Extended Skyrme Equation of State in asymmetric nuclear matter (ANM)

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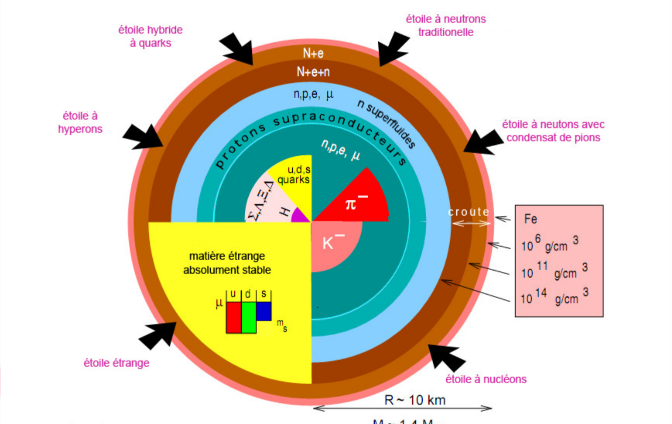
MODE Workshop

Outlines

Neutron star models



Source : Neutrons stars and pulsars (W. Becker)

Brueckner-Bethe-Goldstone (BBG) calculations

EoS derived by Baldo *and al.* (1997) popular

Advantages

- EoS in SNM, ANM, quantities associated,...
- Reach high densities
- Realistic nucleon-nucleon interactions

Drawbacks

- Resource-consuming calculations
- One calculation every (n,T,Y)
- Few finite-nuclei calculations

An alternative: effective interactions

Effective interactions

zero-range → Skyrme interaction

finite-range → Gogny interaction

Gogny interaction → Skyrme interaction: Taylor expansion in SNM

Skyrme interaction → Up to k^2 → Standard

Standard → Up to k^6 → Extended

$$V_{Ext} = \underbrace{V_{k^0} + V_{k^2}}_{Standard} + V_{k^4} + V_{k^6}$$

Advantages

- EoS and quantities
- Analytically obtained
- Finite-nuclei

Drawbacks

- Poor predictive power

Our proposition: Extended Skyrme interaction

$$\left.\begin{aligned} v_{Ex} = \quad & t_0\,(1 + x_0\,P_\sigma) + \frac{1}{2}\,t_1\,(1 + x_1\,P_\sigma)\left[\mathbf{k}'^2 + \mathbf{k}^2\right] \\ & + t_2\,(1 + x_2\,P_\sigma)\,\mathbf{k}'\cdot\mathbf{k} + \frac{1}{6}\,t_3\,(1 + x_3 P_\sigma)\rho^\alpha(R) \end{aligned}\right\}\ \text{Standard Skyrme (LNS)}$$

$$\left.\begin{aligned} & + \frac{1}{4}\,t_1^{(4)}(1 + x_1^{(4)}P_\sigma)\left[(\mathbf{k}^2 + \mathbf{k}'^2)^2 + 4(\mathbf{k}'\cdot\mathbf{k})^2\right] \\ & + t_2^{(4)}(1 + x_2^{(4)}P_\sigma)(\mathbf{k}'\cdot\mathbf{k})(\mathbf{k}^2 + \mathbf{k}'^2) \\ & + \frac{1}{2}\,t_1^{(6)}\left(1 + x_1^{(6)}P_\sigma\right)(\mathbf{k}'^2 + \mathbf{k}^2)\left[(\mathbf{k}'^2 + \mathbf{k}^2)^2 + 12(\mathbf{k}'\cdot\mathbf{k})^2\right] \\ & + t_2^{(6)}\left(1 + x_2^{(6)}P_\sigma\right)(\mathbf{k}'\cdot\mathbf{k})\left[3(\mathbf{k}'^2 + \mathbf{k}^2)^2 + 4(\mathbf{k}'\cdot\mathbf{k})^2\right]. \end{aligned}\right\}\ \text{New contributions}$$

- Include D and F waves
- 8 new parameters
- Finite-nuclei calculations
- A fit: LYVA1

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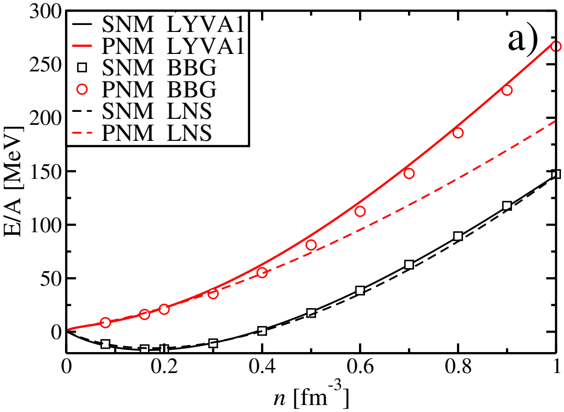
Outlines

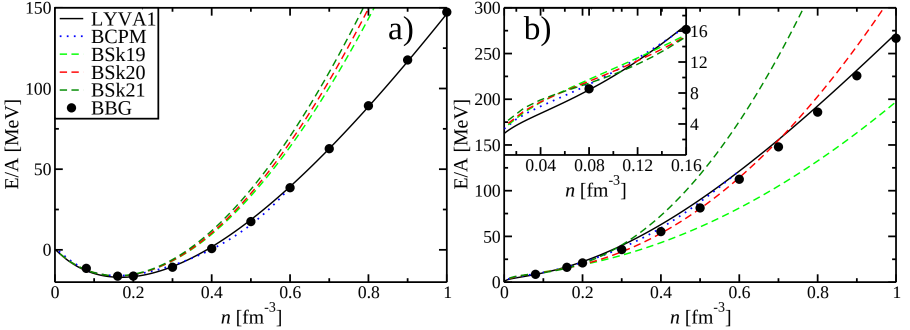
Fit of EoS in SNM and PNM



Properties in SNM :

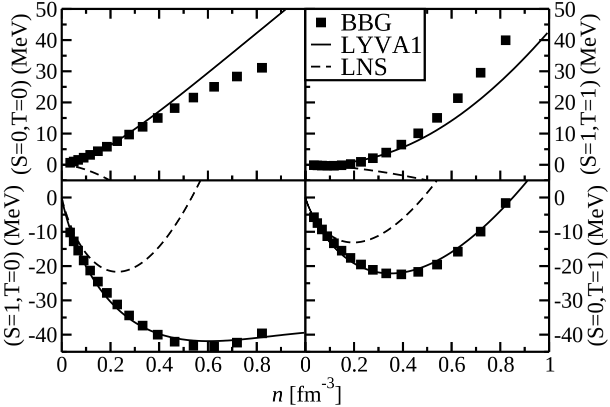
- $n_{sat} = 0.169\ \text{fm}^{-3}$
- $\frac{E}{A}(n_{sat}) = -17.02$ Mev

SNM and PNM LYVA1 vs BCPM vs BSk



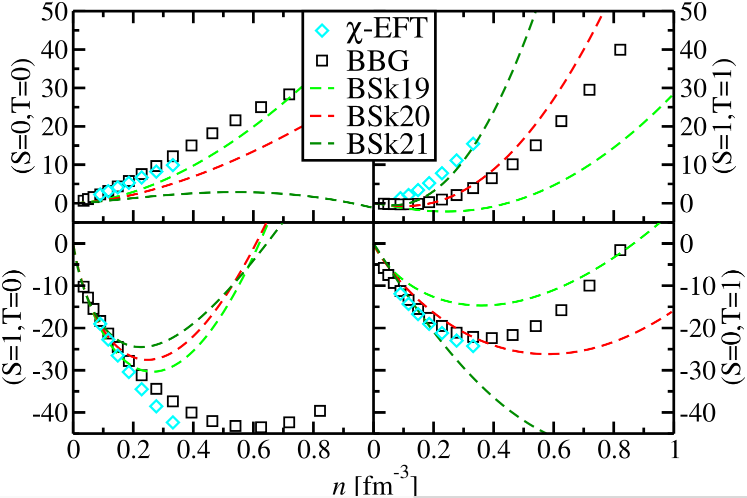
Nuclear channels EoS in SNM

Decomposition of SNM EoS on spin-isospin channels



Lesinski, T. *and al.* (2006) , Phys. Rev. C, 74, 044315

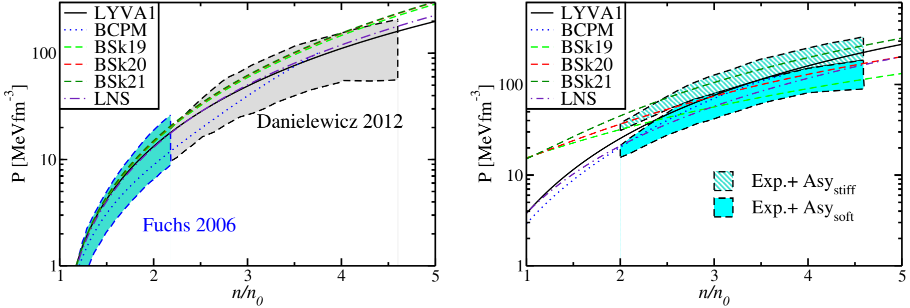
Nuclear channels EoS for LYVA1 and BSk



Outlines

Pressure

Constraints from heavy-ion collisions and experiments on kaons

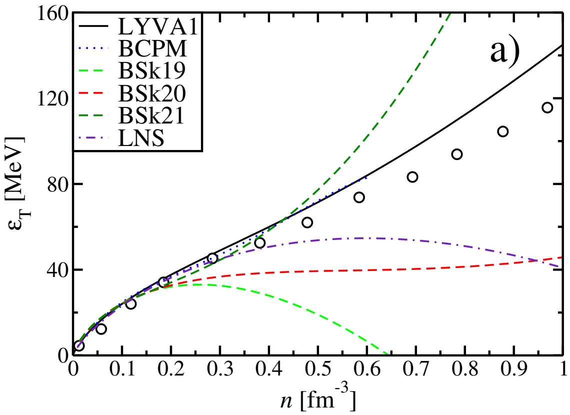


The pressure is obtained by:

$$P = n^2 \left. \frac{\partial (E/A)}{\partial n} \right|_{n=n_0}$$

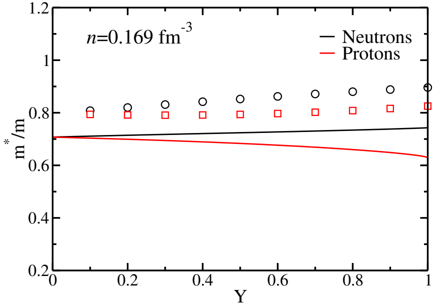
Symmetry energy

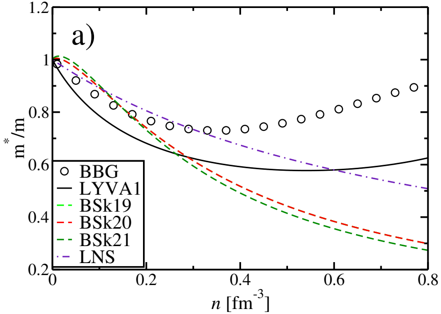
LYVA1 vs BSk vs BCPM vs LNS



$$\boxed{\epsilon_T(n_0) = J = 33.8\text{MeV}}$$

Effective mass



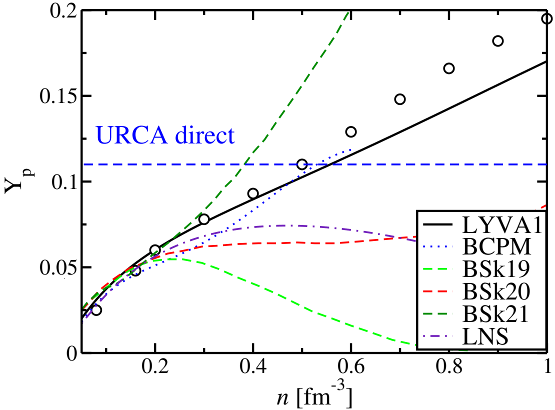


Y, asymmetry parameter

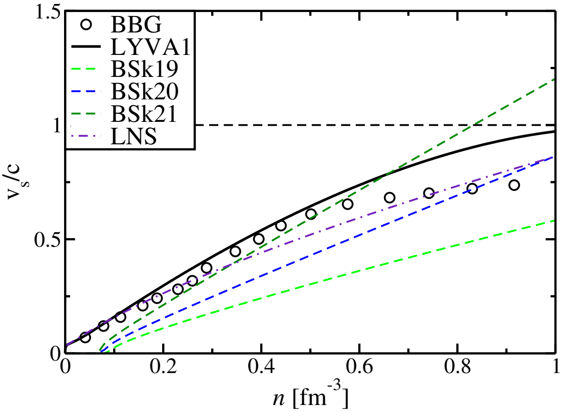
- Y=0 $\rightarrow$ SNM
- Y=1 $\rightarrow$ PNM

Outlines

URCA process
Allowing fast cooling

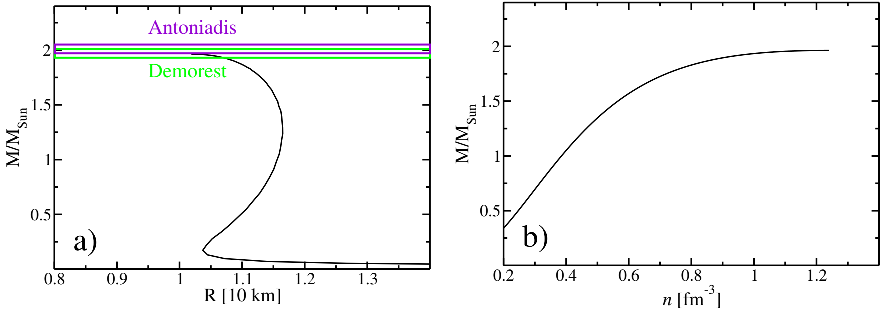


Causality Principle



Neutron Star Masses

Solving the TOV equations,



LYVA1 is **compatible** with 2 $M_\odot$ NS.
Here, we have M=1.96 $M_\odot$.

Outlines

Conclusion

Main results of LYVA1

- Grasp quantities of interest in BBG calc.Easy and analytical.
- EoS for both SNM and PNM
- Causality principle and URCA process
- Available code for stellar calculations

Perspectives

- Properties about polarized matter
- Easy finite-temperature expansion
- Describe both ground state and excited states
- Available code for (neutron-rich) finite-nuclei
- New BBG calcultations on the way

To go further (A&A 585, A83 (2016))

Extended Skyrme Equation of State in asymmetric nuclear matter

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8th June 2015

ABSTRACT

We present a new equation of state for infinite systems (symmetric, asymmetric and neutron matter) based on an *extended* Skyrme functional constrained by microscopic Brueckner-Bethe-Goldstone results. The resulting equation of state reproduces with very good accuracy the main features of microscopic calculations and it is compatible with recent measurements of two times Solar-mass neutron stars. We provide all necessary analytical expressions to facilitate a quick numerical implementation of quantities of astrophysical interest.

Key words. Effective interaction, Equation of state

1. Introduction

A key ingredient for many astrophysical calculations is a reliable Equation of State (EoS) for isospin asymmetric matter, covering

From Gogny to Skyrme in SNM (Submitted)

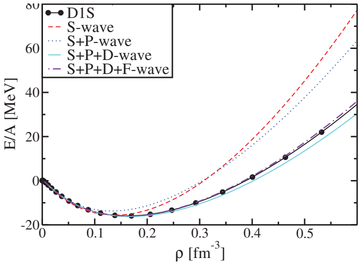


Figure 5: (Colors online) EoS in SNM for the D1S interaction (dots) and sum of different partial waves (left). Comparison between the EoS obtained using full HF calculations Eq. 17 (symbols) and the Eq. 20 truncated (dashed-lines) at $L = 3$ (right). See text for details.

Parametrisation of LYVA1

D. Davesne et al.: Extended Skyrme Equation of State in asymmetric nuclear matter

Table 1. Parameters of the extended LYVA1 Skyrme interaction, with $\alpha = 1/6$, $t_3^{(0)} = 13763$ [MeVfm$^{3+\alpha}$], and $x_3^{(0)} = 0.3$.

$t_0^{(0)}$ [MeVfm^5]	$t_1^{(2)}$ [MeVfm^5]	$t_2^{(2)}$ [MeVfm^5]	$t_1^{(4)}$ [MeVfm^7]	$t_2^{(4)}$ [MeVfm^7]	$t_1^{(6)}$ [MeVfm^9]	$t_2^{(6)}$ [MeVfm^9]
-2518.240	207.300	527.930	-23.691	-68.263	0	0.690
$x_0^{(0)}$	$x_1^{(2)}$	$x_2^{(2)}$	$x_1^{(4)}$	$x_2^{(4)}$	$x_1^{(6)}$	$x_2^{(6)}$
0.2537	-0.1688	-1.0131	0.5650	-1.2022	0	-1.2500

Properties in INM

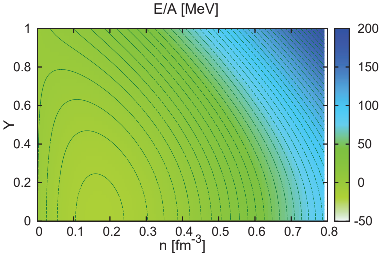


Fig. 4. (Colors online) Equations of states in asymmetric nuclear matter as a function of the density n and asymmetry parameter Y.

Table 2. Basic SNM properties calculated with the LYVA1 parametrization given in Tab.1, the BCPM and the BSk19-21 functionals at saturation density n_0.

	LYVA1	BCPM	BSk19	BSk20	BSk21	LNS
n_0[fm^{-3}]	0.169	0.160	0.160	0.160	0.158	0.175
E/A[MeV]	-17.02	-16.00	-16.08	-16.80	-16.05	-15.31
K[MeV]	231	214	237	241	246	211
m^*/m	0.707	1	0.80	0.80	0.80	0.825
Q[MeV]	-463	-881	-298	-282	-274	-384
J[MeV]	33.8	31.9	30.0	30.0	30.0	33.4
L[MeV]	64.5	53.0	31.9	37.4	46.6	61.5
K_{sym}[MeV]	-75.6	-98.1	-191.4	-136.5	-37.2	-127.7
Q_{sym}[MeV]	464	877	473	550	710	303

Polarized matter of LYVA1

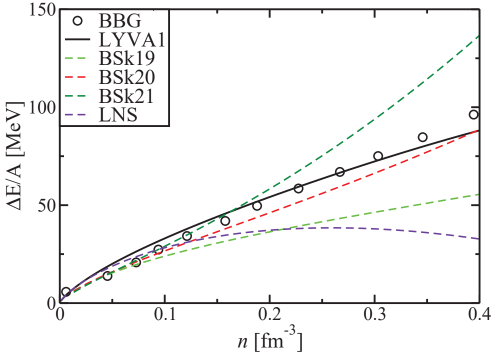
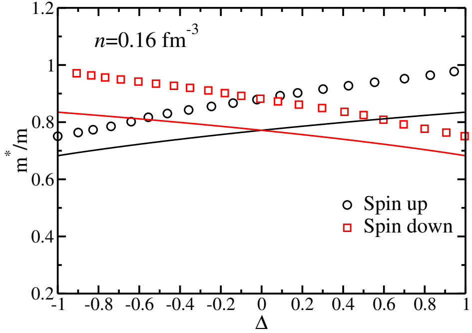
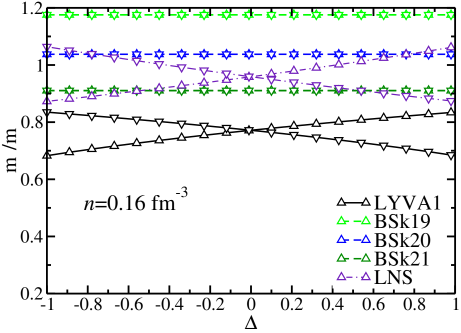


Fig. 9. (Colors online) Energy difference between PolPNM and PNM for the different models considered in the text.

Polarized matter effective mass





(S,T) channels Equation of state in SNM:

$$E/A\,(S=0,T=0) = \frac{3}{160}(1-x_2)\,t_2\,\rho\,k_f^2 + \frac{9}{560}(1-x_2^{(4)})\,t_2^{(4)}\,\rho\,k_f^4$$

$$E/A\,(S=0,T=1) = 3\left[t_0(1-x_0)\frac{\rho}{16} + \frac{t_3}{96}(1-x_3)\rho^{\alpha+1} + \frac{3}{160}t_1(1-x_1)\rho\,k_f^2 + \frac{9}{560}\,t_1^{(4)}(1-x_1^{(4)})\,\rho\,k_f^4\right]$$

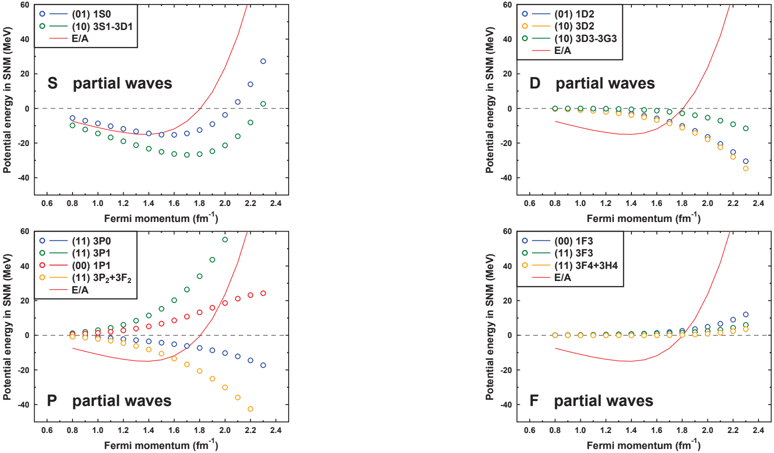
$$E/A\,(S=1,T=0) = 3\left[t_0(1+x_0)\frac{\rho}{16} + \frac{t_3}{96}(1+x_3)\rho^{\alpha+1} + \frac{3}{160}t_1(1+x_1)\rho\,k_f^2 + \frac{9}{560}\,t_1^{(4)}(1+x_1^{(4)})\,\rho\,k_f^4\right]$$

$$E/A\,(S=1,T=1) = 9\left[\frac{3}{160}(1+x_2)\,t_2\,\rho\,k_f^2 + \frac{9}{560}(1+x_2^{(4)})\,t_2^{(4)}\,\rho\,k_f^4\right]$$

This calculations has also been done for neutron matter, asymmetric matter and polarized matter.

See D.Davesne, J.Navarro and al., Phys. Rev. C 91 064303 (2015)

Justification of D and F waves



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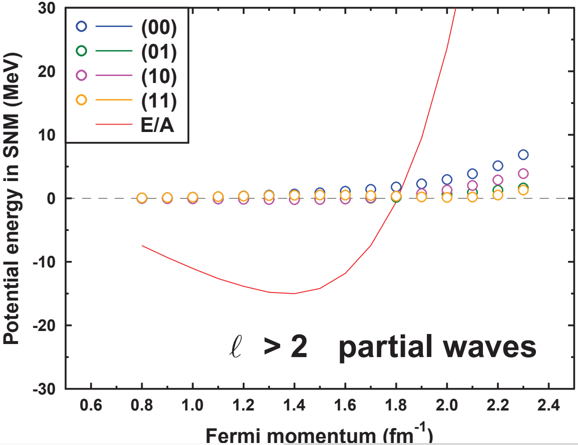

More comments about our interaction

- The effective mass with the D-wave has a dependency in momentum ($\propto \rho^{\frac{1}{3}}$ in INM):

$$\left(\frac{m}{m^*}\right) = 1 + \frac{2m}{\hbar^2}\rho_0 \left[C_0^\tau + \frac{1}{2}C_0^{(4)M\rho}k_F^2\right] \quad . \qquad (1)$$

  We could obtain peaked effective mass, meaning good spectroscopic properties.
- Interaction parameters linked to Laudau parameters
- Stability of the Extended Skyrme functional in INM can be checked with LR.
- See Becker, P. *and al.*, (2014), J. Phys. G: Nucl. Part. Phys, 42, 034001

Where is the D-wave?

A term such as $(\mathbf{k}' \cdot \mathbf{k})^2$ in the differential equation cause a $\cos^2 \omega_{kk'}$ to appear. We use:

$$P_\ell(\cos \omega_{12}) = \frac{4\pi}{2\ell + 1} \sum_m Y^*_{\ell m}(\Omega_1) Y_{\ell m}(\Omega_2) \tag{2}$$

Contribution in ℓ=2:

$$\begin{aligned} \cos^2\,(\omega_{kk'}) &= \frac{2}{3} P_2(\cos\ \omega_{kk'}) + \frac{1}{3} \\ &= \frac{8\pi}{15} \sum_m Y^*_{2m}(\Omega_1) Y_{2m}(\Omega_2) + \frac{1}{3} \end{aligned} \tag{3}$$