Probing Nuclear Superfluidity With Pulsar Timing Observations

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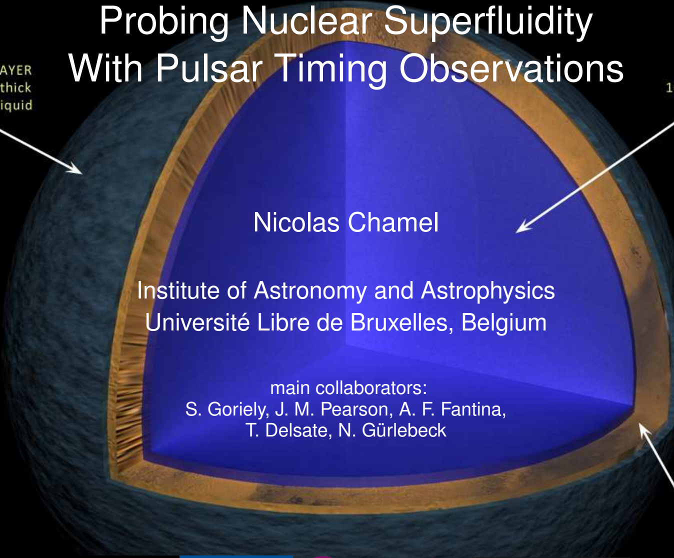

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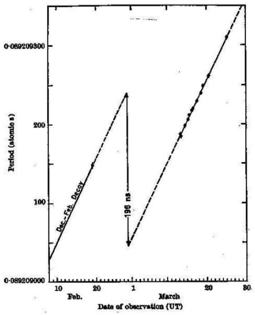
Pulsar glitches

Pulsars are spinning very rapidly with extremely stable periods. The variations $\dot{P}$ of the rotation period of some pulsars do not exceed 10^{-21}, as compared to 10^{-18} for the most accurate atomic clocks.
Hinkley et al., Science 341, 1215 (2013).

Still, some pulsars have been found to suddenly spin up. So far, 472 glitches have been detected in 165 pulsars.
http://www.jb.man.ac.uk/pulsar/glitches.html



The first glitch was detected in Vela:

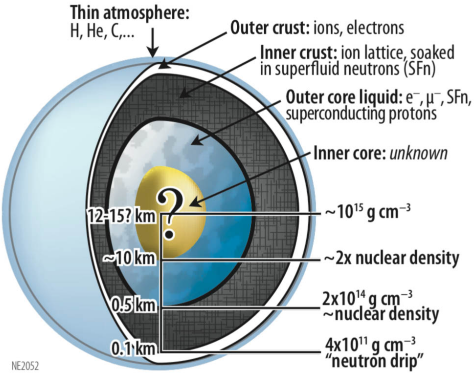
The rotational frequency had increased by $\Delta\Omega/\Omega \simeq 2 \times 10^{-6}$.

The increase in the spin-down rate was even larger $\Delta\dot{\Omega}/\dot{\Omega} \simeq 7 \times 10^{-3}$.

Radhakrishnan& Manchester, Nature 222, 228 (April 1969); Reichley& Downs, ibid. 229

Neutron-star superfluidity

Because of the **long relaxation times**, giant pulsar glitches have long been thought to be the manifestation of neutron-star superfluids:
Baym, Pethick, Pines, Nature 224, 673 (1969)

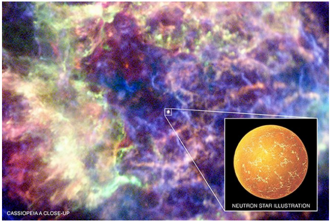


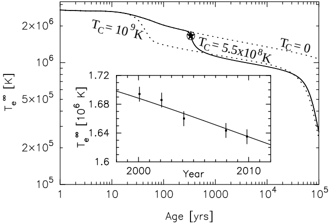
Nuclear superfluids are theoretically well-established, but their properties (e.g. T_c) remain uncertain.

Neutron-star superfluidity

Apart from glitches, other independent observations support the existence of neutron-star superfluids:

- Observations of Cassiopeia A provide strong evidence for neutron-star core superfluidity.
  Page et al., PRL 106, 081101; Shternin et al.,MNRAS 412, L108.

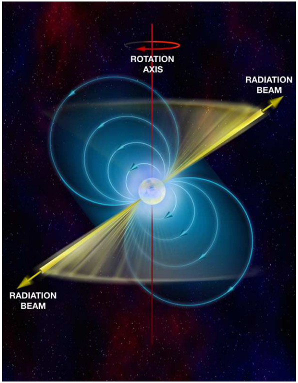




- Observations of quasi-persistent soft X-ray transients provide evidence for neutron-star crust superfluidity.
  Shternin et al., Mon. Not. R. Astron. Soc.382(2007), L43.
  Brown and Cumming, ApJ698 (2009), 1020.

Pulsar braking indices

Monitoring pulsars spin evolution can potentially shed light on nuclear superfluidity, through the **braking index** $n \equiv \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2}$.



The rotating magnetic dipole model predicts $n = 3$.
Assuming in addition that some superfluid component with moment of inertia $I_s = I - I_n$ grows over time yields

$$n = 3 - \frac{2\Omega}{|\dot{\Omega}|}\frac{\dot{I}_s}{I_n} < 3$$

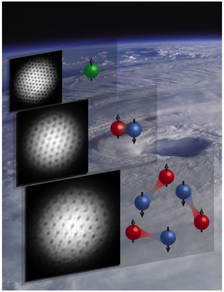
Crab	2.5
Vela	1.4

However, other mechanisms may be at play like magnetic field evolution, pulsar wind, etc. (see J. Pétri's talk).

Vortices and glitches

A rotating superfluid is threaded by a regular array of **quantized vortex lines**. Each line carries an angular momentum $\hbar$. The surface density of vortices depends on the rotation rate.

The surface density of vortices in a neutron star is given by $n_v(\mathrm{km}^{-2}) \sim 10^{14}/P(\mathrm{s})$.



Vortices in cold gases (MIT)

Vortex-mediated glitch theory in a nut shell

Vortices move outwards as the superfluid spins down with the rest of the star due to mutual friction forces.

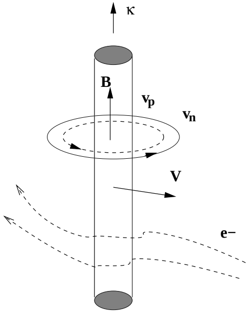
Vortex pinning by nuclei gives rise to crustal stress until:

- vortices are suddenly unpinned (Anderson&Itoh)
- the crust cracks (Ruderman).

Vortex creep leads to the long-term relaxation (Alpar&Pines).

Entrainment and dissipation in neutron-star cores

Historically the **long post-glitch relaxation** provided the first evidence of neutron-star superfluidity. But...



picture from K. Glampedakis

Due to (non-dissipative) mutual entrainment effects, neutron vortices carry a **fractional magnetic quantum flux**

Sedrakyan and Shakhabasyan, Astrofizika 8 (1972), 557; Astrofizika 16 (1980), 727.

The core superfluid is strongly coupled to the crust due to electrons scattering off the magnetic field of the vortex lines.

Alpar, Langer, Sauls, ApJ282 (1984) 533

Glitches are therefore expected to originate from the crust.

Entrainment in neutron-star crusts

Despite the absence of viscous drag, the crust can still resist the flow of the neutron superfluid due to **non-local and non-dissipative entrainment effects**.
Chamel, PhD thesis, Université Paris 6, France (2004)
Carter, Chamel & Haensel, Nucl.Phys.A748,675(2005).

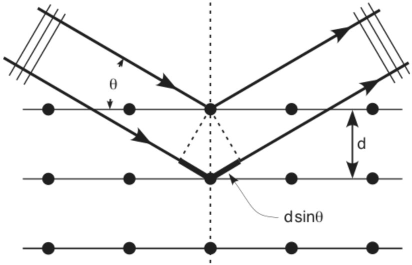
What is the origin of entrainment?

A neutron with wavevector $\boldsymbol{k}$ can be **coherently scattered** by the lattice if

$$d \sin\theta = N\pi/k$$

where $N = 0, 1, 2, ...$ (Bragg's law).

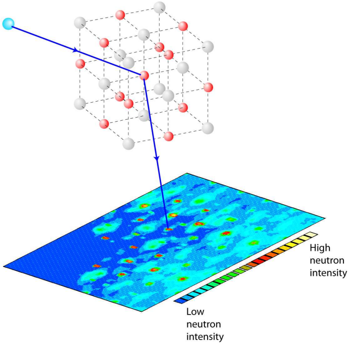
In this case, it does not propagate in the crystal: it is therefore entrained!



Neutron diffraction

For decades, neutron diffraction experiments have been routinely performed to explore the structure of materials.

The main difference in neutron-star crusts is that **neutrons are highly degenerate**



- A neutron can be coherently scattered if $k > \pi/d$ (Bragg's law).
- In neutron stars, neutrons have momenta up to k_{F}. Typically $k_{\mathrm{F}} > \pi/d$ in all regions of the inner crust but the shallowest.

Therefore, Bragg scattering should be taken into account!

How “free” are neutrons in neutron-star crusts?

Imparting a momentum $\boldsymbol{p_n}$ to “free” neutrons induces a neutron current $\boldsymbol{j_n} = n_n^c \boldsymbol{p_n}$ with $n_n^c < n_n^f$. Equivalently $\boldsymbol{p_n} = m_n^\star \boldsymbol{v_n}$ with $m_n^\star = m_n n_n^f / n_n^c > m_n$.

n_n^c (or $m_n^\star$) can be determined by averaging over all occupied $\boldsymbol{k}$ from **band-structure calculations**:

$\bar{n}$ (fm^{-3})	n_n^f/n_n (%)	n_n^c/n_n^f (%)
0.0003	20.0	82.6
0.001	68.6	27.3
0.005	86.4	17.5
0.01	88.9	15.5
0.02	90.3	7.37
0.03	91.4	7.33
0.04	88.8	10.6
0.05	91.4	30.0
0.06	91.5	45.9

$\bar{n}$ is the average baryon density
n_n is the total neutron density
n_n^f is the “free” neutron density
n_n^c is the “conduction” neutron density

In many layers, most neutrons are entrained by the crust!
Chamel,PRC85,035801(2012)

Entrainment impacts our understanding of pulsar glitches.

Giant pulsar glitches and the inertia of neutron-star superfluids

Giant pulsar glitches are usually interpreted as **sudden tranfers of angular momentum between the crustal superfluid and the rest of star**.

Because the superfluid is entrained (even in the absence of interactions), its angular momentum can be written as

$$J_s = I_{ss}\Omega_s + (I_s - I_{ss})\Omega_c$$

(Ω_s and Ω_c being the angular velocities of the superfluid and of the "crust", I_s is the moment of inertia of the superfluid), leading to the following constraint:

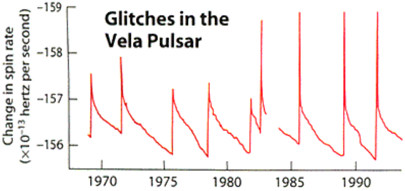
$$\frac{I_s}{I} \geq \mathcal{G}\frac{\bar{m}_n^\star}{m_n}\,, \qquad \mathcal{G} = 2\tau_c A_g$$

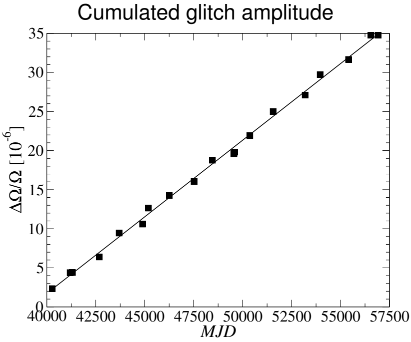
where $\dfrac{\bar{m}_n^\star}{m_n} = \dfrac{I_{ss}}{I_s}$, $\quad \tau_c = \dfrac{\Omega}{2|\dot{\Omega}|}$ and $A_g = \dfrac{1}{t}\sum_i \dfrac{\Delta\Omega_i}{\Omega}$.

Chamel&Carter,MNRAS368,796(2006)

Vela pulsar glitch constraint

Since 1969, 19 glitches have been regularly detected. The latest one occurred in September 2014.





A linear fit of $\frac{\Delta\Omega}{\Omega}$ vs t yields

$A_g \simeq 2.25 \times 10^{-14}\ \mathrm{s}^{-1}$

$$\mathcal{G} = 2\tau_c A_g \simeq 1.62\%$$

Glitch puzzle

The ratio $\bar{m}_n^\star/m_n = I_{ss}/I_s$ depends mainly on the physics of neutron-star crusts:

$$\frac{I_{\rm ss}}{I_{\rm crust}} \approx \frac{1}{P_{\rm cc}}\int_{P_{\rm drip}}^{P_{\rm cc}} \frac{n_n^{\rm f}(P)^2}{\bar{n}(P)n_n^{\rm c}(P)}\,{\rm d}P\,, \qquad \frac{I_{\rm s}}{I_{\rm crust}} \approx \frac{1}{P_{\rm cc}}\int_{P_{\rm drip}}^{P_{\rm cc}} \frac{n_n^{\rm f}(P)}{\bar{n}(P)}\,{\rm d}P\,.$$

where $P_{\rm drip}$ is the pressure at the neutron-drip transition, and $P_{\rm cc}$ the pressure at the crust-core interface.

Using our crust model, we found $I_{\rm ss} \approx 4.6 I_{\rm crust}$ and $I_{\rm s} \approx 0.89 I_{\rm crust}$ leading to $\bar{m}_n^\star/m_n \approx 5.1$.

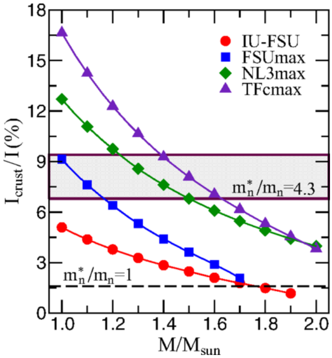
The Vela glitch constraint thus becomes $\dfrac{I_s}{I} \geq 8.3\%$, or $\dfrac{I_{\rm crust}}{I} \geq 9.3\%$

The superfluid in the crust of a neutron star with a mass $M > M_\odot$ does not carry enough angular momentum!
Andersson et al., PRL 109, 241103; Chamel, PRL 110, 011101 (2013).

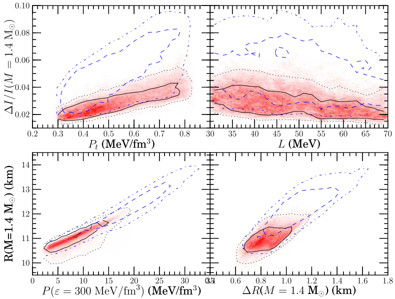
This conclusion has been confirmed by more recent works, e.g.
Newton et al, MNRAS 454, 4400 (2015); Ang Li et al, ApJS 223, 16 (2016).

Do nuclear uncertainties allow for thick enough crusts?

It has been recently argued that nuclear physics uncertainties may allow for thick enough crust to explain Vela pulsar glitches:



Fine-tuned nuclear models
Piekarewicz et al.PRC 90, 015803 (2014).

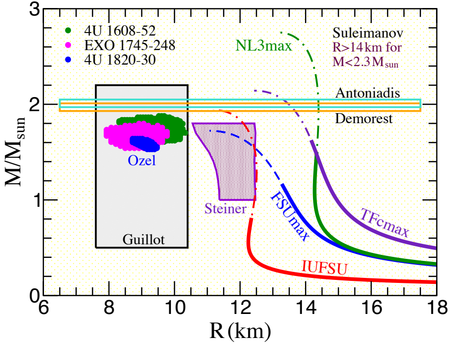


Monte Carlo simulations
of parametrized equations of state
Steiner et al.PRC 91, 015804 (2015).

Do nuclear uncertainties allow for thick enough crusts?

However, these equations of state

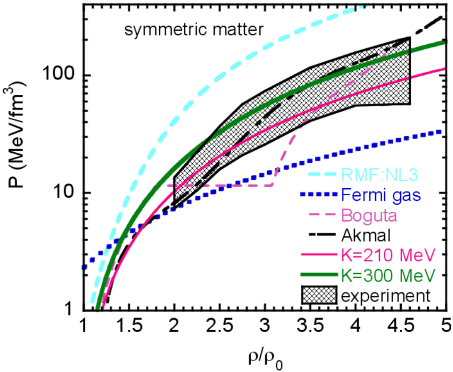
- are not thermodynamically consistent (this may lead to large errors on the neutron-star structure, see M. Fortin's talk),
- are incompatible with terrestrial experiments and observations:



Piekarewicz et al.PRC 90, 015803 (2014).

FSUmax is too soft ($M < 2M_\odot$)

Do nuclear uncertainties allow for thick enough crusts?



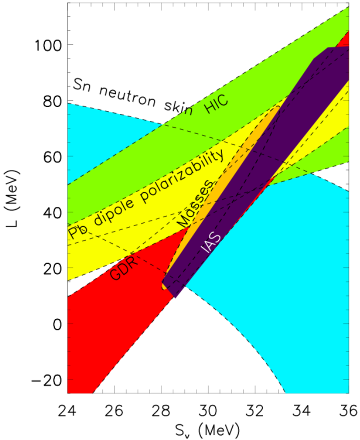
Danielewicz et al., Science 298, 1592 (2002).

Todd-Rutel and Piekarewicz, Phys. Rev. Lett. 95, 122501 (2005).

NL3max is too stiff

(contraints from heavy-ion collisions and giant resonances in nuclei)

Do nuclear uncertainties allow for thick enough crusts?



TFcmax is incompatible with symmetry energy constraints. This model yields $J = 38.3$ **MeV and** $L = 74.0$ **MeV.**

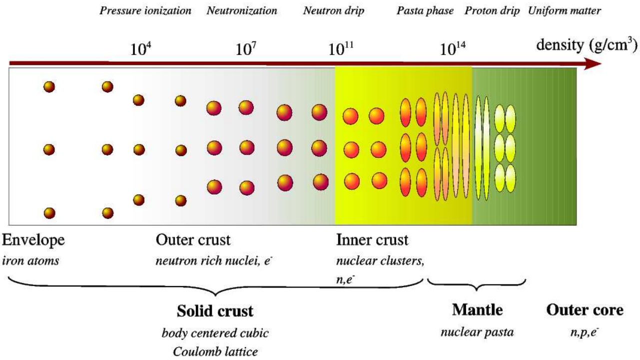
Note that the crust-core boundary is generally correlated with L: the larger is L, the thicker the crust is.

Taken from Lattimer (2014).

Is it possible to construct an equation of state compatible with nuclear-physics experiments, and which allows for thick enough crusts?

Unified equations of state

We have recalculated the structure of neutron-stars using *unified* equations of state that can describe consistently all regions with the *same* nuclear model.



To this end, we have employed the density functional theory with very accurately calibrated functionals.

Nuclear energy density functional theory in a nut shell

The energy $E[n_q(\boldsymbol{r}), \widetilde{n_q}(\boldsymbol{r})]$ of a nuclear system ($q = n, p$ for neutrons, protons) can be expressed as a (universal) *functional* of

- "normal" densities $n_q(\boldsymbol{r})$,
- "abnormal" densities $\widetilde{n_q}(\boldsymbol{r})$ (roughly the density of paired nucleons of charge q).

In turn these densities are written in terms of auxiliary quasiparticle wave functions $\varphi_{1k}^{(q)}(\boldsymbol{r})$ and $\varphi_{2k}^{(q)}(\boldsymbol{r})$ as

$$n_q(\boldsymbol{r}) = \sum_{k(q)} \varphi_{2k}^{(q)}(\boldsymbol{r})\varphi_{2k}^{(q)}(\boldsymbol{r})^* , \qquad \widetilde{n_q}(\boldsymbol{r}) = -\sum_{k(q)} \varphi_{2k}^{(q)}(\boldsymbol{r})\varphi_{1k}^{(q)}(\boldsymbol{r})^*$$

The *exact* ground-state energy can be obtained by minimizing the energy functional $E[n_q(\boldsymbol{r}), \widetilde{n_q}(\boldsymbol{r})]$ under the constraint of fixed nucleon numbers (and completeness relations on $\varphi_{1k}^{(q)}(\boldsymbol{r})$ and $\varphi_{2k}^{(q)}(\boldsymbol{r})$).

Duguet, Lecture Notes in Physics 879 (Springer-Verlag, 2014), p. 293

Dobaczewski & Nazarewicz, in "50 years of Nuclear BCS" (World Scientific Publishing, 2013), pp.40-60

Brussels-Montreal Skyrme functionals (BSk)

These functionals were constructed so as to account for nuclear-physics uncertainties.

In particular, they were fitted to both experimental data and N-body calculations using realistic forces.

Experimental data:

- all atomic masses with $Z, N \geq 8$ from the Atomic Mass Evaluation (root-mean square deviation: 0.5-0.6 MeV)
  `http://www.astro.ulb.ac.be/bruslib/`
- charge radii
- incompressibility $K_v = 240 \pm 10$ MeV (ISGMR)
  Colò et al., Phys.Rev.C70, 024307 (2004).

N-body calculations using realistic forces:

- equation of state of pure neutron matter
- $^1S_0$ pairing gaps in nuclear matter
- effective masses in nuclear matter

Phenomenological corrections for atomic nuclei

For atomic nuclei, we add the following corrections to the HFB energy:

- Wigner energy

$$E_W = V_W \exp\left\{ -\lambda\left(\frac{N-Z}{A}\right)^2\right\} + V'_W|N-Z|\exp\left\{-\left(\frac{A}{A_0}\right)^2\right\}$$

$V_W \sim -2$ MeV, $V'_W \sim 1$ MeV, $\lambda \sim 300$ MeV, $A_0 \sim 20$

- rotational and vibrational spurious collective energy

$$E_{\rm coll} = E_{\rm rot}^{\rm crank}\Big\{b\ \tanh(c|\beta_2|) + d|\beta_2|\ \exp\{-l(|\beta_2| - \beta_2^0)^2\}\Big\}$$

This latter correction was shown to be in good agreement with calculations using 5D collective Hamiltonian.
Goriely, Chamel, Pearson, Phys.Rev.C82,035804(2010).

In this way, these collective effects do not contaminate the parameters (≤ 20) of the functional.

Brussels-Montreal Skyrme functionals

Main features of the latest functionals:
Chamel et al., Acta Phys. Pol. B46, 349(2015)

- **fit to realistic $^1S_0$ pairing gaps (no self-energy)** (BSk16-17)
  Chamel, Goriely, Pearson, Nucl.Phys.A812,72 (2008)
  Goriely, Chamel, Pearson, PRL102,152503 (2009).
- **removal of spurious spin-isospin instabilities** (BSk18)
  Chamel, Goriely, Pearson, Phys.Rev.C80,065804(2009)
- **fit to realistic neutron-matter equation of state** (BSk19-21)
  Goriely, Chamel, Pearson, Phys.Rev.C82,035804(2010)
- **fit to different symmetry energies** (BSk22-26)
  Goriely, Chamel, Pearson, Phys.Rev.C88,024308(2013)
- **optimal fit of the 2012 AME** - rms 0.512 MeV (BSk27*)
  Goriely, Chamel, Pearson, Phys.Rev.C88,061302(R)(2013)
- **generalized spin-orbit coupling** (BSk28-29)
  Goriely, Nucl.Phys.A933,68(2015).
- **fit to realistic $^1S_0$ pairing gaps with self-energy** (BSk30-32)
  Goriely, Chamel, Pearson, Phys.Rev.C93,034337(2016).

Description of neutron star crust below neutron drip

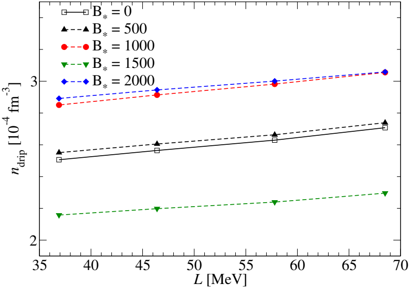
- For the outermost regions (up to about 8×10^6 g cm^{-3}) made of iron, we employed the equation of state from
  Lai et al., Astrophys. J. 377 612 (1991).
- At higher densities, the composition changes. The crust consists of nuclei coexisting with a relativistic gas of electrons.

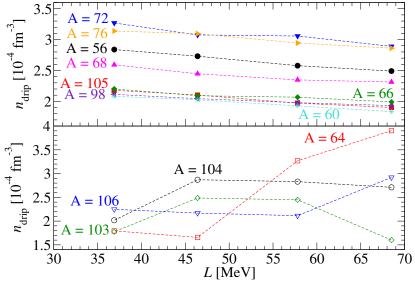
  The only microscopic inputs are nuclear masses. We have made use of the experimental data (Atomic Mass Evaluation) complemented with our HFB mass tables.
  Pearson,Goriely,Chamel,Phys.Rev.C83,065810(2011).

With increasing density, nuclei become progressively more neutron rich until at some point, free neutrons appear.
Chamel, Fantina, Zdunik, Haensel, Phys. Rev. C91,055803(2015).

Neutron-drip transition: role of the symmetry energy

The lack of knowledge of the symmetry energy translates into uncertainties in the neutron-drip density:





In accreted crusts, the neutron-drip transition may be more sensitive to nuclear-structure effects than the symmetry energy.

Fantina et al.,Phys.Rev.C93,015801(2016).

Description of neutron star crust beyond neutron drip

We use the **Extended Thomas-Fermi+Strutinsky Integral (ETFSI)** approach with the *same* functional as in the outer crust:

- the nucleon densities $n_q(\boldsymbol{r})$ are taken as basic variables
- proton shell effects are added perturbatively (neutron shell effects are much smaller and therefore neglected).

In order to further speed-up the calculations, clusters are supposed to be spherical (no pastas) and $n_q(\boldsymbol{r})$ are parametrized.

Pearson,Chamel,Pastore,Goriely,Phys.Rev.C91, 018801 (2015).
Pearson,Chamel,Goriely,Ducoin,Phys.Rev.C85,065803(2012).
Onsi,Dutta,Chatri,Goriely,Chamel,Pearson, Phys.Rev.C77,065805 (2008).

Advantages of the ETFSI method:

- very fast approximation to the full HF+BCS equations
- avoids the difficulties related to boundary conditions
  Chamel et al.,Phys.Rev.C75(2007),055806.

Unified equations of state of neutron stars

The same functionals used in the crust can be also used in the core (n, p, e^-, μ^-) thus providing a **unified and thermodynamically consistent description of neutron stars**.

Tables of the full equations of state for HFB-19, HFB-20, and HFB-21:

`http://vizier.cfa.harvard.edu/viz-bin/VizieR?-source=J/A+A/559/A128`

Fantina, Chamel, Pearson, Goriely, A&A 559, A128 (2013)

Analytical representations of the full equations of state:

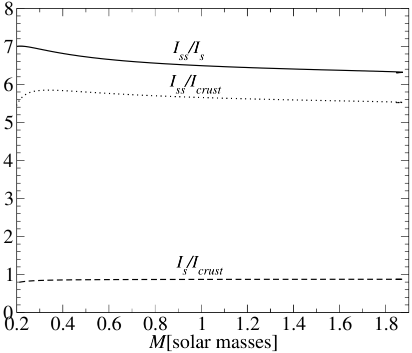
`http://www.ioffe.ru/astro/NSG/BSk/`

Potekhin, Fantina, Chamel, Pearson, Goriely, A&A 560, A48 (2013)

Equations of state for our latest functionals will appear soon.

Refined estimate of the mean effective neutron mass

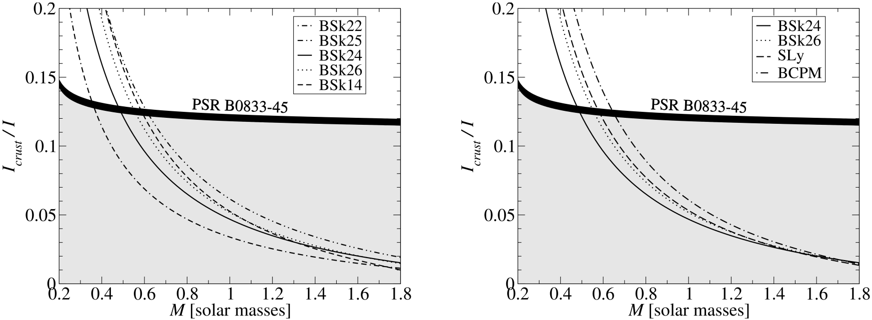
We have calculated I_s and I_{ss} in the slow-rotation approximation using the unified equation of state based on BSk14 (the only model for which $m_n^\star$ was calculated):



$\bar{m}_n^\star / m_n = I_{ss}/I_s$ is almost independent of the global stellar structure, as expected from the thin-crust approximation. However, the ratio is increased by $\sim 30\%$. We use the same value for all models.

Nuclear uncertainties and glitch puzzle

We have recalculated the fractional moment of inertia of the crust considering various *unified* equations of state based on accurately calibrated nuclear models:



The inferred mass of Vela is at most $0.66 M_{\odot}$, corresponding to central baryon densities $\bar{n} \approx 0.23 - 0.33$ fm^{-3}. At such densities, the equation of state is fairly well constrained by laboratory experiments.

Summary

- Pulsar timing can allow us to probe nuclear superfluidity in neutron stars.
- According to the standard vortex-mediated glitch theory, pulsar timing data of Vela requires that $I_{\rm crust}/I \gtrsim 12\%$.
- This condition cannot be fulfilled considering both astrophysical and laboratory experimental data.
- Even if crustal entrainment is ignored, the glitch theory has been challenged by glitches in PSR 2334+6 and PSR J1119−6127.

Possible answers to the glitch puzzle:

- Crustal entrainment is overestimated (see N. Martin's talk).
- The core plays some role (vortex pinning to fluxoids).

These scenarios could be further tested by future observations of the glitch rise time (See A. Sourie's talk).

Nuclear uncertainties in the mass-radius

Mass-radius relation of nonrotating neutron stars for various *unified* equations of state based on accurately calibrated nuclear models: