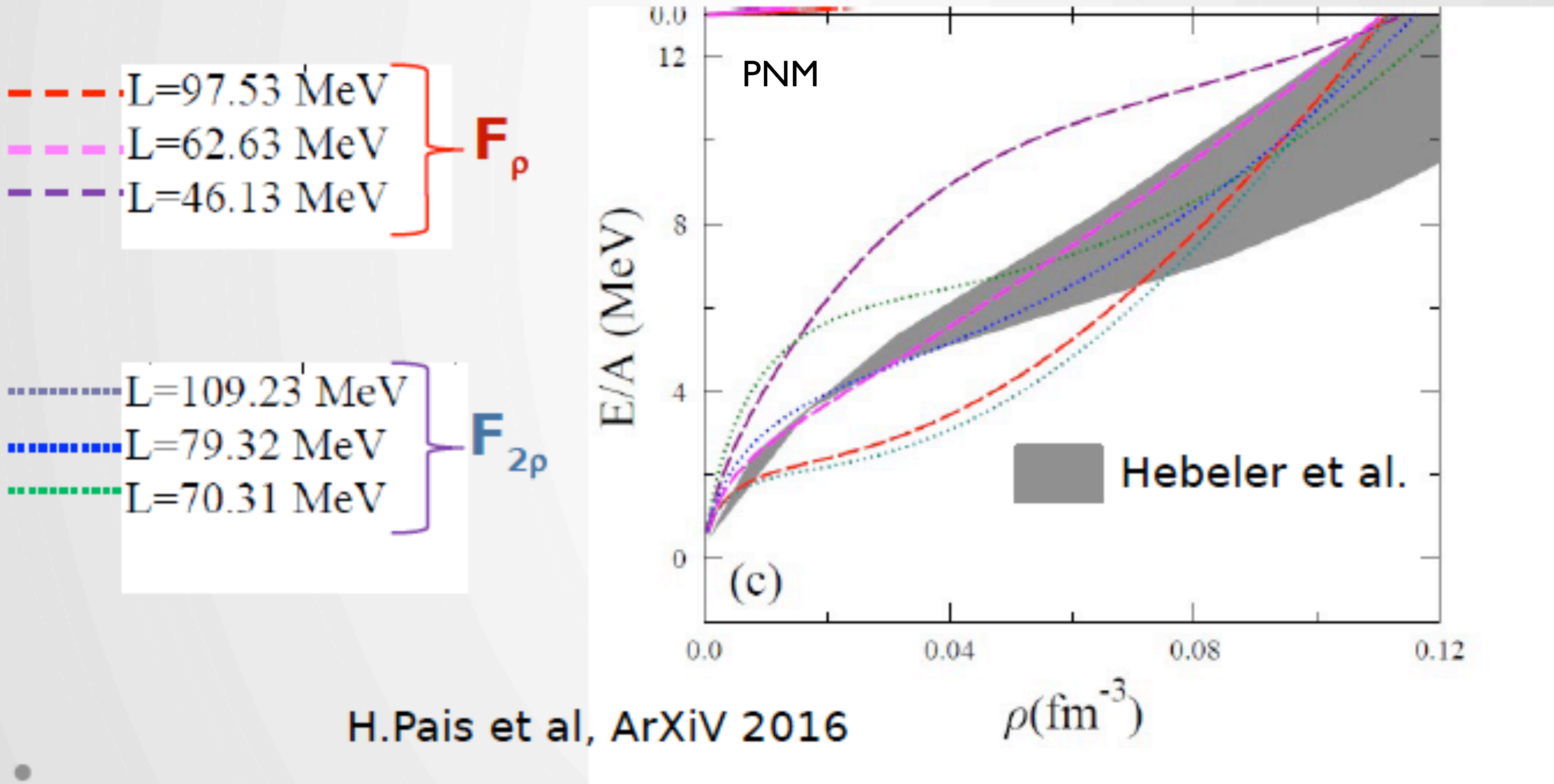
- *Our aim is to develop a model independent (based on empirical constraints) "unified" EOS to describe core (homogeneous nuclear matter) + crust (asymmetric nuclei)*
- *To extend the calculations to describe asymmetric nuclei in outer crust and then to inner crust*
- *To model the composition of the crust and core of a neutron star in beta equilibrium*
- *To study the influence of the empirical parameters on the properties related to the crust-core transition :*
  - *mass*
  - *radius*
  - *moment of inertia*
  - *glitches*

  using the model-independent unified EoS

THANK YOU! MERCI!

HIGHER ORDER TERMS

$$e(\rho,\delta) \cong \left(E_0 + \frac{1}{18}K_0x^2 + O(x^3)\right) + \left(J_0 + \frac{1}{3}Lx + \frac{1}{18}K_{sym}x^2 + O(x^3)\right)\delta^2$$



Influence of density dependence of symmetry energy on properties of crust-core transition analysed for 2 families of RMF models, differing in the isovector channel (same isoscalar properties)